Problem H2.1 The steady state cosmological model In the 1940's, the *perfect cosmological principle* was considered. This principle states that the universe is homogeneous and isotropic in *space* but that it is also homogeneous in *time*, $i.e.$ the Universe has always looked as it looks now. This implies that the mean energy density of the Universe, ρ_0 does not change with time. In turn, the expansion rate, H_0, is also a constant. In such a realisation, the expansion law is

$$\dot{a} = H_0 a$$

(a) Solve the above equation for $a\,(t)$, with initial condition $a(t_0) = a_0$.

(b) Under the perfect cosmological principle, matter has to be spontaneously created to keep the mean energy density of the Universe constant. Determine the expression for the time-dependent volume of a comoving sphere of volume a and the time-dependent mass $M\,(t)$ inside that sphere.

(c) Determine how many hydrogen atoms would be created (i) during a cosmology lecture within the volume of D6-135 and (ii) in one cubic kilometer over one year. Take ρ_0 to be the critical density.

(d) Which observations can/do disprove the validity of the perfect cosmological principle? Is our Universe in a steady state?

Problem H2.2 Age of the Universe in (standard) Newtonian cosmology The evolution of law of the scale factor is given by (see lecture)

$$H^2 - \frac{2E}{a^2} = \frac{2GM}{a^3}$$

Assuming that $E = 0$ obtain an equation for $\dot{a}$. Integrate it from $t = 0$, defined such that $a(0) = 0$, to today ($t = t_0$) to obtain an expression for the present age of our Universe. Compute its numerical value and compare it to the currently accepted figure of 13.79 billion years. Why do you think it is quite close/very far off?