Problem C12.1 Dark matter freeze-in Dark matter could have very feeble interactions with the Standard Model. In such scenario, the dark matter relic abundance can be set via a non-thermal production mechanism dubbed freeze-in. At temperatures much larger than the dark matter mass, $T \gg m_X$, the thermal production cross-section of dark matter can be approximated by

$$\langle \sigma_{\mathsf{SM+SM}\to XX} v \rangle \approx \sigma_{\mathsf{SM+SM}\to XX} \approx \frac{\alpha_X^2}{T^2}$$

with α_X the dark matter-Standard Model coupling. Assume that $\alpha_X \ll 1$ such that dark matter never enters equilibrium, $\Gamma_{\mathsf{SM+SM}\to XX}/H \ll 1$. The amount of dark matter produced by unit time is, in that scenario,

$$\frac{dn_X}{dt} + 3Hn_X = 2 \times \langle \sigma_{\mathsf{SM+SM}\to XX} v \rangle \times (n_X^{\mathsf{eq.}})^2$$

where the reverse process (dark matter annihilation) has been (safely) neglected.

(a) Do a back-of-the-envelope calculation to estimate the value of $\alpha_X^{\mathsf{eq.}}$ at which $\Gamma_{\mathsf{SM+SM}\to XX}/H = 1$, *i.e.* the required interaction strength for dark matter to reach equilibrium with the Standard Model.

The Boltzmann equation can be rewritten in terms of the yield $Y \equiv n/s$ and the inverse temperature $z \equiv m_X/T$ to obtain

$$\frac{dY_X}{dz} = 2\frac{s}{H \times z} \langle \sigma_{\mathsf{SM+SM}\to XX} v \rangle \times (Y_X^{\mathsf{eq.}})^2$$

(b) How does Y_X scale with z, with T?

(c) Assume that dark matter production stops abruptly at $T = m_X$. What is the final yield of dark matter as a function of α_X? Does the relic abundance of dark matter scale with α_X in the same way as in the freeze-out scenario?

(d) Sketch how Y_X evolves in the freeze-in and freeze-out scenarios.

(e) The observed relic abundance of dark matter can be expressed in terms of yield

$$Y_{\mathsf{DM}} = 4.1 \times 10^{-9} \left(\frac{\mathsf{GeV}}{m}\right)$$

For $m_{\mathsf{DM}} = 1$ GeV, which α_X gives you the observed dark matter relic abundance? Is it much smaller than $\alpha_X^{\mathsf{eq.}}$?