Chapter 3

WAVE PROPAGATION AND REFRACTIVE INDEX
AT EUV AND SOFT X-RAY WAVELENGTHS

\[ n(\omega) = 1 - \frac{n_a r e \lambda^2}{2\pi} \left( f_1^0 - i f_2^0 \right) \]  
(3.9)

\[ n(\omega) = 1 - \delta + i\beta \]  
(3.12)

\[ l_{\text{abs}} = \frac{\lambda}{4\pi\beta} \]  
(3.22)

\[ \sigma_{\text{abs.}} = 2r e \lambda f_2^0(\omega) \]  
(3.28)

\[ \Delta \phi = \left( \frac{2\pi \delta}{\lambda} \right) \Delta r \]  
(3.29)

\[ \theta_c = \sqrt{2\delta} \]  
(3.41)

\[ R_{s,\perp} \simeq \frac{\delta^2 + \beta^2}{4} \]  
(3.50)

\[ \phi_B \simeq \frac{\pi}{4} - \frac{\delta}{2} \]  
(3.60)
The transverse wave equation is

\[
\left( \frac{\partial^2}{\partial t^2} - c^2 \nabla^2 \right) E_T(\mathbf{r}, t) = -\frac{1}{\epsilon_0} \frac{\partial J_T(\mathbf{r}, t)}{\partial t}
\]

(3.1)

For the special case of forward scattering the positions of the electrons within the atom (\(\Delta k \cdot \Delta r_s\)) are irrelevant, as are the positions of the atoms themselves, \(n(\mathbf{r}, t)\). The contributing current density is then

\[
J_0(\mathbf{r}, t) = -en_a \sum_s g_s \mathbf{v}_s(\mathbf{r}, t)
\]

(3.2)

where \(n_a\) is the average density of atoms, and

\[
\sum_s g_s = Z
\]
The oscillating electron velocities are driven by the incident field \( \mathbf{E} \)

\[
\mathbf{v}(\mathbf{r}, t) = \frac{e}{m} \frac{1}{(\omega^2 - \omega_s^2) + i \gamma \omega} \frac{\partial \mathbf{E}(\mathbf{r}, t)}{\partial t}
\]  

(3.2)

such that the contributing current density is

\[
\mathbf{J}_0(\mathbf{r}, t) = -\frac{e^2 n_a}{m} \sum_s \frac{g_s}{(\omega^2 - \omega_s^2) + i \gamma \omega} \frac{\partial \mathbf{E}(\mathbf{r}, t)}{\partial t}
\]  

(3.4)

Substituting this into the transverse wave equation (3.1), one has

\[
\left( \frac{\partial^2}{\partial t^2} - c^2 \nabla^2 \right) \mathbf{E}_T(\mathbf{r}, t) = \frac{e^2 n_a}{\epsilon_0 m} \sum_s \frac{g_s}{(\omega^2 - \omega_s^2) + i \gamma \omega} \frac{\partial^2 \mathbf{E}_T(\mathbf{r}, t)}{\partial t^2}
\]

Combining terms with similar operators

\[
\left[ \left( 1 - \frac{e^2 n_a}{\epsilon_0 m} \sum_s \frac{g_s}{(\omega^2 - \omega_s^2) + i \gamma \omega} \right) \frac{\partial^2}{\partial t^2} - c^2 \nabla^2 \right] \mathbf{E}_T(\mathbf{r}, t) = 0
\]  

(3.5)
Written in the standard form of the wave equation as

\[
\left[ \frac{\partial^2}{\partial t^2} - \frac{c^2}{n^2(\omega)} \nabla^2 \right] E_T(\mathbf{r}, t) = 0
\]  

(3.6)

The frequency dependent refractive index \( n(\omega) \) is identified as

\[
n(\omega) \equiv \left[ 1 - \frac{e^2 n_a}{\epsilon_0 m} \sum_s \frac{g_s}{(\omega^2 - \omega_s^2) + i \gamma \omega} \right]^{1/2}
\]  

(3.7)

For EUV/SXR radiation \( \omega^2 \) is very large compared to the quantity \( e^2 n_a/\epsilon_0 m \), so that to a high degree of accuracy the index of refraction can be written as

\[
n(\omega) = 1 - \frac{1}{2} \frac{e^2 n_a}{\epsilon_0 m} \sum_s \frac{g_s}{(\omega^2 - \omega_s^2) + i \gamma \omega}
\]  

(3.8)

which displays both positive and negative dispersion, depending on whether \( \omega \) is less or greater than \( \omega_s \). Note that this will allow the refractive index to be more or less than unity, and thus the phase velocity to be less or greater than \( c \).
Refractive Index in the Soft X-Ray and EUV Spectral Region (continued)

\[ n(\omega) = 1 - \frac{1}{2} \frac{e^2 n_a}{\varepsilon_0 m} \sum_s \frac{g_s}{(\omega^2 - \omega_s^2) + i \gamma \omega} \]  \hspace{1cm} (3.8)

Noting that

\[ r_e = \frac{e^2}{4\pi \varepsilon_0 mc^2} \]

and that for forward scattering

\[ f^0(\omega) = \sum_s \frac{g_s \omega^2}{\omega^2 - \omega_s^2 + i \gamma \omega} \]

where this has complex components

\[ f^0(\omega) = f_1^0(\omega) - i f_2^0(\omega) \]

The refractive index can then be written as

\[ n(\omega) = 1 - \frac{n_a r_e \lambda^2}{2\pi} \left[ f_1^0(\omega) - i f_2^0(\omega) \right] \]  \hspace{1cm} (3.9)

which we write in the simplified form

\[ n(\omega) = 1 - \delta + i \beta \]  \hspace{1cm} (3.12)
Refractive Index from the IR to X-Ray Spectral Region

\[ n(\omega) = 1 - \delta + i\beta \]  
\[ \delta = \frac{n_a r_e \lambda^2}{2\pi} f_1^0(\omega) \]  
\[ \beta = \frac{n_a r_e \lambda^2}{2\pi} f_2^0(\omega) \]

- \( \lambda^2 \) behavior
- \( \delta \) & \( \beta \ll 1 \)
- \( \delta \)-crossover

Refractive index, \( n \) vs. Ultraviolet

Infrared \( \omega_{IR} \)  
Visible \( \omega_{UV} \)  
Ultraviolet \( \omega_{K,L,M} \)  
X-ray
The wave equation can be written as

\[
\left( \frac{\partial}{\partial t} - \frac{c}{n(\omega)} \nabla \right) \left( \frac{\partial}{\partial t} + \frac{c}{n(\omega)} \nabla \right) \mathbf{E}_T(\mathbf{r}, t) = 0 \quad (3.10)
\]

The two bracketed operators represent left and right-running waves

\[
\left( \frac{\partial}{\partial t} - \frac{c}{n} \frac{\partial}{\partial z} \right) E_x = 0
\]

\[
\left( \frac{\partial}{\partial t} + \frac{c}{n} \frac{\partial}{\partial z} \right) E_x = 0
\]

where the phase velocity, the speed with which crests of fixed phase move, is not equal to \( c \) as in vacuum, but rather is

\[
v_\phi = \frac{c}{n(\omega)} \quad (3.11)
\]
Recall the wave equation

\[
\left( \frac{\partial}{\partial t} - \frac{c}{n(\omega)} \nabla \right) \left( \frac{\partial}{\partial t} + \frac{c}{n(\omega)} \nabla \right) E_T(\mathbf{r}, t) = 0 \quad (3.10)
\]

Examining one of these factors, for a space-time dependence

\[
E_T = E_0 \exp[-i(\omega t - kz)]
\]

\[-i \left( \omega - \frac{ck}{n} \right) = 0\]

Solving for \( \omega/k \) we have the phase velocity

\[
V_\phi = \frac{\omega}{k} = \frac{c}{n}
\]
Phase Variation and Absorption of Propagating Waves

For a plane wave \( \mathbf{E}(\mathbf{r}, t) = \mathbf{E}_0 e^{-i(\omega t - \mathbf{k} \cdot \mathbf{r})} \) in a material of refractive index \( n \), the complex dispersion relation is

\[
\frac{\omega}{k} = \frac{c}{n} = \frac{c}{1 - \delta + i\beta}
\]  

(3.15)

Solving for \( k \)

\[
k = \frac{\omega}{c} (1 - \delta + i\beta)
\]  

(3.16)

Substituting this into (3.14), in the propagation direction defined by \( \mathbf{k} \cdot \mathbf{r} = kr \)

\[
\mathbf{E}(\mathbf{r}, t) = \mathbf{E}_0 e^{-i[\omega t - (\omega/c)(1-\delta+i\beta)r]}
\]

or

\[
\mathbf{E}(\mathbf{r}, t) = \mathbf{E}_0 e^{\frac{-i\omega(t-r/c)}{\text{vacuum propagation}}} e^{\frac{-i(2\pi \delta/\lambda)r}{\phi\text{-shift}}} e^{\frac{-(2\pi \beta/\lambda)r}{\text{decay}}}
\]  

(3.17)

where the first exponential factor represents the phase advance had the wave been propagating in vacuum, the second factor (containing \( 2\pi \delta r/\lambda \)) represents the modified phase shift due to the medium, and the factor containing \( 2\pi \beta r/\lambda \) represents decay of the wave amplitude.
For complex refractive index $n$

$$
\mathbf{H}(\mathbf{r}, t) = n \sqrt{\frac{\varepsilon_0}{\mu_0}} \mathbf{k}_0 \times \mathbf{E}(\mathbf{r}, t)
$$

(3.18)

The average intensity, in units of power per unit area, is

$$
\bar{I} = |\mathbf{S}| = \frac{1}{2} |\text{Re}(\mathbf{E} \times \mathbf{H}^*)| 
$$

(3.19)

or

$$
\bar{I} = \frac{1}{2} \text{Re}(n) \sqrt{\frac{\varepsilon_0}{\mu_0}} |\mathbf{E}|^2 
$$

(3.20)

Recalling that

$$
\mathbf{E}(\mathbf{r}, t) = \mathbf{E}_0 e^{-i\omega (t-r/c)} e^{-i(2\pi \delta/\lambda) r} e^{-(2\pi \beta/\lambda) r}
$$

(3.17)

$$
\bar{I} = \frac{1}{2} \text{Re}(n) \sqrt{\frac{\varepsilon_0}{\mu_0}} |\mathbf{E}_0|^2 e^{-2(2\pi \beta/\lambda) r}
$$

or

$$
\bar{I} = \bar{I}_0 e^{-(4\pi \beta/\lambda) r}
$$

(3.21)

the wave decays with an exponential decay length

$$
I_{\text{abs}} = \frac{\lambda}{4\pi \beta}
$$

(3.22)
Absorption Lengths

\[ l_{\text{abs}} = \frac{\lambda}{4\pi \beta} \]  

(3.22)

Recalling that \( \beta = n_a r_e \lambda^2 f_2^0(\omega)/2\pi \)

\[ l_{\text{abs}} = \frac{1}{2n_a r_e \lambda f_2^0(\omega)} \]  

(3.23)

In Chapter 1 we considered experimentally observed absorption in thin foils, writing

\[ \frac{\bar{I}}{I_0} = e^{-\rho \mu r} \]  

(3.24)

where \( \rho \) is the mass density, \( \mu \) is the absorption coefficient, \( r \) is the foil thickness, and thus \( l_{\text{abs}} = 1/\rho \mu \). Comparing absorption lengths, the macroscopic and atomic descriptions are related by

\[ \mu = \frac{2r_e \lambda}{A m_u} f_2^0(\omega) \]  

(3.26)

where \( \rho = m_a n_a = A m_u n_a \), \( m_u \) is the atomic mass unit, and \( A \) is the number of atomic mass units
For a wave propagating in a medium of refractive index $n = 1 - \delta + i\beta$

$$E(r, t) = E_0 e^{-i\omega(t-r/c)} e^{-i(2\pi\delta/\lambda)r} e^{-(2\pi\beta/\lambda)r}$$  \hspace{1cm} (3.23)

the phase shift $\Delta\phi$ relative to vacuum, due to propagation through a thickness $\Delta r$ is

$$\Delta\phi = \left(\frac{2\pi\delta}{\lambda}\right) \Delta r$$  \hspace{1cm} (3.29)

- Flat mirrors at short wavelengths
- Transmissive, flat beamsplitters
- Bonse and Hart interferometer
- Diffractive optics for SXR/EUV
Reflection and Refraction at an Interface

incident wave: \( E = E_0 e^{-i(\omega t - k \cdot r)} \) \hspace{1cm} (3.30a)
refracted wave: \( E' = E'_0 e^{-i(\omega t - k' \cdot r)} \) \hspace{1cm} (3.30b)
reflected wave: \( E'' = E''_0 e^{-i(\omega t - k'' \cdot r)} \) \hspace{1cm} (3.30c)

(1) All waves have the same frequency, \( \omega \), and \(|k| = |k''| = \frac{\omega}{c}\)
(2) The refracted wave has phase velocity
\[
V_\phi = \frac{\omega'}{c'} = \frac{c}{n}, \text{ thus } k' = |k'| = \frac{\omega}{c} \left( 1 - \delta + i\beta \right)\]
Boundary Conditions at an Interface

• E and H components parallel to the interface must be continuous

\[ \mathbf{z}_0 \times (\mathbf{E}_0 + \mathbf{E}_0'') = \mathbf{z}_0 \times \mathbf{E}_0' \]  \hspace{1cm} (3.32a)

\[ \mathbf{z}_0 \times (\mathbf{H}_0 + \mathbf{H}_0'') = \mathbf{z}_0 \times \mathbf{H}_0' \]  \hspace{1cm} (3.32b)

• D and B components perpendicular to the interface must be continuous

\[ \mathbf{z}_0 \cdot (\mathbf{D}_0 + \mathbf{D}_0'') = \mathbf{z}_0 \cdot \mathbf{D}_0' \]  \hspace{1cm} (3.32c)

\[ \mathbf{z}_0 \cdot (\mathbf{B}_0 + \mathbf{B}_0'') = \mathbf{z}_0 \cdot \mathbf{B}_0' \]  \hspace{1cm} (3.32d)
Spatial Continuity Along the Interface

Continuity of parallel field components requires

\[(k \cdot x_0 = k' \cdot x_0 = k''_0 \cdot x_0) \text{ at } z = 0 \quad (3.33)\]

\[k_x = k'_x = k''_x \quad (3.34a)\]

\[k \sin \phi = k' \sin \phi' = k'' \sin \phi'' \quad (3.34b)\]

Conclusions:

Since \(k = k''\) (both in vacuum)

\[\sin \phi = \sin \phi'' \quad (3.35a)\]

\[\therefore \quad \phi = \phi'' \quad (3.35b)\]

The angle of incidence equals the angle of reflection

\[k \sin \phi = k' \sin \phi' \quad (3.36)\]

\[k = \frac{\omega}{c} \quad \text{and} \quad k' = \frac{\omega'}{c/n} = \frac{n \omega}{c}\]

\[\sin \phi = n \sin \phi' \quad \sin \phi' = \frac{\sin \phi}{n} \quad (3.38)\]

Snell’s Law, which describes refractive turning, for complex \(n\).
Total External Reflection of Soft X-Rays and EUV Radiation

Snell’s law for a refractive index of \( n \approx 1 - \delta \), assuming that \( \beta \to 0 \)

\[
\sin \phi' = \frac{\sin \phi}{1 - \delta} \quad (3.39)
\]

Consider the limit when \( \phi' \to \frac{\pi}{2} \)

\[
1 = \frac{\sin \phi_c}{1 - \delta}
\]

\[
\sin \phi_c = 1 - \delta \quad (3.40)
\]

\[
\sin(90^\circ - \theta_c) = 1 - \delta
\]

\[
\cos \theta_c = 1 - \delta
\]

\[
1 - \frac{\theta_c^2}{2} + \cdots = 1 - \delta
\]

\[
\theta_c = \sqrt{2\delta} \quad (3.41)
\]

The critical angle for total external reflection.

Glancing incidence (\( \theta < \theta_c \)) and total external reflection

Exponential decay of the fields into the medium
Total External Reflection (continued)

\[ \theta_c = \sqrt{2\delta} \quad (3.41) \]

\[ \delta = \frac{n_a r_e \lambda^2 f_1^0(\lambda)}{2\pi} \]

\[ \theta_c = \sqrt{2\delta} = \sqrt{\frac{n_a r_e \lambda^2 f_1^0(\lambda)}{\pi}} \quad (3.42a) \]

The atomic density \( n_a \), varies slowly among the natural elements, thus to first order

\[ \theta_c \propto \lambda \sqrt{Z} \quad (3.42b) \]

where \( f_1^0 \) is approximated by \( Z \). Note that \( f_1^0 \) is a complicated function of wavelength (photon energy) for each element.
Total External Reflection with Finite $\beta$

Glancing incidence reflection as a function of $\beta/\delta$

- finite $\beta/\delta$ rounds the sharp angular dependence
- cutoff angle and absorption edges can enhance the sharpness
- note the effects of oxide layers and surface contamination

... for real materials

(A) Reflectivity (%)

(B) Reflectivity (%)

(C) Reflectivity (%)

(D) Reflectivity (%)

(Henke, Gullikson, Davis)
The Notch Filter

- Combines a glancing incidence mirror and a filter
- Modest resolution, $E/\Delta E \sim 3-5$
- Commonly used

![Graph showing the mirror reflectivity and filter transmission](image-url)
Reflection at an Interface

$E_0$ perpendicular to the plane of incidence (s-polarization)

tangential electric fields continuous

$$E_0 + E_0'' = E_0'$$  \hspace{1cm} (3.43)

tangential magnetic fields continuous

$$H_0 \cos \phi - H_0'' \cos \phi = H_0' \cos \phi'$$  \hspace{1cm} (3.44)

$$\mathbf{H}(\mathbf{r}, t) = n \sqrt{\frac{\varepsilon_0}{\mu_0}} \mathbf{k}_0 \times \mathbf{E}(\mathbf{r}, t) \quad \Rightarrow \quad H = n \sqrt{\frac{\varepsilon_0}{\mu_0}} E$$

$$\sqrt{\frac{\varepsilon_0}{\mu_0}} E_0 \cos \phi - \sqrt{\frac{\varepsilon_0}{\mu_0}} E_0'' \cos \phi = n \sqrt{\frac{\varepsilon_0}{\mu_0}} E_0' \cos \phi'$$

$$(E_0 - E_0'') \cos \phi = n E_0' \cos \phi'$$  \hspace{1cm} (3.45)

Snell’s Law:

$$\sin \phi' = \frac{\sin \phi}{n}$$

Three equations in three unknowns
$(E_0', E_0'', \phi')$ (for given $E_0$ and $\phi$)
E₀ perpendicular to the plane of incidence (s-polarization)

\[
\frac{E'_0}{E_0} = \frac{2 \cos \phi}{\cos \phi + \sqrt{n^2 - \sin^2 \phi}} \quad (3.47)
\]

\[
\frac{E''_0}{E_0} = \frac{\cos \phi - \sqrt{n^2 - \sin^2 \phi}}{\cos \phi + \sqrt{n^2 - \sin^2 \phi}} \quad (3.46)
\]

The reflectivity R is then

\[
R = \frac{\tilde{I}''}{I_0} = \frac{|\tilde{S}''|}{|\tilde{S}|} = \frac{1}{2} \text{Re}(E''_0 \times H''_0^*) \quad (3.48)
\]

With \( n = 1 \) for both incident and reflected waves,

\[
R = \frac{|E''_0|^2}{|E_0|^2}
\]

Which with Eq. (3.46) becomes, for the case of perpendicular (s) polarization

\[
R_s = \frac{|\cos \phi - \sqrt{n^2 - \sin^2 \phi}|^2}{|\cos \phi + \sqrt{n^2 - \sin^2 \phi}|^2} \quad (3.49)
\]
Normal Incidence Reflection at an Interface

Normal incidence ($\phi = 0$)

$$R_s = \frac{\left| \cos \phi - \sqrt{n^2 - \sin^2 \phi} \right|^2}{\left| \cos \phi + \sqrt{n^2 - \sin^2 \phi} \right|^2}$$  \hspace{1cm} (3.49)

$$R_{s,\perp} = \frac{|1 - n|^2}{|1 + n|^2} = \frac{(1 - n)(1 - n^*)}{(1 + n)(1 + n^*)}$$

For $n = 1 - \delta + i\beta$

$$R_{s,\perp} = \frac{(\delta - i\beta)(\delta + i\beta)}{(2 - \delta + i\beta)(2 - \delta - i\beta)} = \frac{\delta^2 + \beta^2}{(2 - \delta)^2 + \beta^2}$$

Which for $\delta \ll 1$ and $\beta \ll 1$ gives the reflectivity for x-ray and EUV radiation at normal incidence ($\phi = 0$) as

$$R_{s,\perp} \approx \frac{\delta^2 + \beta^2}{4}$$  \hspace{1cm} (3.50)

Example:  Nickel @ 300 eV (4.13 nm)

From table C.1, p. 433

$f_1^\circ = 17.8$  \hspace{1cm} $f_2^\circ = 7.70$

$\delta = 0.0124$  \hspace{1cm} $\beta = 0.00538$

$$R_{\perp} = 4.58 \times 10^{-5}$$
Glancing Incidence Reflection (s-polarization)

\[
R_s = \left| \frac{\cos \phi - \sqrt{n^2 - \sin^2 \phi}}{\cos \phi + \sqrt{n^2 - \sin^2 \phi}} \right|^2 \tag{3.49}
\]

For \( \theta = 90^\circ - \phi \leq \theta_c \)

where \( \theta_c = \sqrt{2\delta} \ll 1 \)

\[
\cos \phi = \sin \theta \simeq \theta
\]

\[
\sin^2 \phi = 1 - \cos^2 \phi = 1 - \sin^2 \theta \simeq 1 - \theta^2
\]

For \( n = 1 - \delta + i\beta \)

\[
n^2 = (1 - \delta)^2 + 2i\beta(1 - \delta) - \beta^2
\]

\[
R_{s,\theta} = \left| \frac{\theta - \sqrt{(\theta^2 - \theta_c^2) + 2i\beta}}{\theta + \sqrt{(\theta^2 - \theta_c^2) + 2i\beta}} \right|^2 \quad (\theta \ll 1)
\]

Day 3
1. \( \beta/\delta = 0 \)
2. \( \beta/\delta = 10^{-2} \)
3. \( \beta/\delta = 10^{-1} \)
4. \( \beta/\delta = 1 \)
5. \( \beta/\delta = 3 \)
Reflection at an Interface

$E_0$ perpendicular to the plane of incidence (p-polarization)

\[
\frac{E''}{E_0} = \frac{n^2 \cos \phi - \sqrt{n^2 - \sin^2 \phi}}{n^2 \cos \phi + \sqrt{n^2 - \sin^2 \phi}} \tag{3.54}
\]

\[
\frac{E'}{E_0} = \frac{2n \cos \phi}{n^2 \cos \phi + \sqrt{n^2 - \sin^2 \phi}} \tag{3.55}
\]

The reflectivity for parallel (p) polarization is

\[
R_p = \left| \frac{E''}{E_0} \right|^2 = \left| \frac{n^2 \cos \phi - \sqrt{n^2 - \sin^2 \phi}}{n^2 \cos \phi + \sqrt{n^2 - \sin^2 \phi}} \right|^2 \tag{3.56}
\]

which is similar in form but slightly different from that for s-polarization. For $\phi = 0$ (normal incidence) the results are identical.
Brewster’s Angle for X-Rays and EUV

For p-polarization

\[ R_p = \left| \frac{E''}{E_0} \right|^2 = \left( \frac{n^2 \cos \phi - \sqrt{n^2 - \sin^2 \phi}}{n^2 \cos \phi + \sqrt{n^2 - \sin^2 \phi}} \right)^2 \]  \hspace{1cm} (3.56)

There is a minimum in the reflectivity where the numerator satisfies

\[ n^2 \cos \phi_B = \sqrt{n^2 - \sin^2 \phi_B} \]  \hspace{1cm} (3.58)

Squaring both sides, collecting like terms involving \( \phi_B \), and factoring, one has

\[ n^2(n^2 - 1) = (n^4 - 1) \sin^2 \phi_B \]

or

\[ \sin \phi_B = \frac{n}{\sqrt{n^2 + 1}} \]

the condition for a minimum in the reflectivity, for parallel polarized radiation, occurs at an angle given by

\[ \tan \phi_B = n \]  \hspace{1cm} (3.59)

For complex \( n \), Brewster’s minimum occurs at

\[ \tan \phi_B = 1 - \delta \]

or

\[ \phi_B \simeq \frac{\pi}{4} - \frac{\delta}{2} \]  \hspace{1cm} (3.60)
Focusing with Curved, Glancing Incidence Optics

The Kirkpatrick-Baez mirror system

- Two crossed cylinders (or spheres)
- Astigmatism cancels
- Fusion diagnostics
- Common use in synchrotron radiation beamlines
- See hard x-ray microprobe, chapter 4, figure 4.14
Determining $f_1^0$ and $f_2^0$

- $f_2^0$ easily measured by absorption
- $f_1^0$ difficult in SXR/EUV region
- Common to use Kramers-Kronig relations

\[
\begin{align*}
  f_1^0(\omega) &= Z - \frac{2}{\pi} P_C \int_0^\infty \frac{u f_2^0(u)}{u^2 - \omega^2} \, du \\
  f_2^0 &= \frac{2\omega}{\pi} P_C \int_0^\infty \frac{f_1^0(u) - Z}{u^2 - \omega^2} \, du
\end{align*}
\] (3.85a) (3.85b)

as in the Henke & Gullikson tables (pp. 428-436)

- Possible to use reflection from clean surfaces; Soufli & Gullikson
- With diffractive beam splitter can use a phase-shifting interferometer; Chang et al.
- Bi-mirror technique of Joyeux, Polack and Phalippou (Orsay, France)