Problem C4.1 Until hydrogen recombination, which occurred when the universe was 1000 times smaller than now, the universe was opaque to photons. Therefore, in practice, our view is limited to the maximum distance light can travel since recombination. This is called the "optical" horizon:

$$d_{\text{opt}}(t) = a(t) \int_{t_r}^{t} \frac{dt'}{a(t')}$$

Calculate t_r in a dust-dominated universe, and verify that the present optical horizon is less than the particle horizon by only a small percentage.

Problem C4.2 In the lecture we found that for matter domination with $k = 0$ the scale factor a is proportional to η^2. Use this relation to determine how a depends on the time t.

Problem H4.1

(a) Compute the scale factor a as a function of time and of conformal time for a flat Universe without matter or radiation but with a positive cosmological constant. The resulting space is called **de Sitter space**.

(b) Compute the present size of the event Horizon d_e in de Sitter space.

(c) What is the size of the event horizon in matter or radiation dominated Universes?

Problem H4.2 Luminosity distance

(a) The scale factor $a\,(t)$ can be expanded around t_0 ($i.e.$ today) to second order

$$a\,(t) \approx a\,(t_0) \left[1 + H_0\,(t - t_0) - \frac{1}{2} q_0 H_0^2\,(t - t_0)^2\right] \tag{1}$$

where q_0 is called the deceleration parameter. Use $1 + z = 1/a$ to obtain z at second order in $t - t_0$.

(b) Invert the power series and obtain an expression for $H_0\,(t_0 - t)$ up to second order in z.

(c) The luminosity distance is given by $d_L = a_0\,(1 + z)\,\chi$ for χ the comoving distance given by $d\chi^2 = \frac{dr^2}{1-kr^2}$. Photon fluxes from far away galaxies are proportional to d_L^{-2}. For a photon, $d\chi = \frac{dt}{a(t)}$. Compute the comoving distance traveled by photons which were emitted recently in the past to second order around t_0.

(d) Finally, find an expression for d_L to second order in z.

There exists astronomical events (for example, type Ia supernovae) for which we know very well the luminosity at the source, L. By measuring the photon flux F at Earth, we can obtain $d_L^2 \propto L/F$ as well as z via spectroscopy. Using the z expansion of d_L, one can therefore deduce if the expansion of the universe accelerates or decelerates.