Problem C9.1 Neutrinos are a class of particle called *hot relic*, they decoupled from the thermal bath while relativistic.

(a) Obtain the hot relic number density $n_{\text{dec.}}$ and its yield $Y_{\text{dec.}} = n_{\text{dec.}}/s$ when it decoupled from the thermal bath at temperature $T_{\text{dec.}}$.

(b) At later times, the hot relic number density will be given by $n\left(T < T_{\text{dec.}}\right) = s\left(T < T_{\text{dec.}}\right) Y_{\text{dec.}}$ due to entropy dilution. Obtain the relic mass density today and compare it to the critical density. Use this to set an upper bound on the allowed mass of hot relics.

(c) What is the allowed mass range for a hot relic which decoupled when $T \gg m_{\text{top}}$? What about the case of neutrinos, do they violate this bound?

Problem C9.2 Verify that after electron-positron annihilation the effective number of relativistic degrees for freedom is

$$g_* = 3.36 + 0.45(N_\nu - 3)$$

where N_ν is the number of neutrino flavors. *Hint:* Recall that the neutrino and photon temperatures are different after electron-positron annihilation.

Problem H9.1 Derive the formula for the equilibrium concentration of ^{4}He and verify that it's of order unity already at $T \sim 0.3$ MeV.

Problem H9.2 At low temperatures, electroweak interactions are mediated by massive vector bosons of mass m_{EW} and are therefore short-ranged. The "electroweak" scalar potential generated by an electroweakly charged particles is given by the Yukawa potential

$$V_{\text{EW}}\left(r\right) = \frac{g_{\text{EW}}}{r} e^{-m_{\text{EW}} r}$$

where g_{EW} is the dimensionless electroweak coupling.

(a) Fourier transform $V_{\text{EW}}\left(r\right)$ to obtain $\tilde{V}_{\text{EW}}\left(r\right)$ and take the $m_{\text{EW}} \to \infty$ limit.

(b) Fourier transform back to the spatial domain and deduce that $V_{\text{EW}} \propto \delta\left(r\right)$. How does the normalization scale with m_{EW}?

(c) In the Born approximation, quantum mechanics tell us that the differential cross-section for scattering off the (spherically symmetric) electroweak potential is given by

$$\frac{d\sigma}{d\Omega} = \left|f\left(\theta\right)\right|^2$$
$$f\left(\theta\right) = \frac{-2m}{q} \int_0^\infty r \sin\left(qr\right) V_{\text{EW}}\left(r\right) dr \quad , \quad q = 2k \sin\frac{\theta}{2}$$

where k is the momentum of the incoming particle. How does the cross-section scale with m_{EW}?