1.6 Propagation of light & particles

light: $\overset{\cdot}{x} \sim\sim\overset{\cdot}{} \; x+dx$ $\qquad d\tau^2 = 0$

FRW: $$d\tau^2 = dt^2 - a^2\left[\frac{(dr)^2}{1-kr^2} + r^2 d\Omega^2\right]$$

consider radial trajectory $d\Omega = 0$

$$d\tau = 0 \iff dt = a\frac{dr}{\sqrt{1-kr^2}}$$

conformal time: $$d\tau^2 = a^2\left\{(d\eta)^2 - \frac{(dr)^2}{1-kr^2} - r^2 d\Omega^2\right\}$$

$$d\tau^2 = 0 \iff d\eta = \frac{dr}{\sqrt{1-kr^2}}$$

A light signal emitted at $r=0$ at time t_i will have the radial coordinate r_f at time t_f with

$$\int_{t_i}^{t_f}\frac{dt}{a(t)} = \int_0^{r_f}\frac{dr}{\sqrt{1-kr^2}}$$

Horizons

If the Universe has a finite age, the light has travelled only a finite distance since the Big Bang.

Thus we can receive information only from a finite volume in space. The boundary of this volume is called particle horizon.

Age of the Universe ~ 15 Gy $\Rightarrow$ present distance of the particle horizon $d_p \sim 15$ G ly

r_p: radial coordinate of particle horizon, determined by

$$\int_0^{t_0} \frac{dt}{a(t)} = \int_0^{r_p} \frac{dr}{\sqrt{1-kr^2}} \qquad t_0 = \text{today}$$

Spatial distance of particle horizon

$$d_p = a_0 \int_0^{r_p} \frac{dr}{\sqrt{1-kr^2}} = a_0 \int_0^{t_0} \frac{dt}{a(t)}$$

d_p looks particularly simple when written in terms of η:

$$dt = a\,d\eta \Rightarrow d_p = a_0\,\eta_0 \qquad \text{for} \quad \eta = 0 \text{ at } t = 0$$

example: radiation domination, $k = 0$

$$a = a_0 \left(\frac{t}{t_0}\right)^{1/2}$$

$$d_p = \int_0^{t_0} dt \left(\frac{t}{t_0}\right)^{-1/2} = t_0^{1/2} \; 2\, t_0^{1/2} = 2t_0 > t_0$$

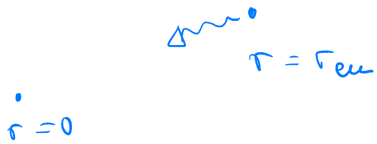
$$H = \frac{1}{2t} \Rightarrow t_0 = \frac{1}{2H_0}, \quad d_p = 2\,\frac{1}{2H_0} = \frac{1}{H_0}$$

There is another horizon, the <u>event horizon</u> d_e. It encloses the volume which can be reached by light signals in the future. Its present size is

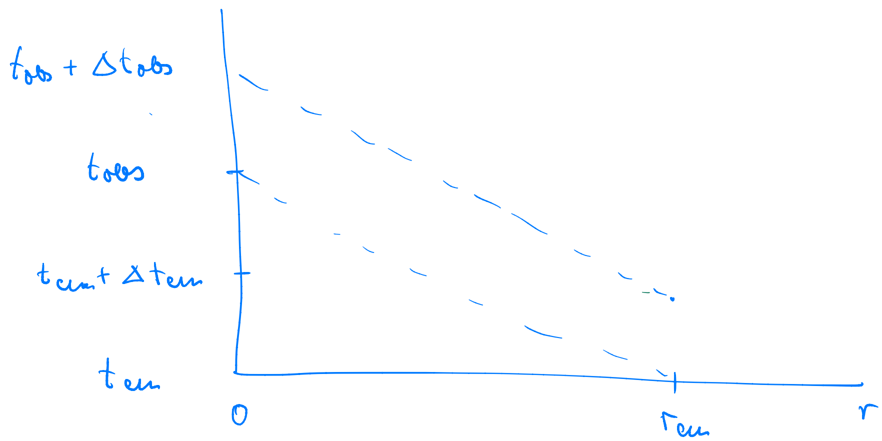
$$d_e = a_0 \int_{t_0}^{t_f} \frac{dt'}{a(t')}$$

If the Universe expands forever, then $t_f = \infty$

Redshift [Mukhanov 2.4]



emission at $t = t_{em}$,
short duration Δt_{em}



$$d\eta = \frac{dr}{\sqrt{1-kr^2}}$$

$$\eta_{obs} - \eta_{em} = \eta_{obs} + \Delta\eta_{obs} - (\eta_{em} + \Delta\eta_{em}) \Rightarrow$$

$$\Delta\eta_{obs} = \Delta\eta_{em}$$

time differences: $dt = a\, d\eta$

$$\Delta t_{em} = a(t_{em})\, \Delta\eta_{em}$$
$$\Delta t_{obs} = a(t_{obs})\, \Delta\eta_{obs} \quad \Rightarrow$$

$$\frac{\Delta t_{obs}}{\Delta t_{em}} = \frac{a(t_{obs})}{a(t_{em})}$$

Now let Δt_{em} be one period of the light wave $\Rightarrow$

$\Delta t_{em} = \lambda_{em}$ λ : wavelength

$$\boxed{\frac{\lambda_{obs}}{\lambda_{em}} = \frac{a(t_{obs})}{a(t_{em})}}$$

redshift : $1 + z = \dfrac{\lambda_{obs}}{\lambda_{em}}$ $\Rightarrow$

$$\boxed{1 + z = \frac{a(t_{obs})}{a(t_{em})}}$$

The wavelength of light is stretched by the expansion of the Universe, $\lambda \propto a(t)$, frequency $\omega \propto \frac{1}{a(t)}$ <u>redshift</u>

Cosmic microwave background : blackbody spectrum

phase space density = Bose - Einstein distribution

of photons which have momentum $|\vec{p}| = E = \omega$

$$f_B(\omega) = \frac{1}{e^{\omega/T} - 1}$$

redshift $\omega \to \dfrac{\omega}{1+z}$ preserves the shape of

of the phase space density but changes the temperature

$$T \to \frac{T}{1+z}$$

so that $T \propto \dfrac{1}{a(t)}$

energy density $\rho_\gamma = 2\int \frac{d^3p}{(2\pi)^3}\,\omega f_B(\omega) \propto T^4 \quad \Rightarrow$

$\rho_\gamma \propto \frac{1}{a^4(t)}$

<u>redshift as Doppler shift</u>

consider two galaxies at a distance $\Delta l \ll H^{-1}$

H^{-1} sets the scale at which curvature becomes important, so for $\Delta l \ll H^{-1}$ we can choose a local inertial frame where gravity can be neglected

$\Delta l = r\,a$

relative speed : $v = r\dot{a} = \Delta l\, H \ll 1$

Doppler shift : $\Delta\omega = -v\,\omega = -\Delta l\, H\omega = -\Delta t\, H\omega$

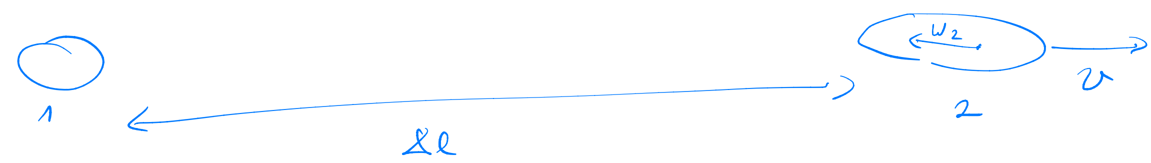
Δt : time between emission and measurement

differential equation for the measured frequency :

$$\dot{\omega} = -H\,\omega = -\frac{\dot{a}}{a}\,\omega$$

solution $\quad \omega \propto \frac{1}{a}$

<u>peculiar velocity</u> = velocity relative to Hubble flow



Consider a massive particle for which the comoving observer measures the peculiar velocity w_2 in the direction of observer 1. Observer 1 will measure its peculiar velocity as

$$w_1 = w_2 - v \quad , \quad \Delta w = -v = -H \Delta l$$

Time between measurements $\Delta t = \dfrac{\Delta l}{w} \Rightarrow$

$$\dot{w} = -\frac{\Delta w}{\Delta l} = -H w \quad \Rightarrow$$

$$\boxed{w \propto \frac{1}{a}}$$

The expansion of the Universe brings particles to rest as measured by comoving observers.