1.7 Measuring a(t)

If one knows properties of distant objects, then the way we observe them today depends on the expansion history since the emission of the signal.

If one knows the absolute brightness or luminosity of some objects they are called standard candles.

Example: type Ia Supernovae

If one knows their size, they are called standard rulers.

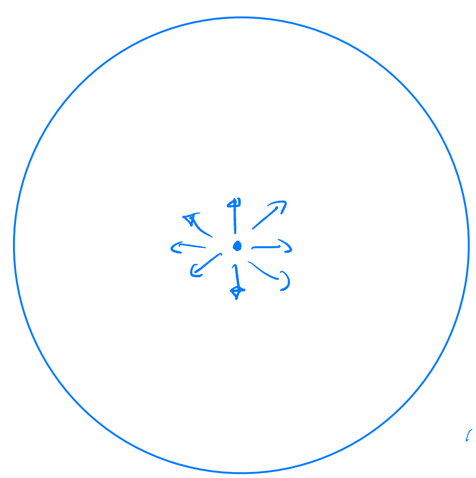
luminosity

Consider an isotropic light source with luminosity L defined as the total energy emitted per unit time and a sphere of radius d around this source

This gives a flux

F = energy per unit time and unit area

through the sphere.



In Minkowski space one would have

$$F = \frac{L}{4\pi d^2}$$

One can use this to measure d of some distant object if L is known.

In cosmology one defines the <u>luminosity distance</u> as

$$d_L^2 = \frac{L}{4\pi F}$$

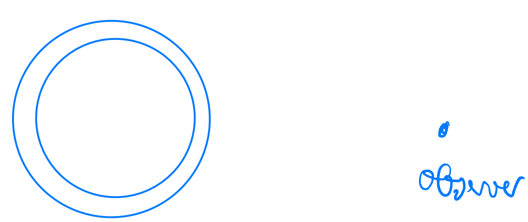
for known L from the measured flux F.

Compute the z-dependence of d_L.

Energy emitted in a time interval Δt_{em}:

$$\Delta E_{em} = L\,\Delta t_{em} = L\,a(t_{em})\,\Delta\eta_{em}$$

This energy sits in some thin shell which expands



We have seen that for 2 light signals emitted successively the conformal-time distance is constant.
Thus the η-width of the shell is $\Delta\eta_{em}$ at any time.
The photons in the shell get red-shifted.
At the time observation the energy in the shell is

$$\Delta E_{obs} = \frac{a(t_{em})}{a_0}\,\Delta E_{em} = \frac{a^2(t_{em})}{a_0}\,L\,\Delta\eta_{em}$$

surface area of the shell at observer at r:

$$A = 4\pi a_0^2 r^2$$

time-width of the shell: $\Delta t_0 = a_0 \Delta \eta_{em}$ $\Rightarrow$

$$F = \frac{\Delta E_{obs}}{A \quad \Delta t_0} = \frac{a^2(t_{em})}{a_0} L \Delta \eta_{em} \frac{1}{4\pi a_0^2 r^2} \frac{1}{a_0 \Delta \eta_{em}}$$

$$= \frac{a^2(t_{em}) L}{4\pi a_0^4 r^2} = \frac{1}{(1+z)^2} \frac{L}{4\pi a_0^2 r^2} \Rightarrow$$

$$d_L = (1+z) a_0 r$$

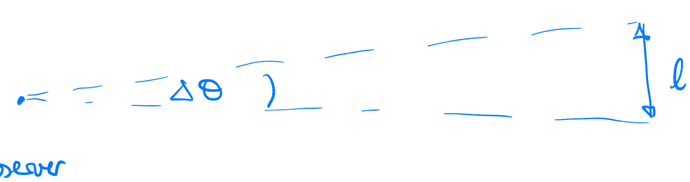
The relation r and z depends on the expansion history $a(t)$:

$$(*) \qquad \int_{t_{em}}^{t_0} \frac{dt}{a(t)} = \int_0^r \frac{dr'}{\sqrt{1-kr'^2}} =: \chi(r)$$

determines t_{em} and thus $1+z = \dfrac{a_0}{a(t_{em})}$ as function of r

inverting this gives $r = r(z)$

angular diameter



object at r, $\varphi = 0$

in Minkowski space: distance $= \dfrac{l}{\Delta\theta}$

define angular diameter distance $d_A := \dfrac{l}{\Delta\theta}$

$$dl = a(t_{em}) r \, d\vartheta \Rightarrow l = a(t_{em}) r \Delta\theta$$

$$d_A = a(t_{em}) r = \frac{a(t_{em})}{a_0} a_0 r = \frac{1}{1+z} \frac{d_L}{1+z} = \frac{d_L}{(1+z)^2}$$

N.B. $d_A \simeq d_L$ only when $z \ll 1$

example: evidence for dark energy [Rubakov 4.6]

subst: $t \to z = \frac{a_0}{a} - 1$ in (*)

$$dz = -\frac{a_0}{a^2}\, da = -\frac{a_0}{a^2}\, \dot{a}\, dt = -\frac{a_0}{a} H\, dt$$

$$dt = -\frac{a}{a_0 H}\, dz$$

$$\chi = \int_{t_{em}}^{t_0} \frac{dt}{a(t)} = \int_0^z \frac{dz'}{a_0\, H(z')}$$

$$H^2 = \frac{8\pi}{3} G \left[\rho_{m0} \left(\frac{a_0}{a}\right)^3 + \rho_{rad0} \left(\frac{a_0}{a}\right)^4 + \rho_\Lambda + \rho_{curv0} \left(\frac{a_0}{a}\right)^2 \right]$$

where $\frac{8\pi}{3} G \rho_{curv} = -\frac{k}{a^2}$

(*) $$H^2 = H_0^2 \left[\Omega_{m0} \left(\frac{a_0}{a}\right)^3 + \Omega_{rad0} \left(\frac{a_0}{a}\right)^4 + \Omega_\Lambda + \Omega_{curv0} \left(\frac{a_0}{a}\right)^2 \right]$$

with $\Omega_i = \frac{\rho_i}{\rho_{crit}}$, $\rho_{crit} = \frac{3}{8\pi G} H_0^2$

Apply this to relatively recent times

We already saw that $\Omega_{\gamma 0}$ is quite small. Assume that there is no large amount of relativistic DM and that Ω_{curv} is small (both will be justified later), so that we can neglect Ω_{rad} and Ω_{curv}.

$$H^2 = H_0^2 \left[\Omega_m \left(\frac{a_0}{a}\right)^3 + \Omega_\Lambda \right] = H_0 \left[\Omega_m (1+z)^3 + \Omega_\Lambda \right]$$

and $x \simeq r \qquad (k=0)$

$$a_0 r = \int_0^z dz' \, H_0^{-1} \left[\Omega_{m0} (1+z')^3 + \Omega_\Lambda \right]^{-1/2}$$

where $\Omega_{m0} + \Omega_\Lambda \simeq 1$

$$d_L = (1+z) a_0 r$$

Larger $\Omega_\Lambda \rightarrow$ larger $[\ldots]^{-1/2} \rightarrow$ faster increase of r with z

$\rightarrow$ objects appear dimmer.

Abb: $m - M = 5 \log_{10} (d_L / \mathrm{Mpc}) + 25$

Fig with $z > 1$ SNae : difference with $\Omega_m = \Omega_\Lambda = 0$, $\Omega_{curv} = 1$

N.B. (*) should be used with care because it makes several assumptions

(i) ideal gas, meaning no interactions (except gravity)

(ii) # relativistic / nonrelativistic particle species (degrees of freedom) are constant

These are oftentimes good approximations but they break down in certain situations

(i) near the QCD crossover $T \simeq T_{QCD} = 130\ MeV$ the interaction of quarks and gluons is very strong

(ii) for $T \gg T_{QCD}$: approximately ideal gas of quarks and gluons, with

$N_{fl} \geq 3 = \#$ flavors

$\underset{\text{color}}{3} \cdot \underset{\text{quark/antiquark}}{2} \cdot \underset{\text{spin}}{2} \cdot N_{fl}$ fermionic degrees of freedom (quarks)

$8 \cdot 2$ bosonic degrees of freedom (gluons)

There are at least $36 + 16$ degrees of freedom

for $T \ll T_{QCD}$: π^+, π^-, π^0 pions   3 bosonic degrees of freedom

3 massless neutrinos at decoupling at $T \sim 1\,MeV$

today: at least 2 neutrinos are nonrelativistic