

Cosmology

evidence for hot big bang

- 1) Hubble expansion [Hubble 1929]
- 2) Cosmic microwave background radiation (CMBR, or CMB)
- 3) Big Bang nucleosynthesis (BBN), 3 min after Big Bang
~ abundance of light elements (H, D, ^3He , ^4He , ^7Li)

Hubble's law

redshift of spectral lines

$$z := \frac{\lambda_o - \lambda_{\text{emitted}}}{\lambda_{\text{emitted}}}$$

λ_o : observed wavelength

In 1929 Hubble found that z is proportional to the distance of the observed object:

$$z = \text{const} \cdot \underset{\substack{\uparrow \\ \text{distance}}}{d}$$

Parsec [Hubble 1.6]

common unit of length used in cosmology & astrophysics



pc = parsec
parallax second

$$1 \text{ pc} = 3.26 \text{ ly}$$

↑
light years

1 astronomical unit (AU) = average earth-sun distance

$$1 \text{ AU} = 1.5 \cdot 10^{11} \text{ m}$$

Hubble interpreted z as Doppler shift due to the motion of the source.

$$\text{For } v \ll c: \quad z = \frac{v}{c}$$

Then

$$v = H_0 d \quad \text{or} \quad z = H_0 \frac{d}{c}$$

↑
Hubble constant

original determination (Hubble) : $H_0 = 550 \text{ km sec}^{-1} \text{ Mpc}^{-1}$

present-day value $H_0 = h \text{ } 100 \text{ km sec}^{-1} \text{ Mpc}^{-1}$

$h = 0.67$ from CMB anisotropies

$h = 0.73$ from cepheid variables, type Ia supernovae

H_0^{-1} is a time scale and gives an estimate of the age of the Universe

$$H_0^{-1} = 14 \text{ Gy}$$

A better understanding of z is provided by GR, which explains z by the expansion of the universe.

The distance between two (sufficiently far away but otherwise arbitrary) observers is proportional to the scale factor $a(t)$ which grows in time.

The wavelength of light is stretched by the same factor:

$$\lambda_0 = \frac{a(t_0)}{a(t_{\text{emission}})} \lambda_{\text{emitted}}, \quad t_0 = \text{present epoch}$$

$$z = \frac{a(t_0)}{a(t_{\text{emission}})} - 1$$

Then the Hubble law is obtained by expanding

$$a(t_{\text{emission}}) \approx a(t_0) + \dot{a}(t_0) \underbrace{(t_{\text{emission}} - t_0)}_{\approx -d/c}$$

so that $z \approx \frac{\dot{a}(t_0)}{a(t_0)} d/c$, and

$$\boxed{H_0 = \frac{\dot{a}(t_0)}{a(t_0)}}$$

Here we have already introduced the concept of two "arbitrary observers". This is based on the idea that the Universe looks the same at any point in space (homogeneous) and in any direction (isotropic).

A very strong argument in favor of isotropy comes from the observation of the

CMB

thermal photon spectrum $T = 2.73 \text{ K}$

interpretation: when the temperature of the Universe was very high, it was filled with

electrons, photons & nuclei in thermal equilibrium.

When T dropped below $T \sim 3000 \text{ K}$, atoms formed and the Universe became transparent.

Since then the photons were redshifted ($z \sim 1000$)

CMB nearly isotropic, $\delta T/T \sim 10^{-5}$

BBN

At still higher T , protons & neutrons were unbound.

Then at $T \approx 1 \text{ MeV}/k_B$ they formed light nuclei

Their abundances only depend on n_0/n_γ

$n_B =$ baryon number density

p, n have baryon number $B = 1$

Units

most of the time we use natural units

$$\hbar = c = k_B = 1$$

Then energy, mass & temperature all have the same unit

$$\text{since } [E] = [mc^2] = [k_B T]$$

A convenient unit is eV

$$\text{proton mass } m_p = 938 \text{ MeV}$$

$$1 \text{ Kelvin } 1K \simeq 10^{-4} \text{ eV}$$

length, time have dimension m^{-1}

$$1 \text{ GeV}^{-1} \simeq 10^{-14} \text{ cm} \simeq 10^{-24} \text{ sec}$$

gravitational potential for masses m_1, m_2

$$V = -G \frac{m_1 m_2}{r}$$

$$[V] = [E], \quad \left[\frac{1}{r}\right] = [E] \Rightarrow [G] = \left[\frac{1}{m^2}\right]$$

$$G = M_{\text{Pl}}^{-2}$$

$$M_{\text{Pl}} = 1.2 \cdot 10^{19} \text{ GeV} \quad \text{Planck mass}$$

If one also puts $M_{\text{Pl}} = 1$, one obtains the

Planck-units.