Problem C7.1 At which temperature/time do neutrino oscillation start becoming important? In the two-flavour case, the probability of a neutrino of flavour α to oscillate into the other flavour β is given by

$$P_{\alpha\to\beta\neq\alpha} = \sin^2(2\theta)\sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

where $\theta \simeq 0.1$ is the mixing angle (irrelevant in this exercise), $\Delta m^2 = m_1^2 - m_2^2$ is the mass squared difference, L is the length of the path travelled since emission and E the energy of the neutrino. Measurements give $\Delta m^2 \simeq 10^{-3}$ eV^2.

Problem C7.2 The neutrino-nucleon scattering cross-section is given by

$$\sigma_{\nu-\text{nucleon}} \simeq G_F^2 m_{\text{nucl.}} E$$

where $G_F \simeq 1.166 \times 10^{-5}$ GeV^{-2} is Fermi's constant, $m_{\text{nucl.}}$ is the mass of a nucleon and E is the energy of the neutrino. The mean free path of a neutrino in a medium of target number density n is

$$\lambda = \frac{1}{\sigma n}$$

What is the mean free path of neutrinos in lead? Consider a reactor neutrino ($E \simeq 1$ MeV) and a relic neutrino today ($E \simeq 1$ meV). For reference, lead has 82 protons and 126 neutrons and has a mass density of $\rho \simeq 11.35$ g/cm^3

Problem H7.1 Estimate the number density of pions at $T = 40$ MeV.

Problem H7.2 It is known that dark matter today contributes $\Omega_{\text{DM0}} \simeq 0.27$ to the total energy density. One interesting dark matter candidate are sterile (right-handed) neutrinos with a mass of about 1 keV. Compute their present number density assuming that they make up all of dark matter, and compare them with photon and proton denities.