THE PRESSURE OF HOT QCD*

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Progress in the field of thermal QCD is obstructed by the presence of severe infrared divergences. We sketch the basic principles of our methods to overcome this problem, merging perturbative calculations with lattice Monte-Carlo estimates, in an effective theory framework.

1. Introduction

The theory of strong interactions, Quantum Chromodynamics (QCD), is guaranteed to be accessible to perturbative methods once one of its parameters, the temperature T, is increased towards asymptotically high values. In practice, however, calculations of corrections to the behavior of an ideal gas of quarks and gluons, the limit that is formally realized at infinite T, where due to asymptotic freedom the gauge coupling $g(T)$ vanishes, are obstructed by severe infrared (long-distance) divergences: As pointed out by Linde[1], for every observable one sets out to compute, there exists an order of the perturbative expansion to which an infinite number of Feynman diagrams contribute.

The aim of this contribution is to review our methods which are designed to deal with the effects of long-range modes on thermodynamic observables in QCD, in particular the pressure, by combining perturbative methods with 3-dimensional (3d) lattice Monte-Carlo techniques. Necessarily, we have to be brief on technical details here, but nevertheless try to be precise in giving the background needed to interpret the main results. Since this also means that we will be particularly unfair by refraining from discussing

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related work using weak-coupling techniques, let us refer to Ref. 2 for a recent review of those.

This paper is organized as follows. In Section 2, we will discuss full 4d approaches to the QCD pressure, summarizing the essence of perturbative as well as lattice Monte-Carlo calculations. Section 3 introduces the concept of dimensionally reduced effective theories, and covers our setup to estimate contributions from the *soft* sector of the theory on the lattice, while treating the *hard* ones perturbatively[3]. Finally, in Section 4 we give an outline of how a further reduction step, leading to an effective theory for the *ultrasoft* modes, singles out a pure number to be measured on the lattice, in order to overcome the infrared problems which so far deny any computation of the pressure at and beyond 4-loop order[4].

2. Four-dimensional treatment

The QCD pressure is defined as the negative free energy density in the thermodynamic (infinite volume) limit:

$$\begin{aligned} p_{\rm QCD}(T) &= \lim_{V\to\infty} \frac{T}{V} \ln \int \mathcal{D}\left[A_\mu, \overline{\psi}, \psi\right] e^{-\int_0^{1/T} {\rm d}\tau \int {\rm d}^3x \mathcal{L}_{\rm Eucl}^{\rm 4d\ QCD}(g^2)} \\ &= T^4 \hat{p}_{\rm QCD}(g^2) \; . \end{aligned} \tag{1}$$

Here, we have defined $\hat{p}$ to be the dimensionless pressure. In this paper, all dimensionless quantities will be marked by hats. The Euclidean QCD Lagrangian reads

$$\mathcal{L}_{\rm Eucl}^{\rm 4d\ QCD} = \frac{1}{4}\, G^a_{\mu\nu} G^a_{\mu\nu} + \bar{q}\gamma_\mu D_\mu q \; , \tag{2}$$

with field strength tensor $G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$ and gauge-covariant derivative $D_\mu = \partial_\mu - igA^a_\mu T^a$.

A perturbative (hard thermal) loop expansion of Eq. (1) has been carried out up to N^{4}LO (that is g^5). Systematically describing the corrections to the ideal gas limit of QCD with n_f fermion flavors

$$p_0(T) = \frac{\pi^2}{45}\, T^4 \left(8 + \frac{21}{4}\, n_f\right) \; , \tag{3}$$

at (asymptotically) large temperatures, the series unfortunately does not behave well at all practically relevant temperatures, as can be seen from Figure 1. In fact, it oscillates wildly, does not converge below astronomically high temperatures, and shows a scale dependence which increases at higher orders. Furthermore, it is well known that, starting at the order g^6, the

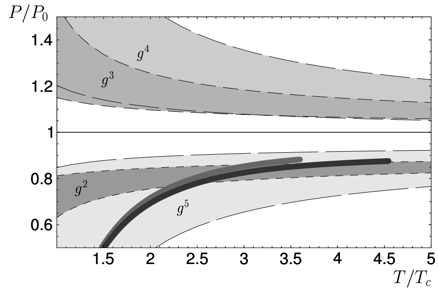

Figure 1. Perturbative results for $p_{\rm QCD}$ with $n_f = 0$, normalized to the ideal-gas value p_0, as a function of T/T_c (assuming $T_c/\Lambda_{\overline{\rm MS}} = 1.14$)[2]. The grey bands correspond to the pressure including order g^2, g^3, g^4, and g^5, respectively, reflecting a variation of the renormalization scale $\pi T < \bar{\mu} < 4\pi T$. The thick dark-grey and light-grey lines show continuum-extrapolated lattice results[5,6].

hard thermal loop resummation still leaves an infinite class of diagrams (infrared) divergent[1]. No practical method is known to resum this class, whence perturbation theory hits an unsurmountable wall at this order, rendering all g^6 (and higher) contributions non-perturbative.

Another strategy to evaluate Eq. (1) as it stands is to estimate it numerically by Monte-Carlo simulations on a space-time lattice. While the partition function and hence the pressure itself is not directly accessible due to the importance sampling property of Monte Carlo simulations, its derivatives are. Concretely, taking a derivative of Eq. (1) with respect to the gauge coupling g^2 relates the derivative of the pressure to the vacuum expectation value of the field strength tensor. Hence, estimating the average plaquette allows to integrate back to the pressure,

$$\hat{p}_{\rm QCD}\left(g^2(T)\right) \propto \int_{g^2(T_0)}^{g^2(T)} \frac{{\rm d}g^2}{g^2}\, \langle {\rm Tr}(\mathbb{1} - \Box)\rangle_{\rm QCD} + \hat{p}_{\rm QCD}\left(g^2(T_0)\right) \ . \quad (4)$$

While this approach is a truly first-principles one, in practice it is limited by the finiteness of computer resources to one's disposal. Approximating the theory on a $V \times 1/T = (aN_\sigma)^3 \times aN_\tau$ lattice worldvolume, taking the continuum limit $a \to 0$ while still working in the thermodynamic limit requires to take $N_\sigma \gg N_\tau \gg 1$. Furthermore, treating the contributions of dynamical fermions is a problem that gets harder exponentially as the volume grows. Finally, the integration constant has to be fixed at low temperatures, leading to a (small) ambiguity.

As can be seen from Figure 2, lattice results are available from around the critical temperature T_c, where the system undergoes a deconfining phase

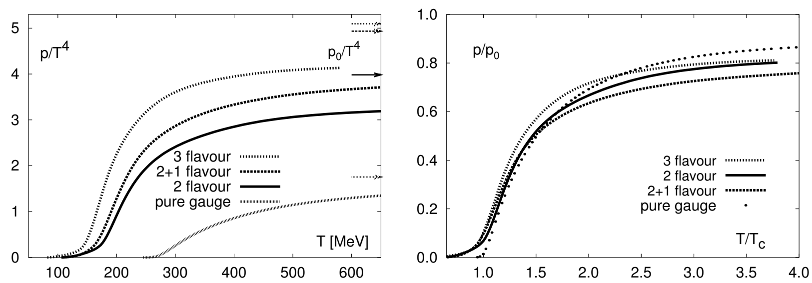

Figure 2. Monte-Carlo results for $p_{\rm QCD}$ with $n_f = 0$, 2 and 3 light quarks as well as two light and a heavier (strange) quark[7]. The $n_f = 0$ data is extrapolated to the continuum limit[5], while the $n_f \neq 0$ data is expected to be close to its continuum limit due to the use of improved lattice actions[8]. Arrows indicate the ideal gas pressure p_0 for each case.

transition, up to a few times T_c. Scaling the data, which has been taken for different fermionic content of the theory, with the appropriate factors p_0 and T_c (see the right panel of Fig. 2), it almost falls on a universal curve. This remarkable feature can be interpreted as signalling the dominant role of the pure gauge sector (and we take it as a motivation to show results for $n_f = 0$ only in the following sections) during the phase transition. Comparing with Fig. 1, it is however far from obvious whether the approach to the ideal gas limit can be treated within a weak-coupling expansion. In the following two sections, we will outline two approaches of how this can be achieved.

3. Dimensionally reduced effective theory I

At high temperatures $T \gg m_{\rm quark}$, the physics of QCD is governed by three distinct scales. First, compactification of the (imaginary) time-axis leads to the occurrence of Matsubara modes, with typical masses of order of the temperature, πT. While this is true for all fermions, gluons possess also a zero mode. Second, dynamical effects lead to a screening of longitudinal (zero-mode) gluons, coined the 'Debye mass', which is of order gT. Third, the non-perturbative infrared sector is screened dynamically by a 'magnetic mass' of order g^2T. It will become clear in Section 4 why this is the only natural choice.

Due to asymptotic freedom of QCD, we are guaranteed that the coupling constant $g(T)$ be small at large temperatures. One then has a hierarchy among the three physical scales, and typically refers to them as *hard*, *soft* and *ultrasoft* scales. It therefore seems very natural to look at contributions to Eq. (1) from the different scales in an effective theory framework.

As a first step, let us exploit the scale separation $\pi T \gg gT$, and integrate out the hard modes $\sim \pi T$. This amounts to treating all fermions as well as all non-zero-mode gluons as heavy. One arrives at an effective theory for the soft (as well as ultrasoft) modes, which is a (3d) gauge + adjoint Higgs theory (labelled AH below),

$$\mathcal{L}_{\rm Eucl}^{\rm 3d\ AH} = \frac{1}{4}\, G_{ij}^a G_{ij}^a + \frac{1}{2}\,(D_i A_0)^a (D_i A_0)^a + V(A_0^a) + \delta\mathcal{L}_{\rm AH}\ , \qquad (5)$$

with magnetostatic field strength tensor $G_{ij}^a = \partial_i A_j^a - \partial_j A_i^a + g_E f^{abc} A_i^b A_j^c$, Higgs potential[a] $V(A_0^a) = \frac{1}{2}\, m_E^2 A_0^a A_0^a + \frac{1}{8}\,\lambda_E (A_0^a A_0^a)^2$ and $\delta\mathcal{L}_{\rm AH}$ collecting all other local gauge-invariant operators of dimension 3 and higher that can be constructed out of A_0 and A_i. Note that in 3d, the couplings $g_{\rm E}^2$ and $\lambda_{\rm E}$ acquire the dimension of a mass.

The QCD pressure can then be rewritten as

$$\begin{aligned} p_{\rm QCD}(T) &= p_{\rm E}(T) + T \lim_{V\to\infty} \frac{1}{V} \ln \int \mathcal{D}\,[A_i, A_0]\, e^{-\int {\rm d}^3x \mathcal{L}_{\rm Eucl}^{\rm 3d\ AH}(g_{\rm E}^2, \lambda_{\rm E}, m_{\rm E}^2, \dots)} \\ &= T^4 \hat{p}_{\rm E}(g^2) + T m_{\rm E}^3 \hat{p}_{\rm AH}\left(\frac{\lambda_{\rm E}}{g_{\rm E}^2}, \frac{m_{\rm E}^2}{g_{\rm E}^4}, \dots\right)\ . \end{aligned} \qquad (6)$$

The ellipsis in Eq. (6) stand for parameters of $\delta\mathcal{L}_{\rm AH}$, whose omission introduces, using a very conservative estimate, maximally an $\mathcal{O}(g^7)$ error in the pressure. Being mainly interested in effects up to order g^6, we will neglect it completely in the following discussion.

The matching coefficients are functions of the parameters of Eq. (2), and can be computed as power series in g^2,

$$\left\{p_{\rm E}, g_{\rm E}^2, \lambda_{\rm E}, m_{\rm E}^2\right\} = \left\{T^4, g^2 T, g^4 T, g^2 T^2\right\} \times (1 + g^2 + \dots)\ . \qquad (7)$$

Note that the effect of fermions resides in these coefficients only. The effective theory is purely bosonic, fermions are 'integrated out'.

Now, the 3d effective theory is confining, hence non-perturbative[b]. The most straightforward idea is then to attempt a lattice-measurement of $\hat{p}_{\rm AH}$. Again, since one cannot measure the partition function directly on the lattice, one has to take derivatives. While in principle the pressure $p_{\rm AH}$ of the 3d theory does depend on two parameters, in the context of dimensional reduction they are both functions of $g(T)$ only, hence related. Introducing

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[a] In general there are two quartic couplings, corresponding to the operators ${\rm Tr}(A_0^2)^2$ and $({\rm Tr}A_0^2)^2$, which are however related in SU(2) and SU(3).

[b] It is instructive to note at this point that, expanding $\hat{p}_{\rm AH}$ perturbatively, the 4d series known up to g^5 can be reproduced exactly, albeit with considerably less effort[9]. Note however that any attempt to go further will again hit the infrared wall mentioned above.

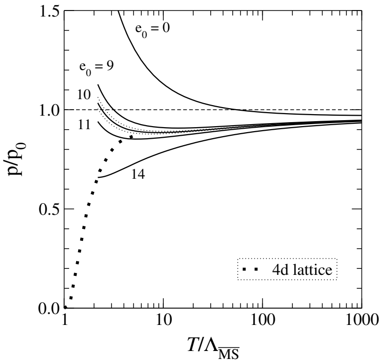

Figure 3. Results for $p_{\rm QCD}$ with $n_f = 0$ according to Eq. (6), allowing for a free parameter e_0 summarizing the effect of unknown higher-order terms[3], compared with 4d lattice data[5]. The dotted lines around the $e_0 = 10$ curve are typical error bars, stemming from estimating the condensate $\langle {\rm Tr} A_0^2 \rangle$ on the lattice. For more details, see Section 3.

the dimensionless couplings $\hat\lambda = \lambda_{\rm E}/g_{\rm E}^2$ and $\hat m^2 = m_{\rm E}^2/g_{\rm E}^4$, the 3d pressure can be related to A_0 condensates

$$\hat p_{\rm AH} \propto \left(\frac{1}{\hat m^2}\right)^{\frac{3}{2}} \int_{\hat m^2(T_0)}^{\hat m^2(T)} {\rm d}\hat m^2 \left[\left\langle {\rm Tr}\hat A_0^2 \right\rangle_{\rm AH} + (\partial_{\hat m^2}\hat\lambda) \left\langle ({\rm Tr}\hat A_0^2)^2 \right\rangle_{\rm AH}\right] \\ + \hat p_{\rm AH}\left(\hat\lambda(T_0), \hat m^2(T_0)\right) \ . \tag{8}$$

The integration constant $\hat p_{\rm AH}(T_0)$ can be fixed at (very) high temperature, where it is purely perturbative, hence systematically improvable. Its expansion is equivalent to $\hat p_{\rm M}$ below, evaluated at a fixed temperature. The lattice measurement is then used to integrate down, from T_0 to T.

In practice, we choose $T_0 = 10^{11}\Lambda_{\overline{\rm MS}}$. The loop expansion parameter for the matching coefficients Eq. (7) is $3g^2/(4\pi)^2 \approx \alpha_s/4 \approx \{0.1 \ldots 0.025\}$ for $T = \{4 \ldots 10^{11}\}\Lambda_{\overline{\rm MS}}$, while for the integration constant $\hat p_{\rm AH}(T_0)$ it reads

$$\hat l_0 \equiv \frac{3g_{\rm E}^2(T_0)}{4\pi m_{\rm E}(T_0)} \approx 0.1 \ , \tag{9}$$

shedding light on the origin of the bad convergence seen in Fig. 1. The other parameter of $\hat p_{\rm AH}(T_0)$ turns out to be truly small at T_0, $\lambda_{\rm E}(T_0)/(3g_{\rm E}^2(T_0)) \approx 0.01$, such that for clarity we will suppress it below.

For results of implementing Eq. (6) see Figure 3. The plot includes:
(a) A lattice measurement of $\langle {\rm Tr} A_0^2 \rangle$ for $2\Lambda_{\overline{\rm MS}} < T < 10^{11}\Lambda_{\overline{\rm MS}}$, matched to the $\overline{\rm MS}$ scheme via a 2-loop calculation in lattice perturbation theory. The effect of $\langle ({\rm Tr} A_0^2)^2 \rangle$ is parametrically subleading, see Eq. (8), and has not

yet been included; it would require 4-loop matching. The data has been taken on $12^3 \dots 80^3$ lattices, for $\beta = 2 \cdot 3/(ag_{\rm E}^2) = 12 \dots 120$, to allow for continuum- and infinite-volume extrapolations.
(b) The matching coefficient $\hat{p}_{\rm E}$ and integration constant $\hat{p}_{\rm AH}(T_0)$ at $T_0 = 10^{11}\Lambda_{\overline{\rm MS}}$ to 3-loop order in the $\overline{\rm MS}$ scheme.
(c) A free parameter e_0 to test the sensitivity on unknown terms of 4-loop and higher order in both $\hat{p}_{\rm AH}(T_0)$ and $\hat{p}_{\rm E}$

$$e_0 = \Big(e_1 \ln \hat{l}_0 + e_2\Big) + e_3 \;, \tag{10}$$

where $\hat{l}_0$ is the effective expansion parameter of $\hat{p}_{\rm AH}$ at T_0, see Eq. (9), and the coefficients e_i are the 4-loop (g^6) terms:

$$\hat{p}_{\rm AH} = \left(\frac{g_{\rm E}^2}{m_{\rm E}}\right)^3 \left([\hat{l}_0^{-3} + \hat{l}_0^{-2} + \hat{l}_0^{-1}] - \frac{8 \cdot 3^3}{(4\pi)^4} \Big(e_1 \ln \hat{l}_0 + e_2\Big) \Big[1 + \mathcal{O}\Big(\hat{l}_0\Big)\Big] \right)$$

$$\hat{p}_{\rm E} = \big[1 + g^2 + g^4\big] - g^6 \frac{8 \cdot 3^3}{(4\pi)^4} (e_3) \big[1 + \mathcal{O}\left(g^2\right)\big] \;. \tag{11}$$

It is interesting to note that, using the recently computed[10] value of $e_1 = -8\alpha_{\rm M} \approx -4.44$, already the first of the three contributions to Eq. (10) gives a number which seems to have the correct order of magnitude expected from Fig. 3: $e_0|_{e_1} \approx 10.2$. Indeed, as could have been expected, a large contribution is made by this logarithmically enhanced term $\propto g_{\rm E}^6 \ln \hat{m} \sim g^6 \ln g$. It remains to be seen, however, whether the other two terms are parametrically small.

Finally, let us point out that, in contrast to full QCD lattice simulations needed in the four-dimensional case, one has a purely bosonic theory here. Fermionic effects are in the coefficients only, and can be treated perturbatively. Obviously, this is a tremendous simplification of lattice measurements according to Eq. (8).

4. Dimensionally reduced effective theory II

As a next step, exploiting the hierarchy $gT \gg g^2T$, the adjoint Higgs field A_0 can also be integrated out, whereby one arrives at an effective theory for the ultrasoft modes, which to lowest order is a (3d) pure Yang-Mills theory

$$\mathcal{L}_{\rm Eucl}^{\rm 3d\ YM} = \frac{1}{4}\, G_{ij}^a G_{ij}^a + \delta\mathcal{L}_{\rm M} \;. \tag{12}$$

$\delta\mathcal{L}_{\rm M}$ includes all possible local gauge-invariant operators of dimension 5 and higher that can be constructed out of A_i^a.

The QCD pressure can then be rewritten as

$$\begin{aligned} p_{\rm QCD}(T) &= p_{\rm E}(T) + p_{\rm M}(T) + T \lim_{V\to\infty} \frac{1}{V} \ln \int \mathcal{D}\,[A_i]\, e^{-\int {\rm d}^3x \mathcal{L}^{\rm 3d\ YM}_{\rm Eucl}(g^2_{\rm M},\dots)} \\ &= T^4 \hat{p}_{\rm E}(g^2) + T m_{\rm E}^3 \hat{p}_{\rm M}\left(\frac{g^2_{\rm E}}{m_{\rm E}}, \frac{\lambda_{\rm E}}{g^2_{\rm E}}, \dots\right) + T g^6_{\rm M} \hat{p}_{\rm G}(\dots)\;. \end{aligned} \tag{13}$$

Again, the ellipsis stand for parameters of higher-order operators collected in the $\delta\mathcal{L}$ which will be neglected below. For the pressure, omission of $\delta\mathcal{L}_{\rm M}$ introduces an $\mathcal{O}(g^9)$ error at most.

The matching coefficients are functions of the parameters of Eq. (5), and can be computed as power series in $g^2_{\rm E}/m_{\rm E} \sim g$ and $\lambda_{\rm E}/g^2_{\rm E} \sim g^2$,

$$\left\{p_{\rm M}, g^2_{\rm M}\right\} = \left\{T m^3_{\rm E}, g^2_{\rm E}\right\} \times \left(1 + \frac{g^2_{\rm E}}{m_{\rm E}}\left(1 + \frac{\lambda_{\rm E}}{g^2_{\rm E}}\right) + \dots\right)\;. \tag{14}$$

Now, the only non-perturbative input needed is $\hat{p}_{\rm G}$, which is a pure number if one neglects the higher-order operators $\delta\mathcal{L}_{\rm M}$. Being the pressure of 3d pure gauge theory, which contains a dimensionful coupling as its only parameter, it is hence measurable exactly,

$$\hat{p}_{\rm G} \propto \langle {\rm Tr}(1\!\!1 - \Box)\rangle_{\rm 3d\ YM}\;. \tag{15}$$

To re-iterate: here, no integration and hence no integration constant are required. Note that in principle, once $\hat{p}_{\rm G}$ is known, the infrared (Linde) problem of thermal QCD is overcome: everything else is perturbative. However, pushing the expansion to order g^9 and beyond will require inclusion of higher-order operators in $\delta\mathcal{L}_{\rm M}$. A detailed analysis of how they can change the general strategy outlined here remains to be carried out.

The key element in the realization of a measurement of $\hat{p}_{\rm G}$ is a purely dimensional argument. As a dimensionless quantity, the lattice-regularized version of $\hat{p}_{\rm G}$ will possess an expansion in powers of $ag^2_{\rm M}$ near the continuum limit $a \to 0$

$$\begin{aligned} &\lim_{V\to\infty}\frac{1}{V}\ln\int \mathcal{D}U e^{-\frac{1}{ag^2_{\rm M}}{\rm Tr}(1\!\!1-\Box)} = \\ &\qquad g^6_{\rm M}\left[\left(\frac{1}{ag^2_{\rm M}}\right)^3 + \dots + \left(\ln\frac{1}{ag^2_{\rm M}} + \#_{\rm G}\right) + \mathcal{O}(a)\right]\;. \end{aligned} \tag{16}$$

All divergences on the right-hand side of Eq. (16) are computable in lattice perturbation theory. As a manifestation of superrenormalizability, there is only a finite number of them. Now, taking logarithmic derivatives with respect to the gauge coupling on both sides of Eq. (16), one finds that the

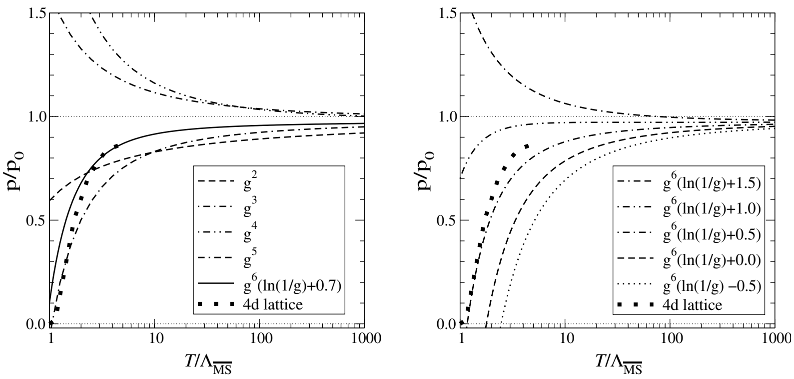

Figure 4. Results for $p_{\rm QCD}$ with $n_f = 0$ according to Eq. (13)[4], compared to 4d lattice results[5]. Left: perturbative results at various orders, including an 'optimal' choice for the $\mathcal{O}(g^6)$ constant. Right: dependence of the $\mathcal{O}(g^6)$ result on the (not yet computed) constant, which contains both perturbative and non-perturbative contributions.

averaged plaquette, after subtraction of divergences, matches the unknown constant in the continuum limit,

$$\langle {\rm Tr}(\mathbb{1} - \Box) \rangle \propto g_{\rm M}^6 \#_{\rm G} + \mathcal{O}(a) \; . \tag{17}$$

Finally, one of course needs to relate lattice and continuum regularization schemes, which amounts to a finite-order (again due to superrenormalizability) computation, in both schemes, at the g^6 level, including constant terms. On the lattice side, an interesting approach of tackling this is stochastic perturbation theory[11].

For results of implementing the strategy Eq. (13) see Figure 4. The plots include the matching parameters up to four-loop logarithms and a free parameter ("0.7") to test the sensitivity of $\hat{p}_{\rm QCD}$ on the unknown g^6 terms. Schematically, the free parameter arises from the three different scales as follows:

$$\begin{array}{lllll}
\hat{p}_{\rm E}: & 1 + g^2 & +g^4 & +g^6 \#_{\rm E} & +\mathcal{O}(g^8) \\
\hat{p}_{\rm M}: & & +g^3 + g^4 \ln g + g^4 + g^5 + g^6 \ln g & +g^6 \#_{\rm M} + \mathcal{O}(g^7) & \\
\hat{p}_{\rm G}: & & & +g^6 \#_{\rm G} & +\mathcal{O}(g^9)
\end{array}$$

Many terms contributing to $\#_{\rm E}, \#_{\rm G}, \#_{\rm M}$ are already known, while others are precisely defined[4]. It seems that the most challenging open computation, apart from the determination of $\hat{p}_{\rm G}$ like outlined above, is the constant part of the 4-loop terms in $\hat{p}_{\rm E}$.

5. Outlook

It should have become clear that it is possible to separate contributions to thermodynamic quantities emerging from different physical scales in a dimensionally reduced effective theory framework. The major benefit of working with the 3d effective theories lies in their universality: being purely bosonic, they encode the effects of fermions in their coefficients. Hence, none of the usual complications that the fermionic sector poses in a non-perturbative treatment of the theory apply.

Still, realizing the numerical extraction of the yet-unknown parameters emerging from the infrared sector is a challenging open problem even in these 3d bosonic effective theories. We have outlined two strategies worth following when tackling this problem. A successful demonstration in the case of the QCD pressure, would establish a nice example of computing an observable beyond the 'infrared wall'. Once this has been achieved, thermal QCD in its high-temperature phase will again be amenable to perturbative calculations, opening up numerous opportunities to precisely compute observables that might become relevant to the RHIC program, to future accelerators, and to cosmology.

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