2.4 Lepton era

Consider $1\,\text{MeV} \lesssim T \lesssim 100\,\text{MeV}$

quarks & gluons are tightly bound into hadrons

lightest hadrons: pions, $m_\pi \sim 140\,\text{MeV} \ll T$ non-relativistic

same for proton ($m_p = 938\,\text{MeV}$), neutrons ($m_n = 939\,\text{MeV}$)

muons ($m_\mu \simeq 113\,\text{MeV}$) and τ-leptons ($m_\tau \simeq 1.78\,\text{GeV}$)

the only relativistic particles left are

$\gamma, e^\pm$, (anti-)neutrinos $\nu_\alpha, \bar{\nu}_\alpha$ $(\alpha = e, \mu, \tau)$

Conserved are el. charge Q, baryon number B, and (for $T \gtrsim 1\,\text{MeV}$) 3 lepton flavor numbers L_α

particle chemical potentials:

$\mu_{\pi^0} = 0, \quad \mu_{\pi^\pm} = \pm \mu_Q$

$\mu_e = -\mu_Q + \mu_{L_e} \qquad \mu_\mu = -\mu_Q + \mu_{L_\mu}, \ldots$

$\mu_p = \mu_Q + \mu_B \qquad \mu_n = \mu_B$

The values of Q, B and L_α together with

$$\frac{d(s a^3)}{dt}$$

determine the functions $T(t), \mu_Q(t), \mu_B(t), \mu_{L_\alpha}(t)$

$Q = 0, \quad \frac{n_B}{s} \sim 10^{-10}$

The lepton numbers are very weakly constrained because it is very difficult to measure densities of neutrinos & antineutrinos.

From observations $n_{L_\alpha} \gg n_B$ cannot be excluded.

On the other hand, most theories which can explain the origin of the value of n_B also predict $n_{L_\alpha} \sim n_B$

For $T \gg m_i$, then $n_i \sim n_{\bar{i}} \sim n_\gamma \sim T^3$

When T drops below m_i, most particles and anti-particles annihilate.

Only the excess due to the asymmetries survives.

Assumption: some asymmetry is carried by one species i only

$$\eta = \frac{\Delta n_i}{s} = \text{const} \quad \text{with } \Delta n_i := n_i - n_{\bar{i}}$$

for nonrelativistic particles we had

$$n_i = g_i \left(\frac{m_i T}{2\pi} \right)^{3/2} e^{(\mu_i - m_i)/T}$$

$$\mu_{\bar{i}} = -\mu_i$$

$$s = g_* \frac{2\pi^2}{45} T^3 \quad \ldots$$

$$\frac{n_i\, n_{\bar{i}}}{s^2} = g_*^{-2}\, \frac{45^2}{2^2\pi^4}\, T^{-3}\, g_i^2 \left(\frac{m_i T}{2\pi}\right)^3 e^{-2m_i/T}$$

$$= c^2 \frac{g_i^2}{g_*^2} \left(\frac{m_i}{T}\right)^3 e^{-2m_i/T}$$

with $c^2 = \frac{45^2}{2^5\pi^7} \simeq 0{,}021$

$n_{\bar{i}} = n_i - \eta s \Rightarrow$

$$\frac{n_i\, n_{\bar{i}}}{s^2} = \frac{n_i}{s}\left(\frac{n_i}{s} - \eta\right) = \left(\frac{n_i}{s}\right)^2 - \eta \frac{n_i}{s}$$

$$\frac{n_i}{s} = \frac{\eta}{2} + \left[\frac{\eta^2}{4} + c^2 \frac{g_i^2}{g_*^2}\left(\frac{m_i}{T}\right)^3 e^{-2m_i/T}\right]^{1/2}$$

exchange $i \leftrightarrow \bar{i} \Leftrightarrow \eta \leftrightarrow -\eta$:

$$\frac{n_{\bar{i}}}{s} = -\frac{\eta}{2} + \left[\frac{\eta^2}{4} + c^2 \frac{g_i^2}{g_*^2}\left(\frac{m_i}{T}\right)^3 e^{-2m_i/T}\right]^{1/2}$$

number of particle - antiparticle pairs negligible compared to the particle excess number when

(*) $$\frac{\eta}{2} \gg c \frac{g_i}{g_*}\left(\frac{m_i}{T}\right)^{3/2} e^{-m_i/T}$$

rough estimate : (*) $\Leftrightarrow \frac{m_i}{T} \gg \ln \frac{2c}{\eta}$

for $\eta = 10^{-10}$: $\ln \frac{2c}{\eta} \simeq 22$, $\quad T \ll \frac{m_i}{22}$

$m_p = 938$ MeV

antibaryons $\ll$ # baryons for $T \lesssim 43$ MeV

$m_e = 511$ keV

e^+ $\ll$ # e^- for $T \lesssim 23$ keV

<u>estimate chemical potentials</u>

for $T \ll 100$ MeV, $m_\mu, m_\tau \gg T$ while $m_{\nu_\mu}, m_{\nu_\tau} \ll T$

$\Rightarrow$ L_μ, L_τ mostly carried by ν_μ, ν_τ

problem H6.1: $\Delta n_i = \frac{\mu_i T^2}{6}$ $\Rightarrow$

$$\frac{\mu_{L\alpha}}{T} \sim \frac{n_{L\alpha}}{s} \quad \text{for } \alpha = \mu, \tau$$

The contribution of e^- to L_e cannot be neglected, because the electrons must compensate the electric charge carried by the protons.

Relativistic electrons.

$$\left.\begin{aligned} \Delta n_{e^-} &= 2\,\frac{\mu_{e^-} T^2}{6} \\ \Delta n_{e^-} &\sim n_B \end{aligned}\right\} \Rightarrow \frac{\mu_{e^-}}{T} \sim \frac{n_B}{s}$$

Furthermore, $\mu_{e^-} = \mu_{L_e} - \mu_Q$

$$n_{L_e} \simeq 2\,\Delta n_{e^-} + \Delta n_{\nu_e} \simeq (2\mu_{e^-} + \mu_{\nu_e})\frac{T^2}{6}$$
$$= (2\mu_{L_e} - 2\mu_Q + \mu_{L_e})\frac{T^2}{6} = (3\mu_{L_e} - 2\mu_Q)\frac{T^2}{6}$$

Thus we see that n_{L_e} can be much larger than $n_{e^-} = n_B$

Furthermore, we see that for relativistic particles $\frac{\mu_i}{T} \simeq \text{const}$

Now consider p, n.

For $T \ll 45\,\text{MeV}$: $\Delta n_p \simeq n_p$, $\Delta n_n \simeq n_n$

$$\eta_B = \frac{n_B}{s} \simeq \frac{n_p + n_n}{s} \simeq \frac{n_p}{s}\left(1 + \frac{n_n}{n_p}\right)$$

$g_p = 2 \Rightarrow$

$$n_p = 2\left(\frac{m_p T}{2\pi}\right)^{3/2} e^{(\mu_p - m_p)/T}$$

$$s = g_* \frac{2\pi^2}{45} T^3$$

assume $n_n \simeq n_p$ (will be checked later)

$$\eta_B \simeq \frac{2}{g_*} \underbrace{2^{-5/2}\,\pi^{-7/2}\,45}_{=c} \left(\frac{m_p}{T}\right)^{3/2} e^{(\mu_p - m_p)/T} \cdot 2$$

$$\ln \eta_B \simeq \frac{\mu_p - m_p}{T} + \ln\left(\frac{4c}{g_*}\right) + \frac{3}{2}\ln\left(\frac{m_p}{T}\right)$$

$$\frac{m_p - \mu_p}{T} = \ln\left(\frac{4c}{\eta_B}\left[\frac{m_p}{T}\right]^{3/2}\right)$$

$T = 40\,\text{MeV}$: $\ln(\dots) = 25 \Rightarrow \mu_p \simeq -100\,\text{MeV}$

$T = 1\,\mathrm{MeV}: \quad \ln(\dots) = 30.4$

$$\mu_p = m_p - 30.4\,\mathrm{MeV} = 908\,\mathrm{MeV}$$

entropy carried by protons:

We had $\quad s_i = n_i \left(\frac{m_i - \mu_i}{T} + \frac{5}{2} \right)$ for NR particles

$$\frac{s_p}{s} \simeq \eta_B \left\{ \ln\left(\frac{4c}{\eta_B} \left[\frac{m_p}{T}\right]^{3/2} \right) + \frac{5}{2} \right\} \sim 10^{-9}$$

tiny fraction of the total entropy!

now estimate

$$\frac{n_n}{n_p} = \exp\left(\frac{1}{T} [\mu_n - m_n - \mu_p + m_p] \right)$$

$\mu_n = \mu_B, \quad \mu_p = \mu_B + \mu_Q, \quad \mu_n - \mu_p = \mu_Q$

$\mu_Q \sim \mu_{L_e} \sim \eta_{L_e} T \ll T$

$\Delta m := m_n - m_p = 1.29\,\mathrm{MeV}$

$\frac{n_n}{n_p} \simeq \exp\left(-\frac{\Delta m}{T}\right) \simeq 1 \qquad$ for $T \gtrsim 1\,\mathrm{MeV}$ $\quad$ ok