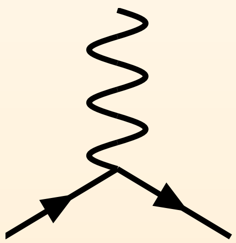
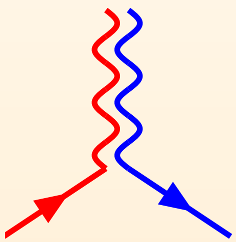
Frontiers in perturbative quantum field theory



York Schröder

(Univ Bielefeld)



Bielefeld, 08 Jan 2007

Motivation

why do we (physicists) do what we do?

- make life meaningful!
- why are we here?
- why are all these strange things happening around us?
  - ▷ *stars, astrophysics, cosmology, universe*
  - ▷ *chemistry, biology, electromagnetism*
  - ▷ *atom, nucleus, protons, quarks*

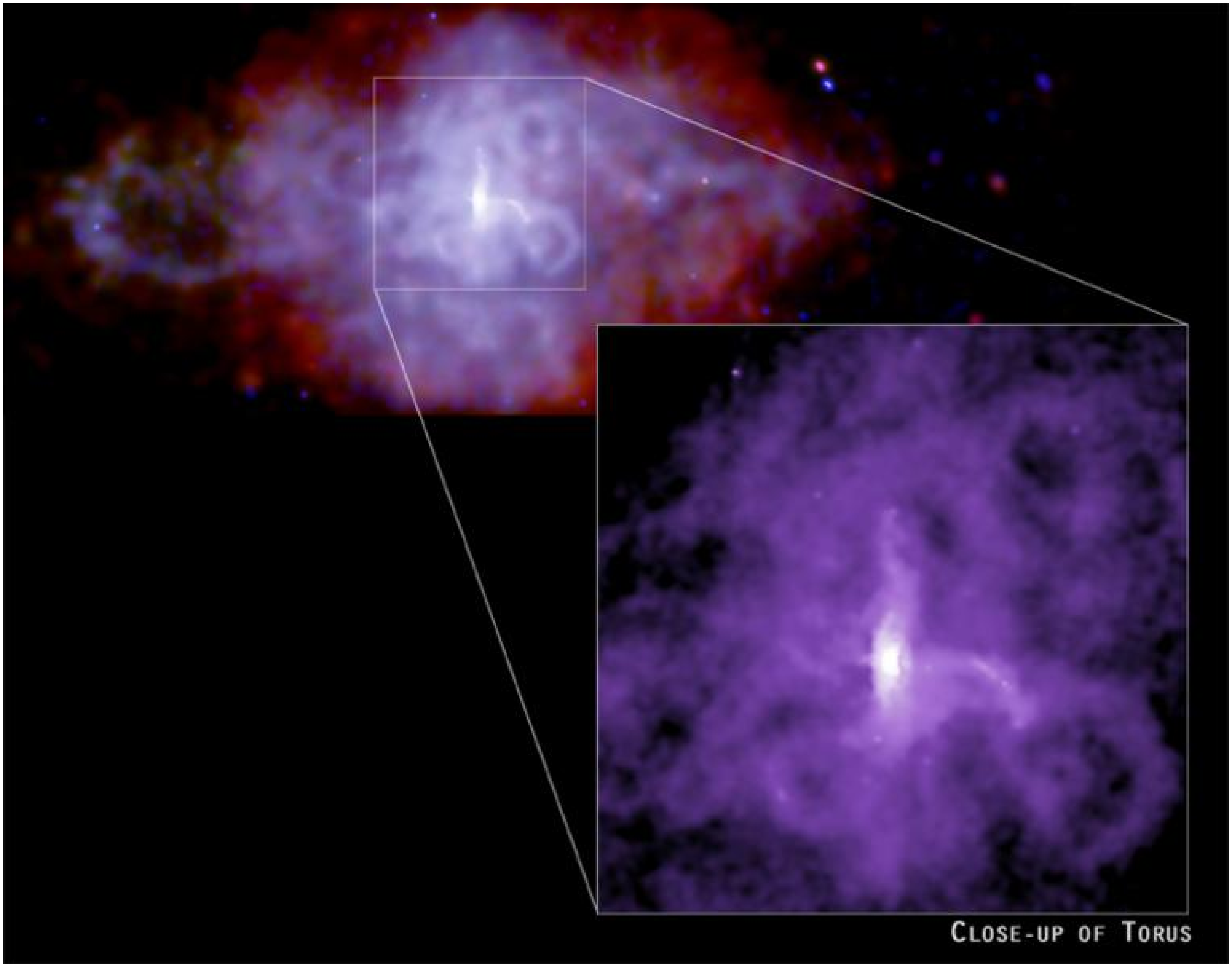
realize it's extremely strange

but also very beautiful

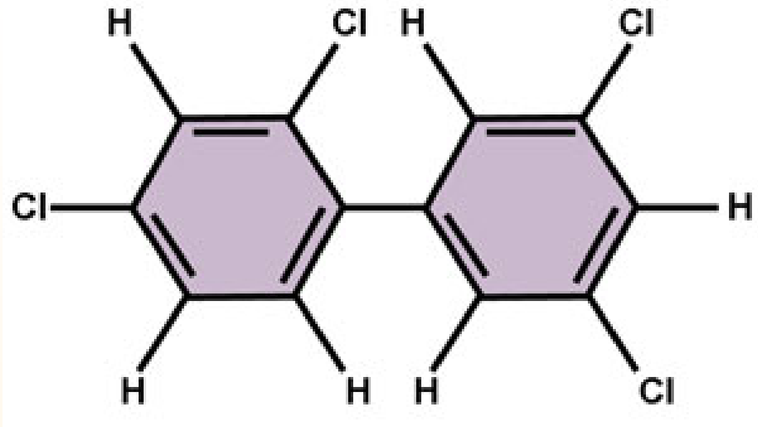
have built a system of understanding based on 3 pillars:

- gauge system
- gravity system
- Higgs system

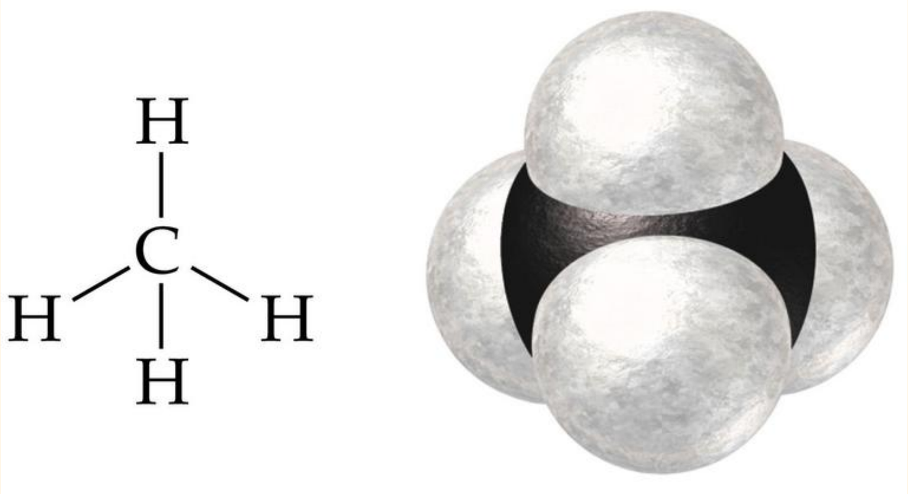
White dwarf, H1505+65. Temperature: 200000 °C



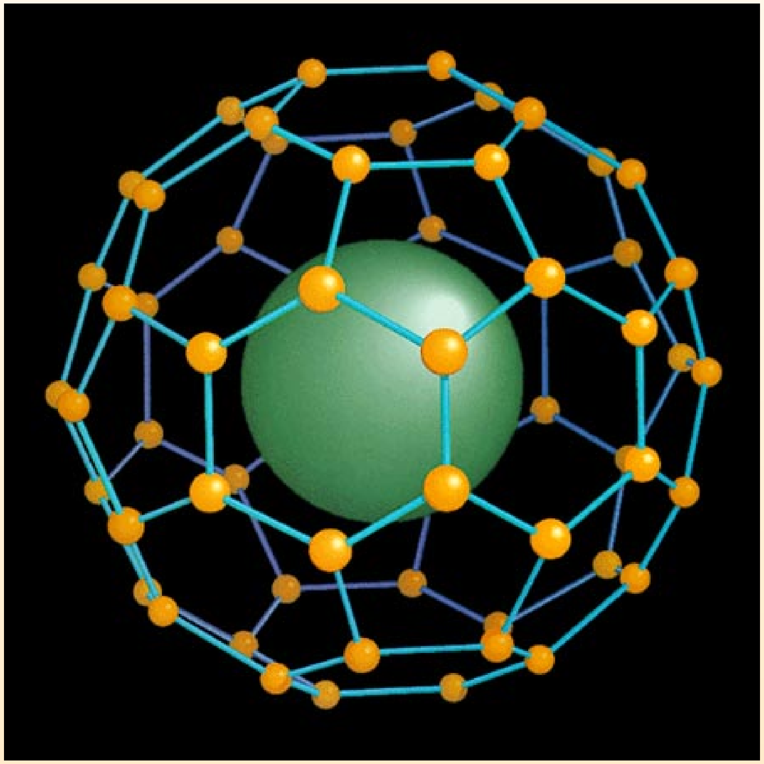
Neutron star 3C58 (rem. of chin. supernova 1181)
10000 lightyears. 1000000 °C. Weight: 1 teaspoon = 1 billion tons

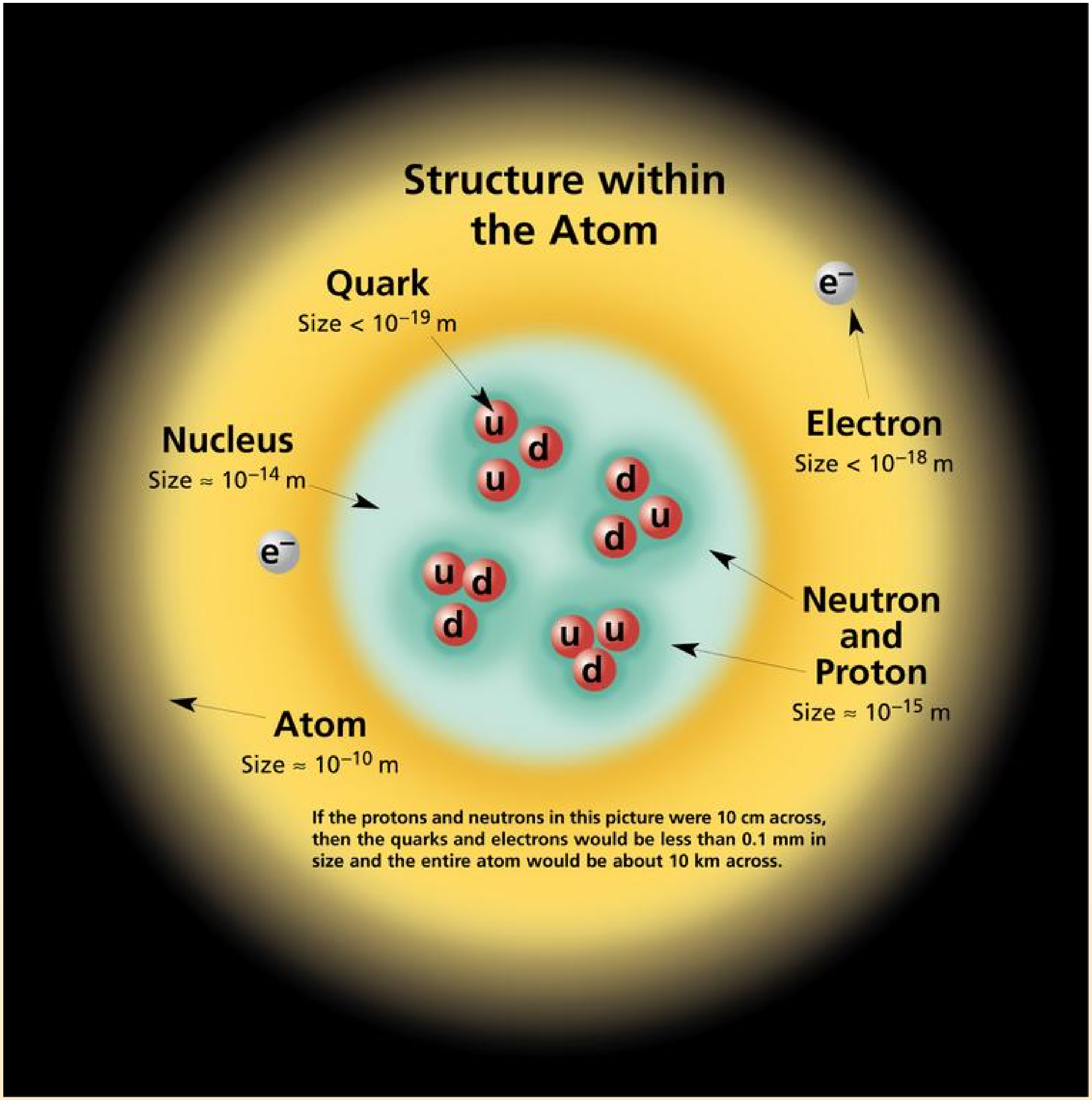


Tetrachlorinated Biphenyl
(a Polychlorinated Biphenyl subcategory, or homolog)



Tetrahedral geometry




Structure within the Atom
Quark
Size < 10^{-19} m
e⁻
Electron
Size < 10^{-18} m
Nucleus
Size ≈ 10^{-14} m
u
d
u
d
d
u
e⁻
u
d
d
u
u
d
Neutron and Proton
Size ≈ 10^{-15} m
Atom
Size ≈ 10^{-10} m
If the protons and neutrons in this picture were 10 cm across, then the quarks and electrons would be less than 0.1 mm in size and the entire atom would be about 10 km across.

Quantum field theory (QFT)

difficult to combine Quantum Mechanics + Special Relativity

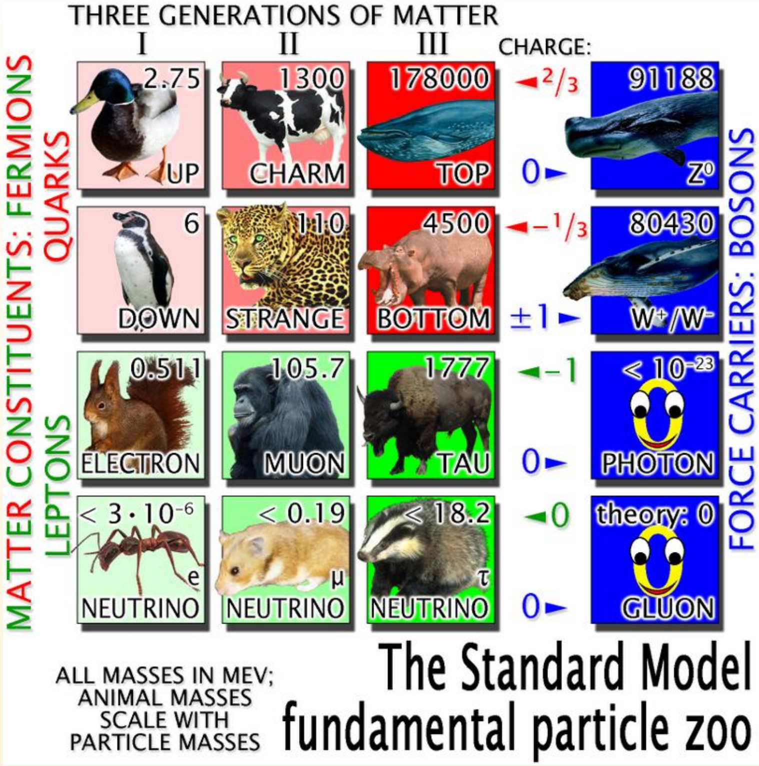
the only known way: QFT

- basic objects: space-filling fields
- we perceive quantized excitations (as particles: leptons, quarks)
- carriers of force: gauge particles
  - ▷ *weak / strong nuclear force: W,Z bosons / gluons*
  - ▷ *physical embodiments of (gauge) symmetry*
  - ▷ *as such, zero mass!*
  - ▷ *BUT* $m_{W,Z} \neq 0$ *(expt.)*
- symmetry (supposedly) spoiled in very special way
  - ▷ *by a form of "cosmic superconductivity"*
  - ▷ *new fields (*$\sim e^-$ *in ordinary supercond.)*
  - ▷ *excitations* $\Rightarrow$ *Higgs particle (not yet observed!)*

"Standard Model" $\hat{=}$ 3 basic conceptual structures

- gauge / gravity / Higgs (deep concepts vs. ad-hoc)
- each concerns interactions of (g/g/H)-particles

gauge system



- based on extensive symmetries among "color" dof's (color: generalization of em charge)
  - ▷ *QCD/weak/em: 3/2/1 color charges*
  - ▷ $SU(3) \times SU(2) \times U(1)$
- gauge symm. + QM + SR
  $\Rightarrow$ powerful!
  $\Rightarrow$ ex. gauge bosons

[D.Dobos, ATLAS collab.]

- basically only 3 parameters
  - ▷ *one coupling for each gauge sector*
  - ▷ *no other "fudge" factors!*
  - ▷ $\Rightarrow$ *precise predictions*
  - ▷ *agreement with numerous experiments* ($\longrightarrow$ *below*)

gravity system

- is essentially Einstein's general relativity
  - ▷ *Einstein-Hilbert action + minimal coupling to matter*

- fails at energies much larger than observable ones
  - ▷ *makes no predictions for ultra-high E particles*
  - ▷ *quantized GR not "renormalizable"*

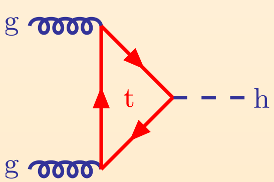
- symmetry principle: Einstein's general covariance
  $\Rightarrow$ ex. graviton

- 2 parameters
  - ▷ G_N: *Newton's grav. const;* $\sqrt{\frac{G_N h}{c^3}}$*=length!*
  - ▷ Λ: *cosmol. term, E density of empty space*

- many tests, e.g.:
  - ▷ *big bang cosmology*
  - ▷ *black hole physics*
  - ▷ *Mercury precession*
  - ▷ *pulsar frequency variation*

Higgs system

- no deep principle!
- many parameters
  - ▷ *infer from mixing of quarks + leptons*
- concept only provisional?
  - ▷ *e.g. CKM matrix (mixing of qu species) almost diagonal*
- searches at colliders
  - ▷ *e.g. LEP:* $m_h > 114$*GeV*
  - ▷ *next: LHC gluon fusion*



- discovery? surprise?

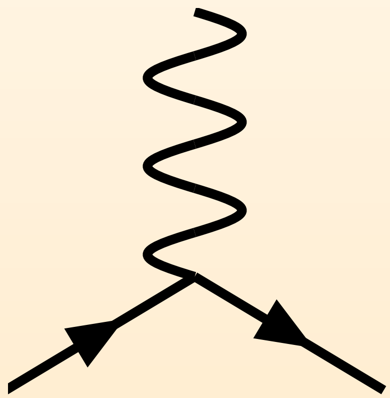
Quantum Chromodynamics (QCD)

''zoom'' into part of gauge system: QCD (the $SU(3)$ above)

what is it?

- QCD is a generalization of QED

- what is QED?
  - ▷ *basic concept:*
    response of photons to el. charge
  - ▷ *space-time picture (see →)*
  - ▷ *em force: via virtual photons*
  - ▷ *''Feynman diagrams'': like puzzle*
  - ▷ *$\hat{=}$ very definite mathematical rules*
  - ▷ *picture encodes:*
    Maxwell eqs for radio waves, light
    Schrödinger eqs for atoms, chemistry
    Dirac eqn (same with spin)
    and more



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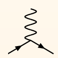
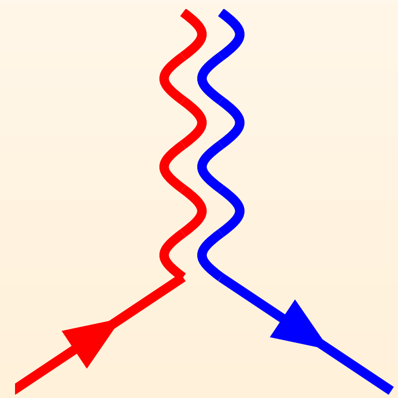
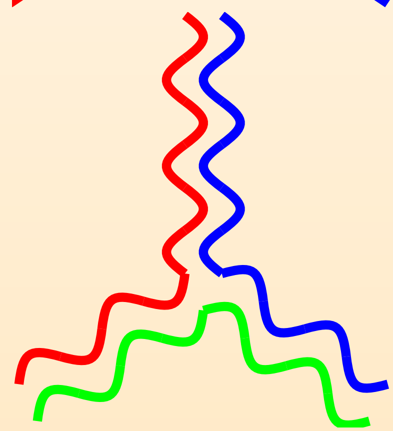
- what is QED? 

- QCD is the same, but bigger
  - ▷ *3 kinds of charge (color), e.g. red, blue, green*
  - ▷ *quarks: 1 unit of one of the color charges*
(+ fractional el. charge)
6 ''flavors''. u,d,c,s,t,b
  - ▷ *8 gluons, respond to color charge*



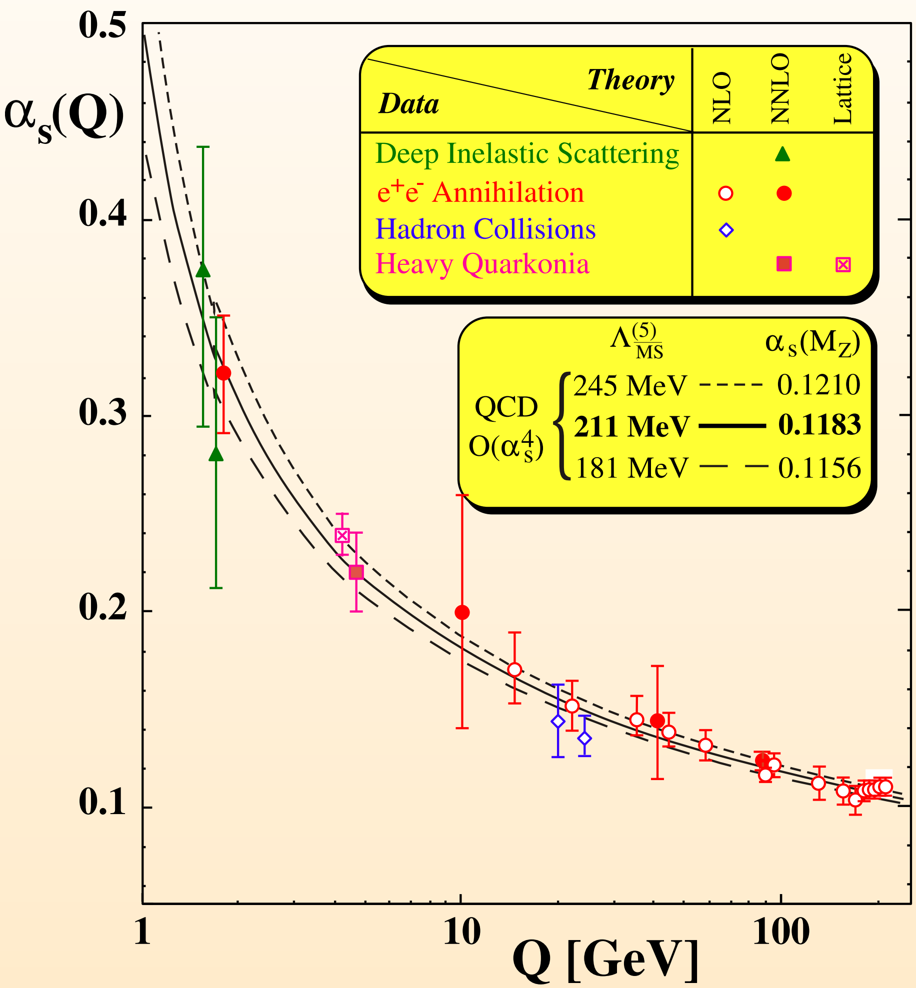


- many puzzle possibilities!
  but large symmetry. red↔blue everywhere (even locally)
  $\longrightarrow$ only one way to assign couplings

- main differences to QED:
  gluons couple stronger / can change charge / interact

Quantum Chromodynamics (QCD)

central feature: asymptotic freedom



[PDG; LEP EWWG]

- smash atoms $\longrightarrow$ e^- get emitted basics of our electronics
- smash protons (p) $\longrightarrow$ get more p + exotic particles; never a quark
- strong force rises with distance
- quarks closer together (high E) $\Rightarrow$ force weaker
- beautiful theory result
- unexpected! (em force opposite)
- Nobel price 2004 G/P/W
- experiment?! ($\leftarrow$ see left)

Quantum Chromodynamics (QCD)

Reality check?!

- outraged claim: none of qu, gl ever seen!
  - ▷ *have to explain confinement*
- phenomenology
  - ▷ *u,d masses tiny ⇒ eqs of QCD possess ''chiral symmetry''*
    (allowing separate trafos among q_R and q_L)
  - ▷ *no such symmetry is observed:*
    strongly int. part. do not come in opposite-parity pairs
- chiral symmetry must be ''spontaneously'' broken (like rotational sy. in ferromagnets)

how to check QCD vs Reality?

- (a) just solve its eqs (⟶ see next slide)
  - ▷ *by computer (lattice); tough; ''oracle''; understand?!*
- (b) consider models ''close to QCD''
  - ▷ *fewer dims; different sy groups; diff particle content*
- (c) consider circumstances in which eqs simplify
  - ▷ *remainder of this talk*

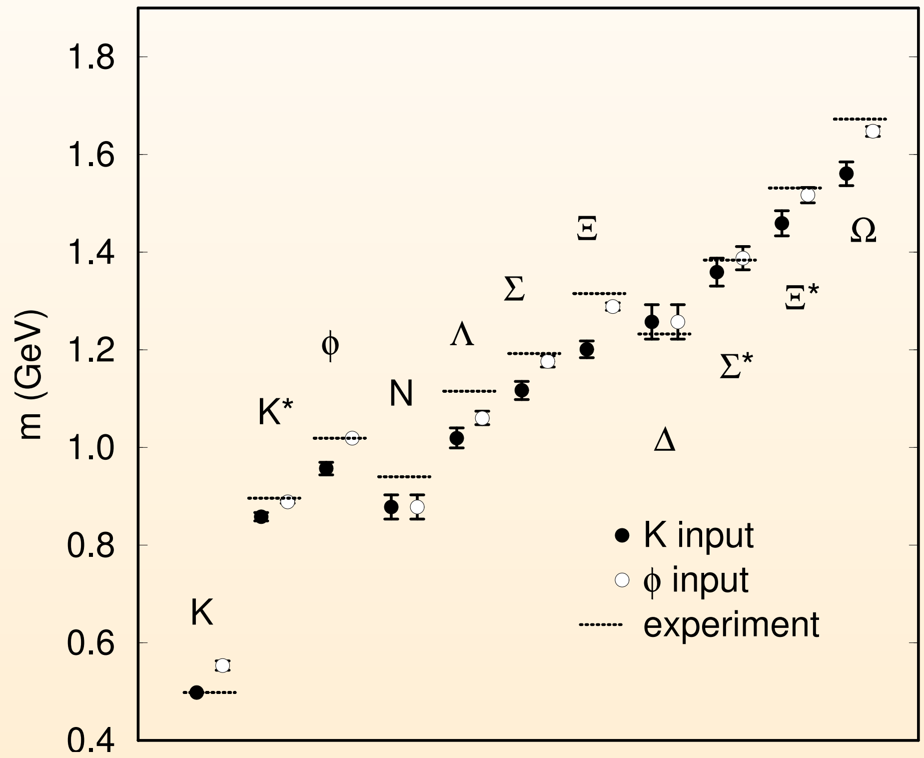
QCD reality check (a:computer)

look at hadron spectrum (hadrons: bound states of quarks; e.g. K=$s\bar{d}$, p=uud, Λ=uds)

- solve QCD eqs by computer
  [e.g. S. Aoki et.al., CP-PACS 1999]

- what does not come out:
  - ▷ *gluons*
  - ▷ *fractional charges*
  - ▷ *enlarged multiplets*

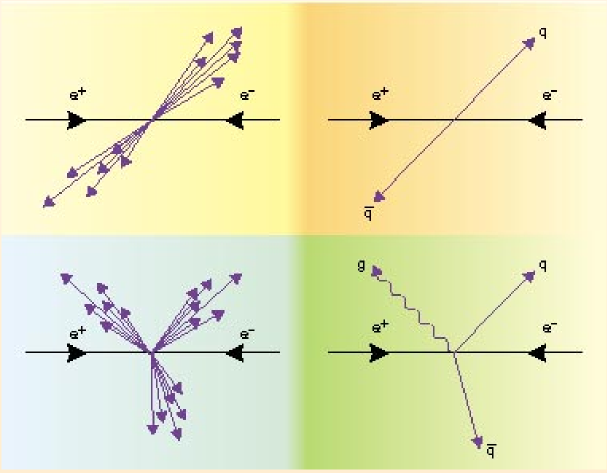
- what one gets:
  - ▷ *just the observed particles + masses*
  - ▷ *no more, no less!*



- punchline: obtain amazingly realistic spectrum, with 10% error
  - ▷ *QCD lite; need to add remaining quark effects + quark masses*
  - ▷ *much development here; teraflop speeds, worldwide effort*

QCD reality check (c:collider)

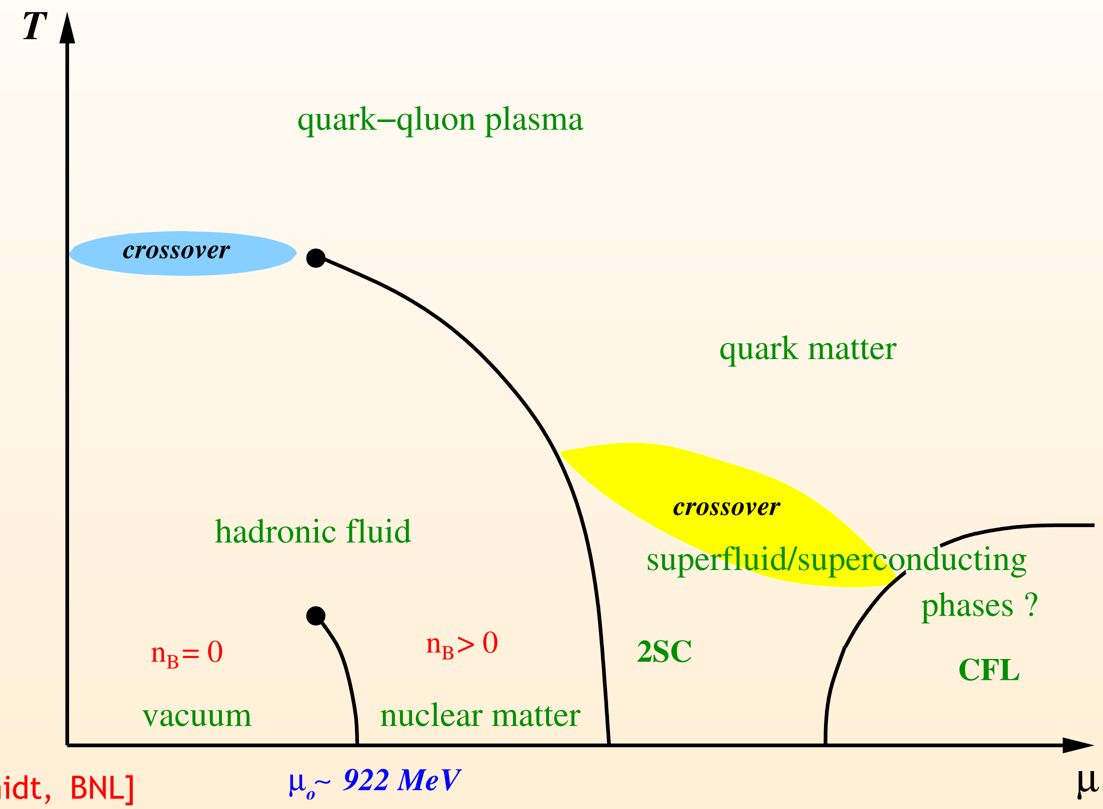
- e.g. LEP, $e^+e^- \to X$ (stuff hitting detector): find 2 broad classes of events (QM!)
- (1) $X = e^+e^-$ or $\tau^+\tau^-$ or ... l^+l^-
  - ▷ *leptons: no color charge → mainly QED interactions*
  - ▷ *simple final state: coupling small ($\alpha = e^2/(4\pi) \approx 1/137$)* *most of the time (99%) nothing happens*
  - ▷ $e^+e^-\gamma \sim$ *1% → check details of QED*
  - ▷ $e^+e^-\gamma\gamma \sim$ *0.01% → ...*
- (2) $X > 10$ particles: $\pi,\ \rho,\ p,\ \bar{p},\ \ldots$
  - ▷ *"greek+latin soup" constructed from qu+gl*
  - ▷ *pattern: flow of E+momentum in "jets"*
  - ▷ *2 jets ∼ 90%; 3 jets ∼ 9%; 4 jets ∼ 0.9%*
  - ▷ *direct confirmation of asy. freedom!*
  - ▷ *hard radiation is rare → # of jets*
  - ▷ *soft radiation is common → broadens jet*



- nowadays: "testing QCD" → "calculating backgrounds" in search for new phenomena

QCD reality check (c:extremes)

childlike questions: what happens when I heat or squeeze matter?

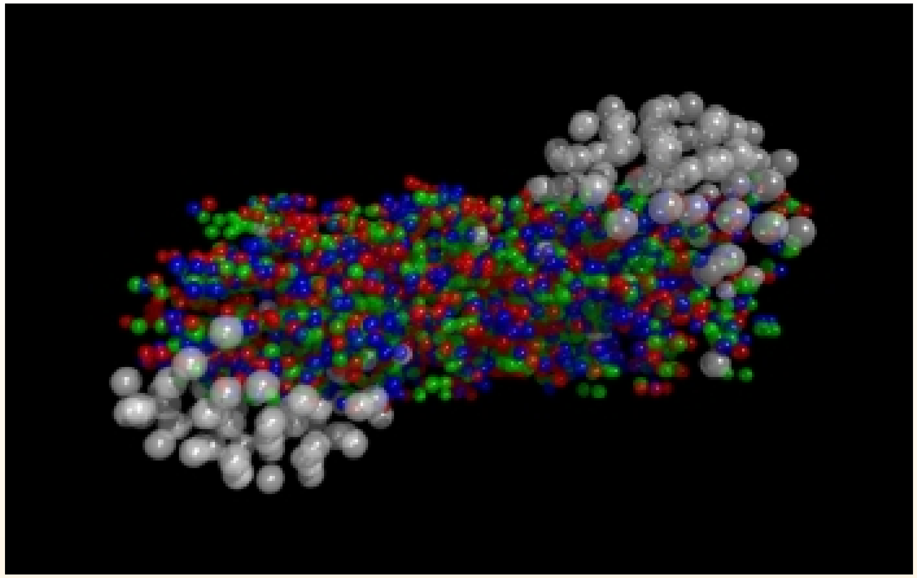


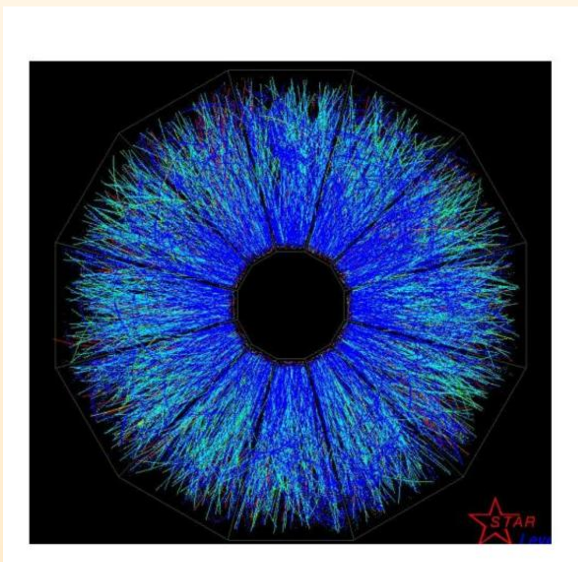
[Ch. Schmidt, BNL]

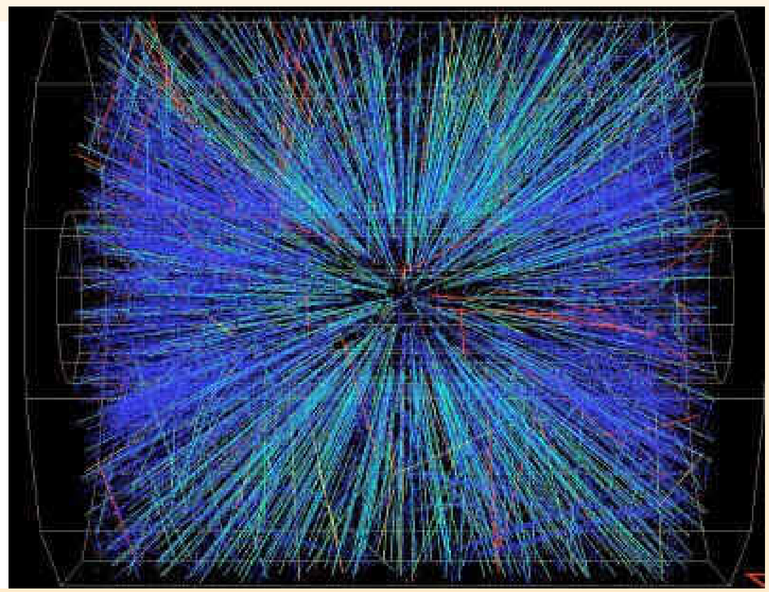
nature: early univ, μ tiny ($\sim \frac{\#baryons}{entropy}$), $T_c \sim 170MeV \sim 10\mu s$
neutron/quark stars

lab expt.: SPS / RHIC $\mu_B \sim \frac{\#baryons}{pions} \sim 45MeV$ / LHC / GSI

Neutron star in Crab nebula. Distance: 6000 lightyears







gold dust. analyze ashes of short-lived nuclear fireball! [N. Borghini, 30 Oct 2006]

basic thermodynamic observable: pressure $p(T)$

$p(T)$ important for cosmology:

- cooling rate of the universe

$$\partial_t T = -\frac{\sqrt{24\pi}}{m_{\mathsf{Pl}}} \frac{\sqrt{e(T)}}{\partial_T \ln s(T)}$$

- with entropy $s = \partial_T\, p$ and energy density $e = Ts - p$
- $\Rightarrow$ cosmol. relics (dark matter, background radiation etc.) originate when an interaction rate $\tau(T)$ gets larger than the age of the universe $t(T)$.
  - ▷ *Ex.: "sterile" ν_R with $m_\nu \sim$ keV can be warm dark matter, and decouple around $T \sim 150$ MeV* *[Abazajian, Fuller 02; Asaka, Shaposhnikov 05]*

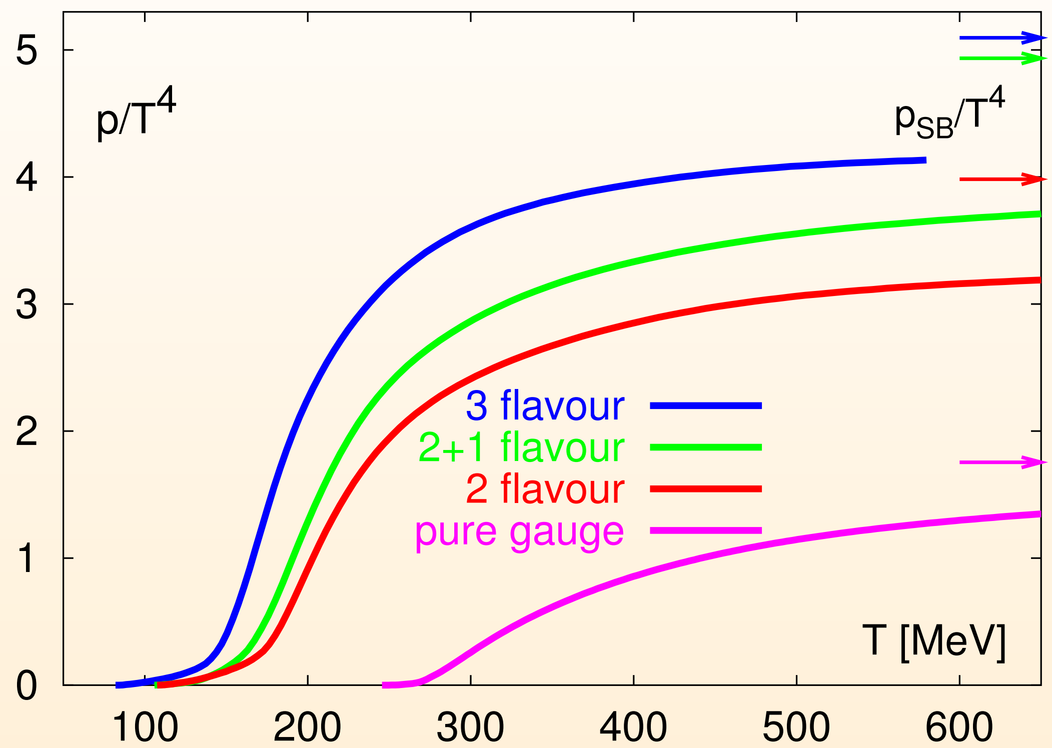
$p(T)$ in heavy ion collisions:

- expansion rate (after thermalization) given by

$$\partial_\mu T^{\mu\nu} = 0 \quad , \quad T^{\mu\nu} = [p(T) + e(T)]\, u^\mu u^\nu - p(T) g^{\mu\nu}$$

- with flow velocity $u^\mu(t, x)$
  - ▷ *hydrodynamic expansion: hadronization at $T \sim 100 - 150$ MeV*
    $\Rightarrow$ observed hadron spectrum depends (indirectly) on $p(T)$

$p(T)$ via (large) computer ($\mu_B = 0$)



[lattice data from Karsch et.al.]

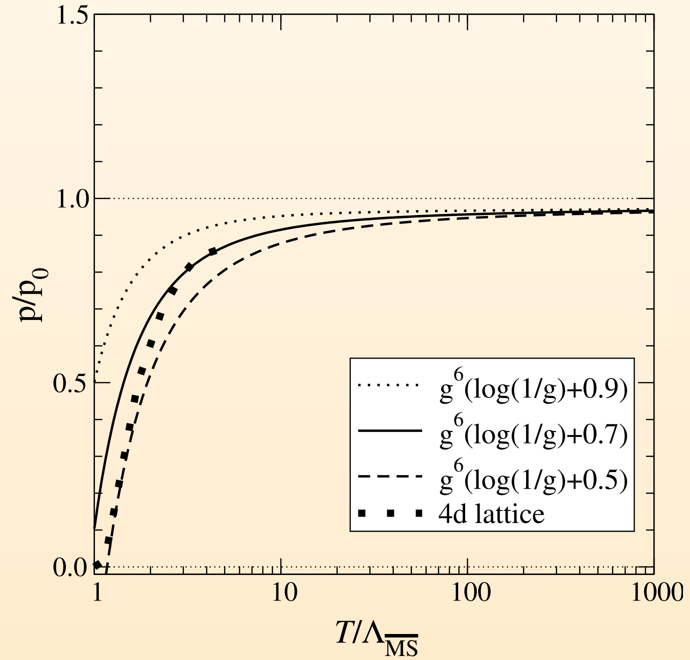
at $T \to \infty$, expect ideal gas: $p_{\text{SB}} = \left(16 + \frac{21}{2} N_f\right) \frac{\pi^2 T^4}{90}$

confirms simplicity: 3 dofs (π) $\to$ 52 ($3 \times 3 \times 2 \times 2$ qu + 8×2 gl)

$p(T)$ via analytical computation

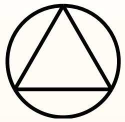
$$p_{\mathsf{QCD}}(T) \equiv \lim_{V\to\infty} \frac{T}{V} \ln \int \mathcal{D}[A^a_\mu, \psi, \bar\psi] \exp\Big(-\frac{1}{\hbar}\int_0^{\hbar/T} d\tau \int d^{3-2\epsilon}x\, \mathcal{L}_{\mathsf{QCD}}\Big)$$

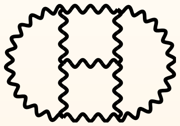
$$\mathcal{L}_{\mathsf{QCD}} = \frac{1}{4} F^a_{\mu\nu}F^a_{\mu\nu} + \bar\psi\gamma_\mu D_\mu\psi + \mathcal{L}_{\mathsf{GF}} + \mathcal{L}_{\mathsf{FP}}$$



asymptotically, expect ideal gas: $p_{\mathsf{QCD}}(T\to\infty) \equiv p_0 = \left(16 + \frac{21}{2}N_f\right)\frac{\pi^2 T^4}{90}$

Methods I: reduction, IBP

can do 4-loop scalar theory on paper:  1 integral

for QCD, need a computer:  25M integrals $(2^9 6^6)$

powerful method: integration by parts (IBP) [Chetyrkin/Tkachov 81]

$\Rightarrow$ systematically use $0 = \int d^d k \, \partial_{k_\mu} f_\mu(k)$

many incarnations: Laporta, Baikov, Gröbner

key idea: lexicographic ordering among all loop integrals [Laporta 00]

arrive at rep in terms of irreducible ($\equiv$ master) integrals

$$\sum_i \frac{\text{poly}_i(d,\xi)}{\text{poly}_i(d)} \text{Master}_i(d)$$

Methods IIa: integration

Evaluating Masters

- numerical integration; cave: precision (MC?)
- explicit integration; can be an ‘‘art’’
- difference equations
  - ▷ *solve directly*
  - ▷ *solve numerically*
  - ▷ *Laplace transform*
- differential equations

Mathematical structure

- interested in the coefficients of an ϵ expansion
- in many cases, these are from a generic class of functions/numbers
- e.g. harmonic polylogarithms HPL(x) [Remiddi/Vermaseren 00]
- e.g. harmonic sums $S(N)$ [Vermaseren 98]

Methods IIb: harmonic sums

find interesting new numbers

$$
\begin{aligned}
S_{a,\bar{m}}(N) &\equiv \sum_{i=1}^{N} \frac{[\mathrm{sgn}(a)]^i}{i^{|a|}} S_{\bar{m}}(i) \quad , \quad S(i \geq 0) = 1 \quad , \quad S(i < 0) = 0 \\
\ln 2 &= -S_{-1}(\infty) \\
\zeta_{n\geq 2} &= S_n(\infty) = \zeta(n) \\
a_{n\geq 4} &= -S_{-1,\bar{1}_{n-1}}(\infty) = Li_n\,(1/2) \\
s_6 &= S_{-5,-1}(\infty) \approx +0.98744142640329971377165000804182021413602714 89 \\
s_{7a} &= S_{-5,1,1}(\infty) \approx -0.95296007575629860340865215892596050767328040 17 \\
s_{7b} &= S_{5,-1,-1}(\infty) \approx +1.02912126296432453422440408804384184300201671 26 \\
s_{8a} &= S_{5,3}(\infty) \approx +1.04178502918279188338999002080231238008156211 01 \\
s_{8b} &= S_{-7,-1}(\infty) \approx +0.99644774839783766598087290122422927214404887 82 \\
s_{8c} &= S_{-5,-1,-1,-1}(\infty) \approx +0.98396667382173367092073025030655946912191093 09 \\
s_{8d} &= S_{-5,-1,1,1}(\infty) \approx +0.99996261346268344769671661371698277760950413 87
\end{aligned}
$$

"the language that Feynman diagrams speak"? [J. Vermaseren]

Methods IIIa: difference equations

repeat reduction with symbolic power x on one line

derive difference equation for generalized master $U(x) \equiv \int \frac{1}{D_1^x D_2 \dots D_N}$

$$\sum_{j=0}^{R} p_j(x) U(x+j) \quad = \quad F(x)$$

compute boundary conditions, e.g. at $x = 0$, $x \gg 1$

typically, want $U(1)$

solve the difference equation

- directly (if 1st order)
- numerically (very general setup)
- Laplace transform

Methods IIIb: numeric solution

very general setup [Laporta 00]

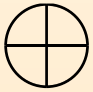
solve via factorial series $U(x) = U_0(x) + \sum_{j=1}^{R} U_j(x)$, where

$$U_j(x) = \mu_j^x \sum_{s=0}^{\infty} a_j(s) \frac{\Gamma(x+1)}{\Gamma(x+1+s-K_j)}$$

plug into difference eq, get μ, $K_j(d)$, and recursion rels for $a_j(s)$

need boundary condition for fixing, say, $a_j(0)$

numerics: truncate sum. example:



$$\begin{aligned} = \quad & +1.2722705418498941993978 8 - 5.67991293994853579036683\epsilon \\ & +17.6797238948173732343788\epsilon^2 - 46.5721846649543261864019\epsilon^3 \\ & +111.658522176214385363568\epsilon^4 - 252.463963901002177432 36\epsilon^5 \\ & +549.301665961614269417 05\epsilon^6 - 1164.51205889715216235 46\epsilon^7 + \mathcal{O}(\epsilon^8) \end{aligned}$$

Towards an answer

- collect contributions to $p(T)$ from all physical scales
  - ▷ *weak coupling, effective field theory setup*
  - ▷ *faithfully adding up all Feynman diagrams*
  - ▷ *get long-distance input from clean lattice observable*

- obtain theory prediction for $p(T)$ [$g^2 = 4\pi\alpha_{strong}$]

$$\frac{p_{\mathsf{QCD}}(T)}{p_{\mathsf{SB}}} = \frac{p_{\mathsf{E}}(T)}{p_{\mathsf{SB}}} + \frac{p_{\mathsf{M}}(T)}{p_{\mathsf{SB}}} + \frac{p_{\mathsf{G}}(T)}{p_{\mathsf{SB}}} \quad , \quad p_{\mathsf{SB}} = \left(16 + \frac{21}{2}N_f\right)\frac{\pi^2 T^4}{90}$$

$$= 1 + g^2 + g^4 + g^6 + \ldots \quad \Leftarrow \text{4d QCD}$$

$$+ g^3 + g^4 + g^5 + g^6 + \ldots \quad \Leftarrow \text{3d adj H}$$

$$+ \frac{1}{p_{\mathsf{SB}}}\frac{T}{V}\int \mathcal{D}[A_k^a]\exp\Big(-S_{\mathsf{M}}\Big) \quad \Leftarrow \text{3d YM}$$

$$= c_0 + c_2 g^2 + c_3 g^3 + (c_4' \ln g + c_4) g^4 + c_5 g^5 + (c_6' \ln g + c_6) g^6 + \mathcal{O}(g^7)$$

[c_2 Shuryak 78, c_3 Kapusta 79, c_4' Toimela 83, c_4 Arnold/Zhai 94, c_5 Zhai/Kastening 95, Braaten/Nieto 96, c_6' KLRS 03]

Conclusions

- we have a working description of nature: gauge/gravity/Higgs systems
- fundamentals of gauge theory (here mostly: QCD) are simple and elegant
- they can directly describe physical behavior of matter under extreme conditions
- QCD contains an extremely rich structure
- thermodynamic quantities of QCD are relevant for cosmology and heavy ion collisons
- these quantities can be determined numerically at $T \sim 200$ MeV, and analytically at $T \gg 200$ MeV
- for precise results, sometimes need very involved mathematical tools interdisciplinary effort ($\longrightarrow$ see next slide)
- there is lots of excitement in advancing our understanding ($\longrightarrow$ LHC) ... and lots to do!

Invitation

International Workshop

14-16 June 2007

ZiF Bielefeld

"Frontiers in perturbative quantum field theory"

Kögerler / Laine / Schröder

Topics:

number theory, algebraic field theory

symbolic and numerical computation

cosmology, heavy ion physics, particle physics