Spatial string tension revisited

(dimensional reduction at work)

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work together with: M. Laine

Motivation

RHIC $\longrightarrow$ QCD at $T \gtrsim$ (a few) 100 MeV

asymptotic freedom $\longrightarrow$ weak coupling expansion

slow convergence, non-trivial structure

problematic dof's are identified

- soft modes $p \sim gT$ $\longrightarrow$ odd powers in g
- ultrasoft modes $p \sim g^2T$ $\longrightarrow$ non-pert coeffs

general picture

- perturbation theory OK for parametrically hard scales $p \sim 2\pi T$
- soft and ultrasoft scales need improved analytic schemes, or non-pert treatment
- starting point: dim red eff. theory, or HTL eff. theory

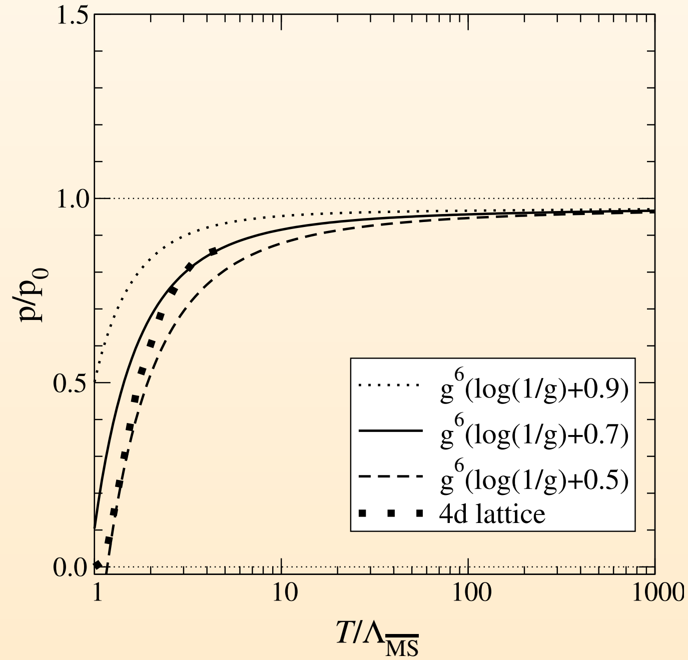
quantitative evidence:

- pick some simple observables
- compare 4d lattice vs soft/ultrasoft eff. theory
- e.g. static correlation lengths $\longrightarrow$ agreement down to $T \sim 2T_c$

Thermal pressure $p(T)$: 4d vs 3d

$$p_{\mathsf{QCD}}(T) \equiv \lim_{V\to\infty} \frac{T}{V} \ln \int \mathcal{D}[A^a_\mu, \psi, \bar\psi] \exp\Big(-\frac{1}{\hbar}\int_0^{\hbar/T} d\tau \int d^{3-2\epsilon}x\, \mathcal{L}_{\mathsf{QCD}}\Big)$$

$$\mathcal{L}_{\mathsf{QCD}} = \frac{1}{4} F^a_{\mu\nu}F^a_{\mu\nu} + \bar\psi\gamma_\mu D_\mu \psi + \mathcal{L}_{\mathsf{GF}} + \mathcal{L}_{\mathsf{FP}}$$



asymptotically, expect ideal gas: $p_{\mathsf{QCD}}(T\to\infty) \equiv p_0 = \left(16 + \frac{21}{2}N_f\right)\frac{\pi^2 T^4}{90}$

Spatial string tension σ_s

study an observable allowing an unambiguous comparison

take rectangular Wilson loop $W_s(R_1, R_2)$ in (x_1, x_2) plane

def potential $V_s(R_1) = -\lim_{R_2\to\infty} \frac{1}{R_2} \ln W_s(R_1, R_2)$

def spatial string tension $\sigma_s \equiv \lim_{R_1\to\infty} \frac{V_s(R_1)}{R_1}$

σ_s has been measured in SU(3) on the (4d) lattice

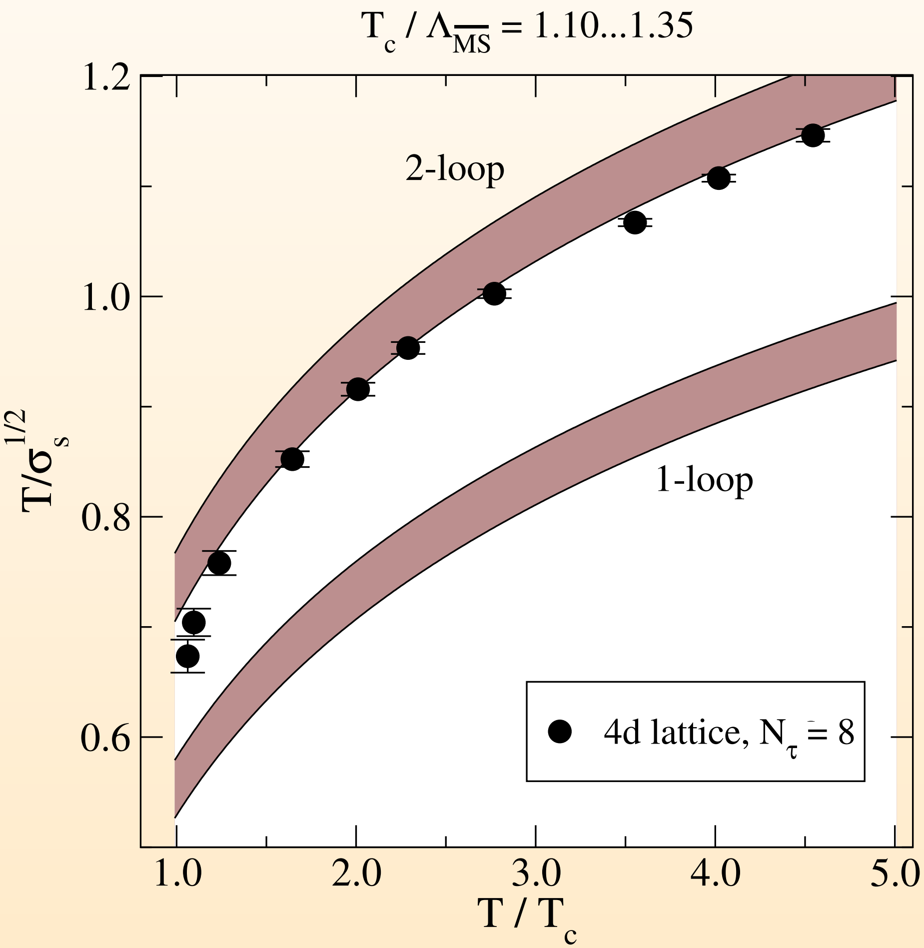
as e.g. $\frac{\sqrt{\sigma_s}}{T} = \phi\left(\frac{T}{T_c}\right)$ [Boyd et al, 96]

aim: get the eff. theory prediction for σ_s

- effective theory setup
- σ_s from 3d lattice
- perturbative matching to 4d
- $\Lambda_{\overline{MS}}$ vs T_c

Spatial string tension σ_s: 4d vs 3d

(anticipating all that follows)



[4d lattice data from Boyd et al, 96] (cave: $32^3 \times 8$, no cont. extrapolation: $N_T = 8$, $T = 1/aN_T$)

Effective theory setup: QCD $\to$ EQCD

high T: QCD dynamics contained in 3d EQCD

$$\mathcal{L}_E = \frac{1}{2} Tr\, F_{kl}^2 + Tr\, [D_k, A_0]^2 + m_E^2 Tr\, A_0^2 + \lambda_E^{(1)} (Tr\, A_0^2)^2 + \lambda_E^{(2)} Tr\, A_0^4 + \ldots$$

matching coefficients [E. Braaten, A. Nieto, 95; M. Laine, YS, 05]

$$\begin{aligned} m_E^2 &= T^2 \{\#g^2 + \#g^4 + \ldots\} \\ \lambda_E^{(1/2)} &= T \{\#g^4 + \#g^6 + \ldots\} \\ g_E^2 &= T \{g^2 + \#g^4 + \#g^6 + \ldots\} \end{aligned}$$

higher order operators do not (yet) contribute [S. Chapman, 94; Kajantie et al, 97, 02]

$$\delta\mathcal{L}_E \sim g^2 \frac{D_k D_l}{(2\pi T)^2} \mathcal{L}_E \sim g^2 \frac{(g^2 T)^2}{(2\pi T)^2} \mathcal{L}_E \qquad \text{(i.e. } g^8 \text{ for } g_E^2\text{)}$$

Digression: g_{E}^2 numerically

in practice, need to renormalize: $g^2 = g^2(\bar\mu)$ is $\overline{\mathsf{MS}}$ coupling

from soln of RGE at 2-loop level, define as usual

$$\Lambda_{\overline{\mathsf{MS}}} \equiv \lim_{\bar\mu\to\infty} \bar\mu \Big[b_0 g^2(\bar\mu) \Big]^{-b_1/2b_0^2} \exp\Big[-\frac{1}{2b_0 g^2(\bar\mu)} \Big]$$

hence $g_{\mathsf{E}}^2 = g_{\mathsf{E}}^2\,(\bar\mu, \Lambda_{\overline{\mathsf{MS}}}, T) = T\,\phi\left(\frac{\bar\mu}{T}, \frac{T}{\Lambda_{\overline{\mathsf{MS}}}}\right)$

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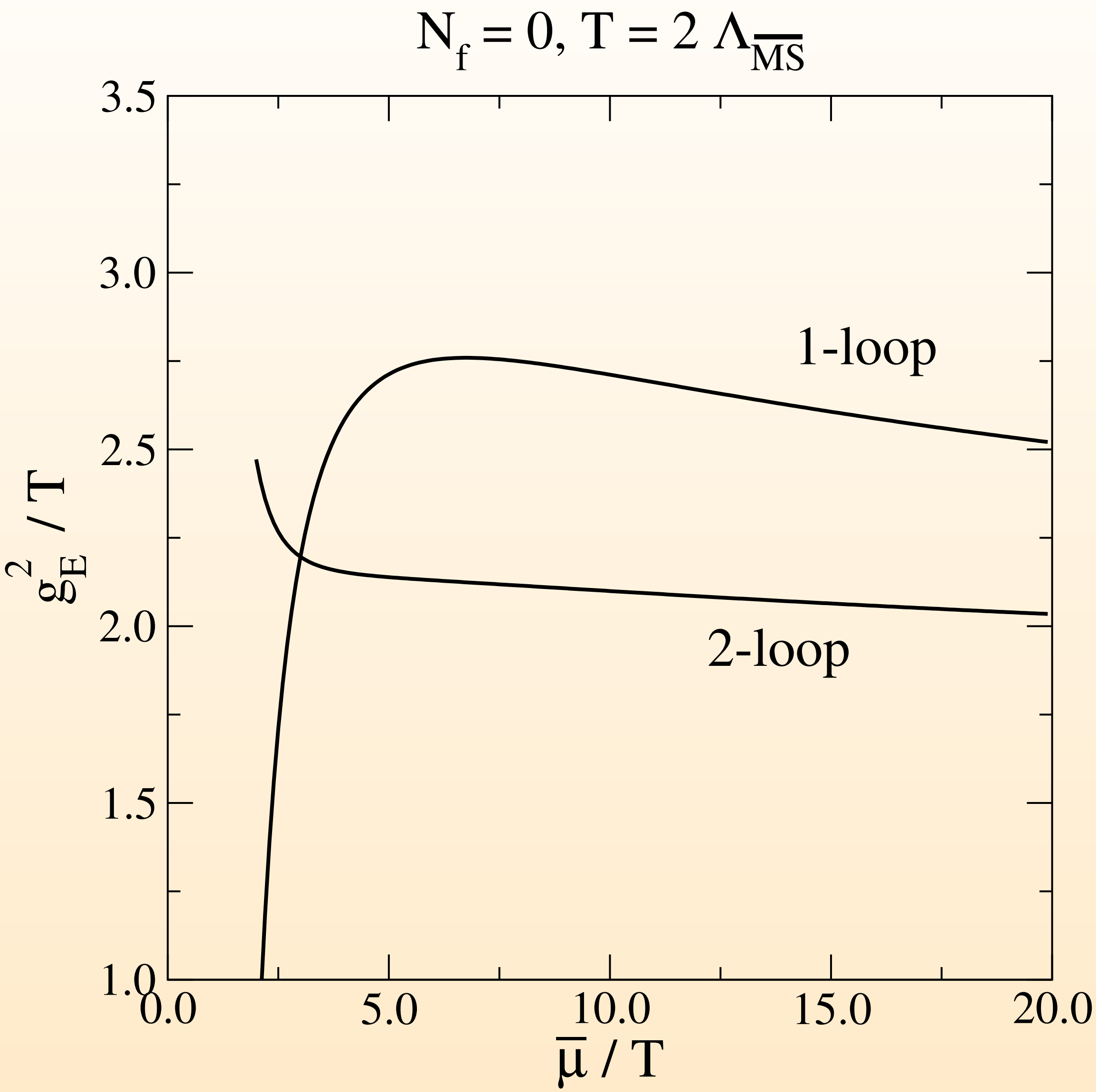
ren. scale dependence

- formally, $\bar{\mu}$ dependence is of higher order
- numerically, there is $\bar{\mu}$ dependence
- free to choose some optimisation procedure, e.g. PMS
  - ▷ *choose $\bar{\mu}_{opt}$ at extremum of 1-loop*
  - ▷ *vary scale within $\bar{\mu} = (0.5\ldots2.0) \times \bar{\mu}_{opt}$*

Choosing $\bar{\mu}_{\text{opt}}$ via PMS



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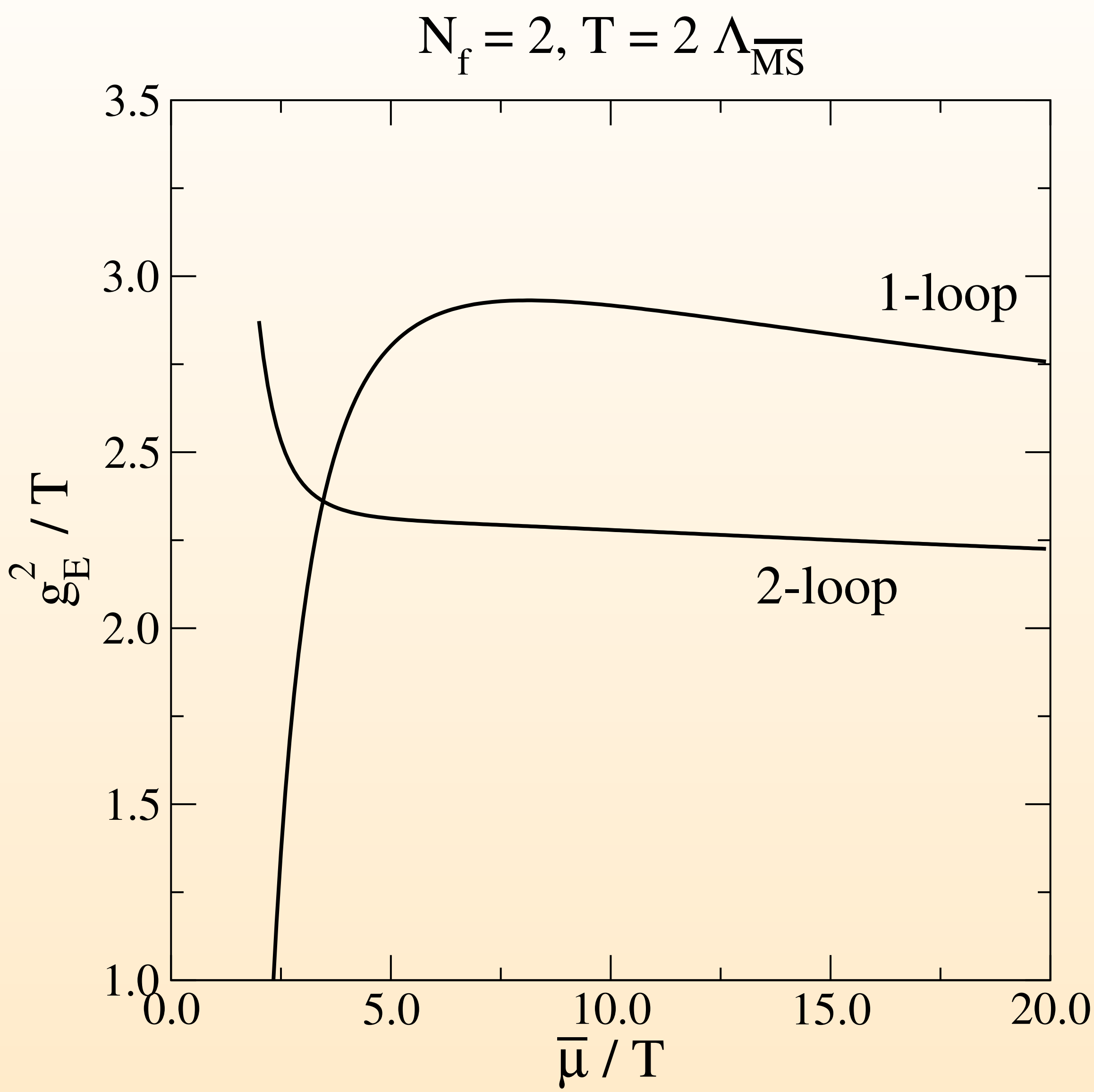

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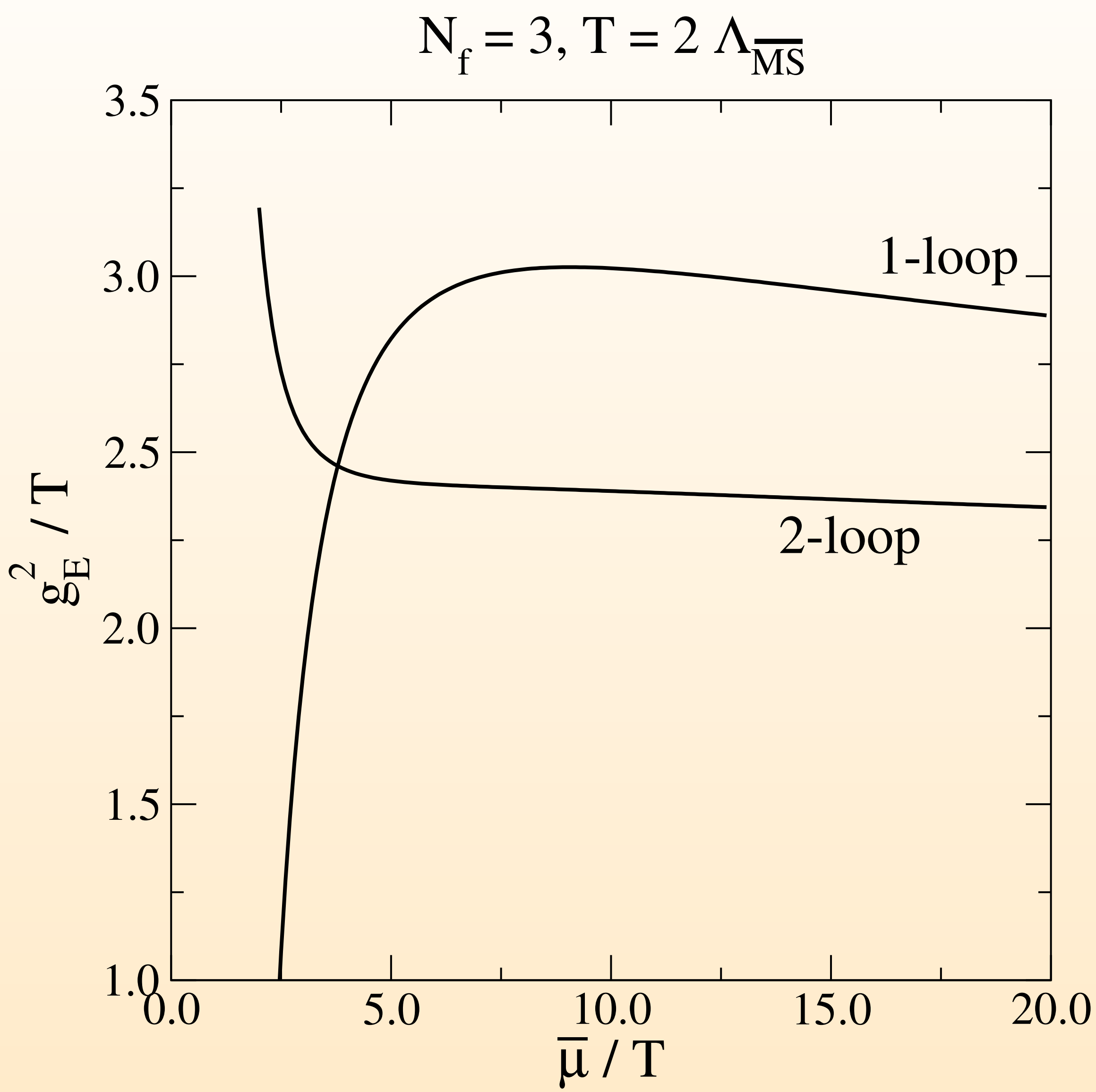

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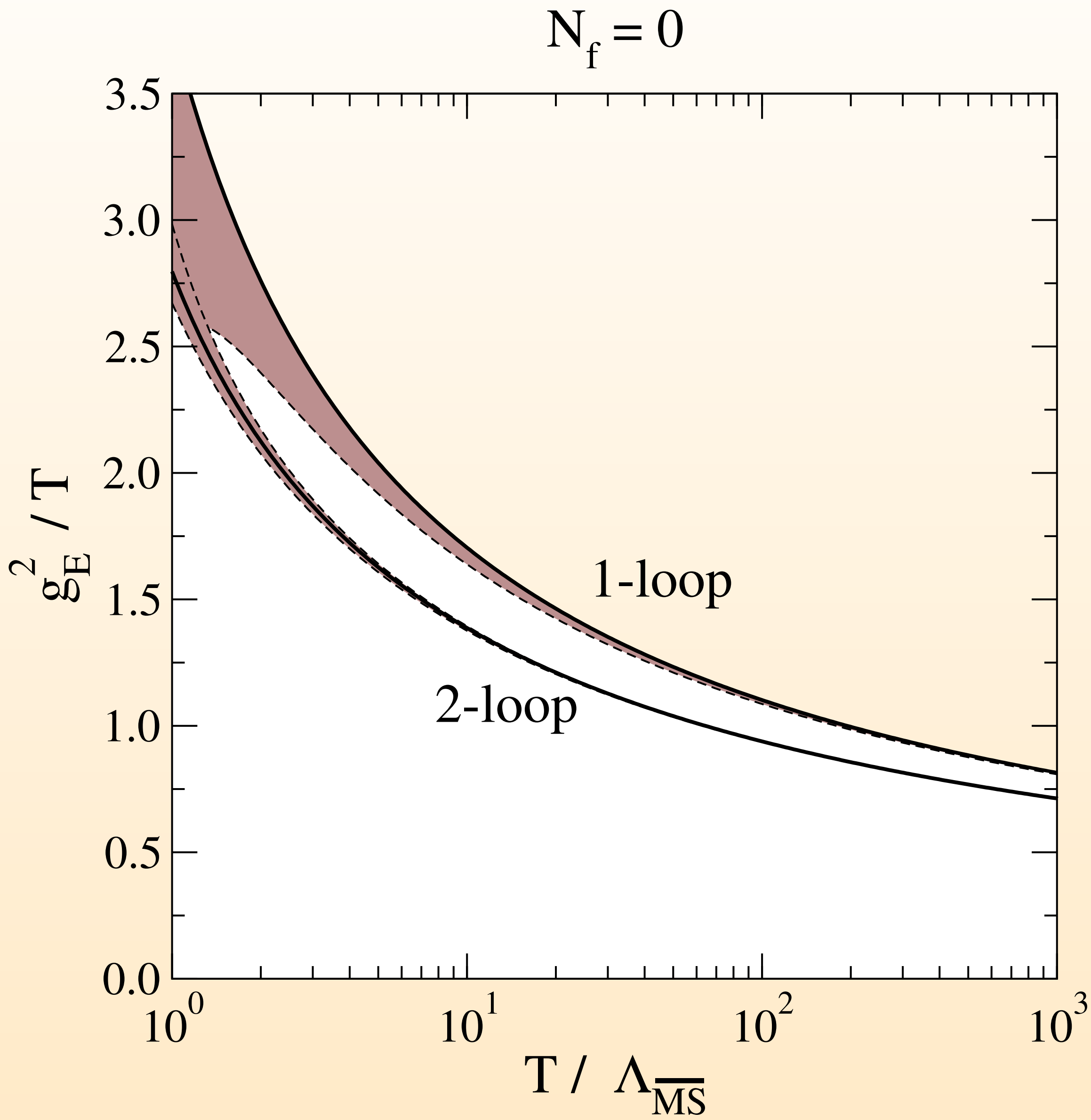

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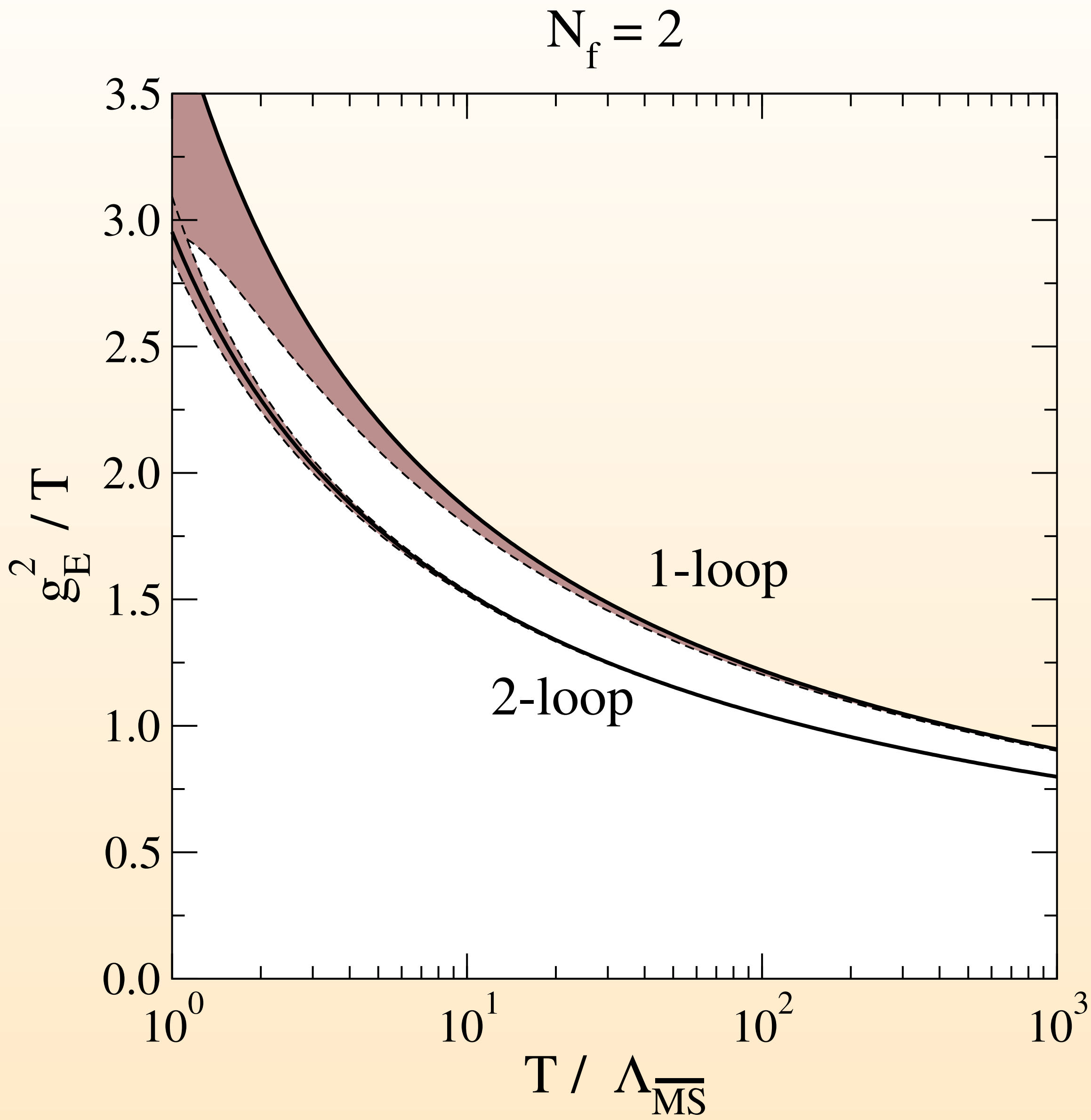

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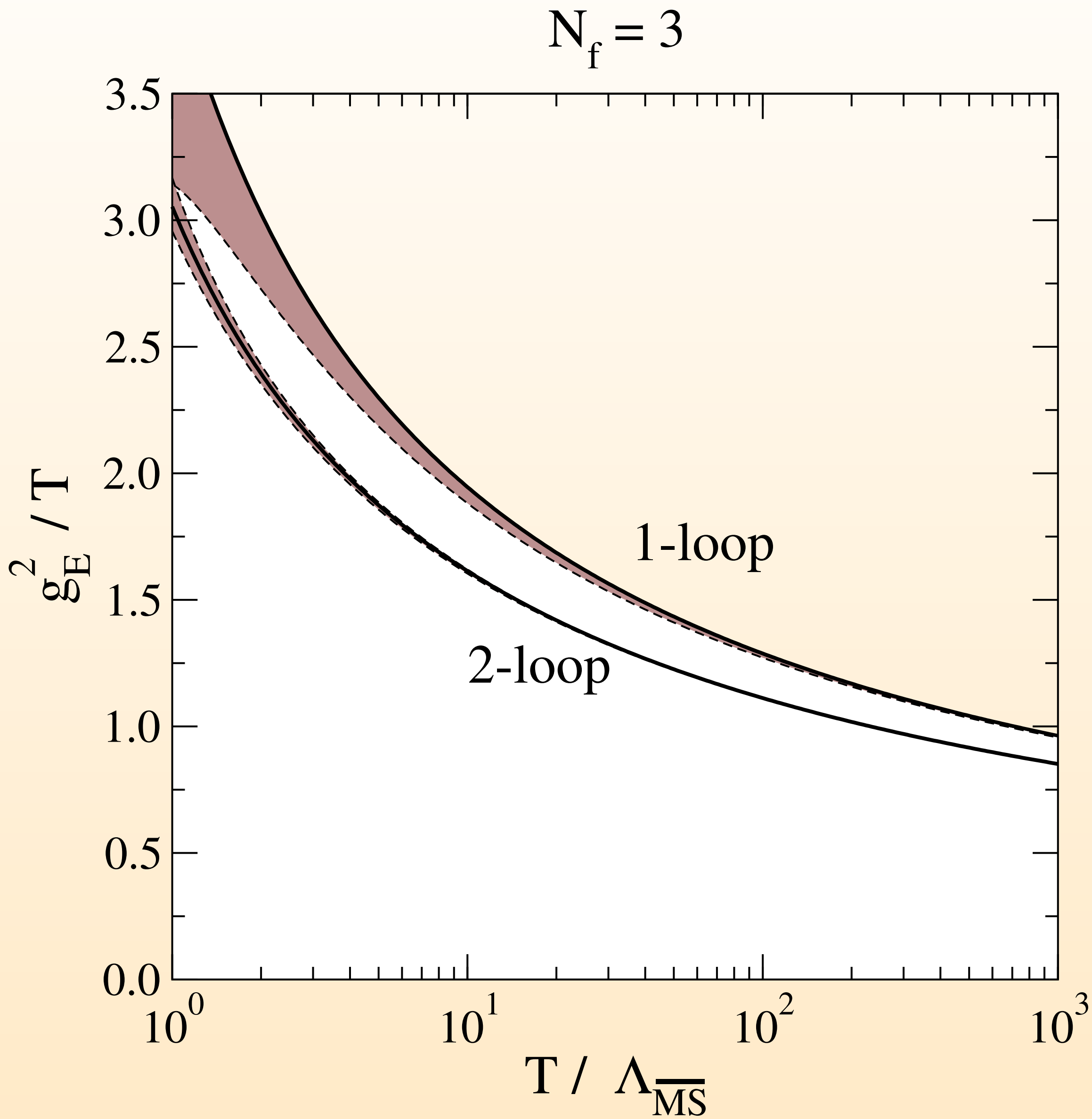
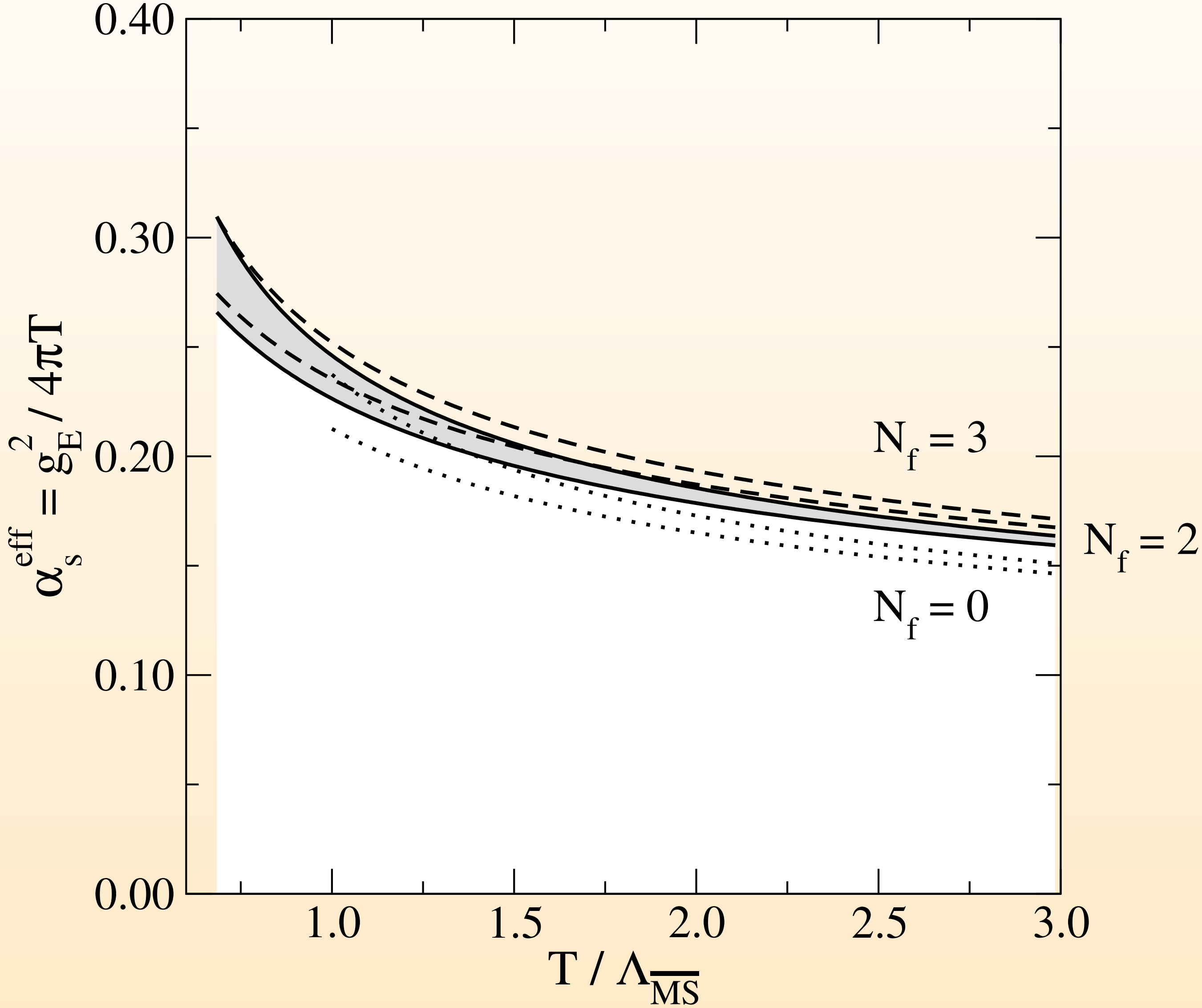


Digression: g_E^2 numerically

[Laine, Schröder, hep-ph/0503061]



Effective theory setup: QCD → EQCD → MQCD

3d EQCD is contained in 3d MQCD

$$\mathcal{L}_{\mathsf{M}} = \frac{1}{2} Tr\, F_{kl}^2 + \ldots$$

matching coefficient [P. Giovannangeli, 04; M. Laine, YS, 05]

$$g_{\mathsf{M}}^2 \quad = \quad g_{\mathsf{E}}^2 \left\{ 1 + \# \frac{g_{\mathsf{E}}^2}{m_{\mathsf{E}}} + \# \frac{g_{\mathsf{E}}^4}{m_{\mathsf{E}}^2} + \# \frac{g_{\mathsf{E}}^2 \lambda_{\mathsf{E}}^{(1/2)}}{m_{\mathsf{E}}^2} + \ldots \right\}$$

expansion converges extremely well, even close to T_c

can safely ignore higher loop corrections for g_{M}^2

higher order operators could contribute

$$\delta \mathcal{L}_{\mathsf{M}} \sim g_{\mathsf{E}}^2 \frac{D_k D_l}{m_{\mathsf{E}}^3} \mathcal{L}_{\mathsf{M}} \sim g_{\mathsf{E}}^2 \frac{(g^2 T)^2}{m_{\mathsf{E}}^3} \mathcal{L}_{\mathsf{M}}$$

Effective theory prediction for σ_s

observable σ_s exists in 3d SU(3) gauge theory (MQCD)

- dimensionful gauge coupling $\rightarrow$ $\sigma_s = \# g_{\mathsf{M}}^4$
- most recent lattice data $\frac{\sqrt{\sigma_s}}{g_{\mathsf{M}}^2} = 0.553(1)$ [M. Teper, B. Lucini, 02]

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- $\frac{\sqrt{\sigma_s}}{T} = 0.553(1)\, \frac{g_{\mathsf{M}}^2}{g_{\mathsf{E}}^2}\, \frac{g_{\mathsf{E}}^2}{T} = \phi\left(\frac{T}{\Lambda_{\overline{\mathsf{MS}}}}\right)$

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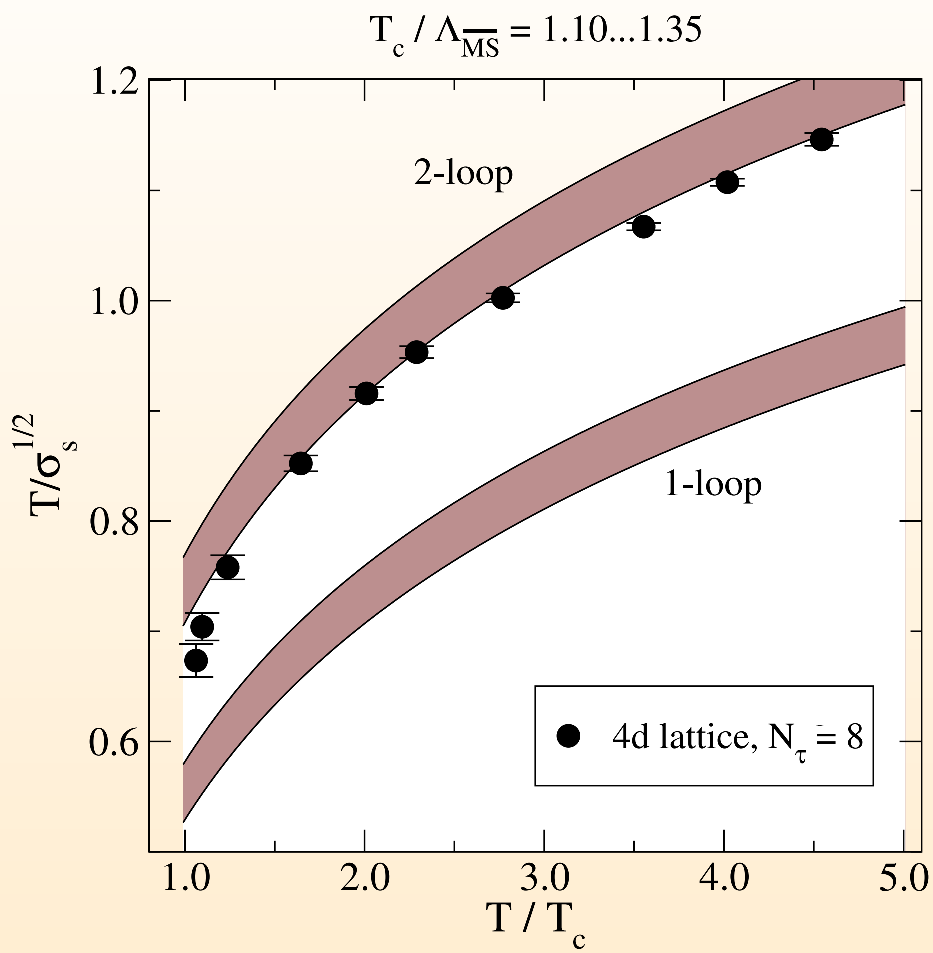
finally, need to relate $\Lambda_{\overline{\mathsf{MS}}}$ and T_c

- e.g. via $T=0$ string tension $\frac{T_c}{\Lambda_{\overline{\mathsf{MS}}}} = \frac{T_c/\sqrt{\sigma}}{\Lambda_{\overline{\mathsf{MS}}}/\sqrt{\sigma}} = 1.16(4)$ [Teper et al, 03; Bali, Schilling, 92]
- e.g. via Sommer scale $\frac{T_c}{\Lambda_{\overline{\mathsf{MS}}}} = \frac{r_0 T_c}{r_0 \Lambda_{\overline{\mathsf{MS}}}} = 1.25(10)$ [S. Necco, 03; ALPHA coll, 98]
- e.g. via scaling at crit. point $\frac{T_c}{\Lambda_{\overline{\mathsf{MS}}}} = 1.15(5)$ [S. Gupta, 00]
- to be conservative, consider the interval 1.10 ...1.35

Spatial string tension σ_s: 4d vs 3d



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parameter-free comparison!

support for hard/soft+ultrasoft picture of thermal QCD