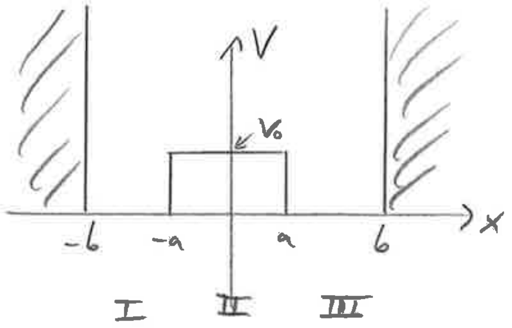
1



zeitunabh. SG , $k_I = \sqrt{\frac{2mE}{\hbar^2}}$, $k_{II} = \sqrt{\frac{2m(E-V_0)}{\hbar^2}}$

$\psi_I'' = -k_I^2 \psi_I$; $\psi_{II}'' = -k_{II}^2 \psi_{II}$; $\psi_{III}'' = -k_I^2 \psi_{III}$

Lsg. $\psi_I = c_1 e^{ik_I x} + c_2 e^{-ik_I x}$;

$$\left.\begin{array}{l} \psi_{II} = c_3\left(e^{ik_{II}x} + e^{-ik_{II}x}\right) \\ \left(\psi_{III} = c_1 e^{-ik_I x} + c_2 e^{ik_I x}\right) \end{array}\right\} \text{ wegen } \psi(x) = \psi(-x) \text{ gerade}$$

Randbed.: $0 \doteq \psi_I(-b) = c_1 e^{-ik_I b} + c_2 e^{ik_I b}$ (1)

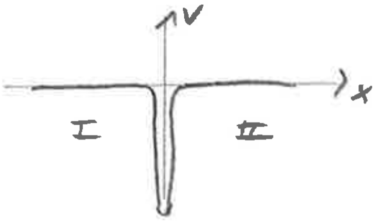
ψ stetig @ $-a$: $c_1 e^{-ik_I a} + c_2 e^{ik_I a} = c_3\left(e^{-ik_{II}a} + e^{ik_{II}a}\right)$ (2)

ψ' stetig @ $-a$: $ik_I\left(c_1 e^{-ik_I a} - c_2 e^{ik_I a}\right) = ik_{II} c_3\left(e^{-ik_{II}a} - e^{ik_{II}a}\right)$ (3)

teile (3) durch (2), benutze (1), $e^{ix} = \cos x + i \sin x$, ...

$\Rightarrow k_I \cot\left(k_I(b-a)\right) = k_{II} \tan\left(k_{II} a\right)$

2a



zeitunabh. SG, $H\psi = E\psi$, $(\partial_x^2 + V_0\delta)\psi = -\frac{2mE}{\hbar^2}\psi$

$$\Rightarrow \int_{-\varepsilon}^{\varepsilon} dx\,(\psi'' + V_0\delta\psi) = -\frac{2mE}{\hbar^2}\int_{-\varepsilon}^{\varepsilon} dx\,\psi$$

$\varepsilon \to 0$: $\psi'(0^+) - \psi'(0^-) + V_0\psi(0) = 0$;

ψ stetig bei $x=0$ (denn sonst $\psi' \ni \delta$, $\psi'' \ni \delta'$, aber $V \not\ni \delta'$)

$\Rightarrow \psi(0^-) = \psi(0^+)$

2b

Bindungszust., $E<0$, $\kappa = \sqrt{-\frac{2mE}{\hbar^2}} > 0$,

SG $(\partial_x^2 + V_0)\psi = \kappa^2\psi$

Lsg $\psi_I = c_1 e^{\kappa x}$ (wegen $\psi_I(-\infty)=0$ kein $e^{-\kappa x}$)

$\psi_{II} = c_2 e^{-\kappa x}$ (wegen $\psi_{II}(\infty)=0$ kein $e^{+\kappa x}$)

aus 2a: $\psi(0^-) = \psi(0^+) \Rightarrow c_1 = c_2 \equiv c$

$\psi'(0^-) = c\kappa = \psi'(0^+) + V_0\psi(0) = c(-\kappa) + V_0 c$

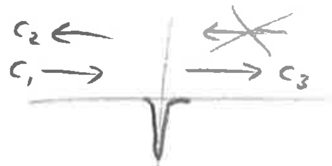
$\Rightarrow \kappa = V_0/2$, $E = -\frac{\hbar^2 V_0^2}{8m}$,

Norm $\frac{1}{2\cdot|c|^2} = \int_0^\infty dx\, e^{-2\kappa x} = \frac{1}{-2\kappa} e^{-2\kappa x}\Big|_0^\infty = 0 - \frac{1}{-2\kappa} = \frac{1}{2\kappa}$

$\Rightarrow \psi(x) = \sqrt{\frac{V_0}{2}}\, e^{-V_0|x|/2}$

2c

jetzt $E>0$, $k = \sqrt{\frac{2mE}{\hbar^2}} > 0$,



SG $(\partial_x^2 + V_0\delta)\psi = -k^2\psi$

$\psi_I = c_1 e^{ikx} + c_2 e^{-ikx}$, $\psi_{II} = c_3 e^{ikx}$

aus 2a: $c_1 + c_2 = c_3$; $ik(c_1 - c_2) = ik c_3 + V_0 c_3$

$\Rightarrow 2c_1 = c_3\left(2 + \frac{V_0}{ik}\right)$, $|c_1|^2 = |c_3|^2\left(1 + \frac{V_0^2}{4k^2}\right)$, $T = \frac{1}{1 + V_0^2/4k^2}$

$R = 1 - T = \frac{1}{1 + 4k^2/V_0^2}$

3a zeitabhängige SG, $i\hbar \partial_t \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = -\mu B \frac{\hbar}{2} \begin{pmatrix} \psi_1 \\ -\psi_2 \end{pmatrix}$

$$\Rightarrow \psi(t) = \begin{pmatrix} \cos\frac{\alpha}{2} \cdot e^{i\mu Bt/2} \\ \sin\frac{\alpha}{2} \cdot e^{-i\mu Bt/2} \end{pmatrix}$$

3b

$$\langle S_x \rangle = \frac{\hbar}{2} (\psi_1^*, \psi_2^*) \begin{pmatrix} \psi_2 \\ \psi_1 \end{pmatrix} = \frac{\hbar}{2} (\psi_1^* \psi_2 + \psi_2^* \psi_1)$$

$$= \frac{\hbar}{2} \cos\frac{\alpha}{2} \sin\frac{\alpha}{2} \left(e^{-i\mu Bt} + e^{i\mu Bt} \right) = \hbar \cos\frac{\alpha}{2} \sin\frac{\alpha}{2} \cos(\mu Bt)$$

$$\langle S_y \rangle = -\frac{i\hbar}{2} (\psi_1^* \psi_2 - \psi_2^* \psi_1) = -\hbar \cos\frac{\alpha}{2} \sin\frac{\alpha}{2} \sin(\mu Bt)$$

$$\langle S_z \rangle = \frac{\hbar}{2} (\psi_1^* \psi_1 - \psi_2^* \psi_2) = \frac{\hbar}{2} \left(\cos^2\frac{\alpha}{2} - \sin^2\frac{\alpha}{2} \right) = \frac{\hbar}{2} \cos\alpha$$

4

$$E_0 \leq E_\varphi = \int dx\, N^* e^{-\alpha^2 x^2/2} \left(-\frac{\hbar^2}{2m} \partial_x^2 + \lambda x^4 \right) N e^{-\alpha^2 x^2/2}$$

Norm $\frac{1}{|N|^2} = \int dx\, e^{-\alpha^2 x^2} = \frac{\sqrt{\pi}}{\alpha}$

$$= \frac{\alpha}{\sqrt{\pi}} \int dx\, e^{-\alpha^2 x^2} \left(-\frac{\hbar^2}{2m} \left[-\alpha^2 + \alpha^4 x^2 \right] + \lambda x^4 \right)$$

$$= \frac{\alpha}{\sqrt{\pi}} \left\{ \frac{\hbar^2 \alpha^2}{2m} \int dx\, e^{-\alpha^2 x^2} + \frac{\hbar^2 \alpha^4}{2m} \underbrace{\int dx\, (-x^2) e^{-\alpha^2 x^2}}_{\stackrel{!}{=} \partial_{\alpha^2} \sqrt{\frac{\pi}{\alpha^2}} = -\frac{\sqrt{\pi}}{2\alpha^3}} + \lambda \underbrace{\int dx\, (-x^2)^2 e^{-\alpha^2 x^2}}_{\stackrel{!}{=} \partial_{\alpha^2}^2 \sqrt{\frac{\pi}{\alpha^2}} = +\frac{3\sqrt{\pi}}{4\alpha^5}} \right\}$$

$$= \frac{\hbar^2 \alpha^2}{4m} + \frac{3\lambda}{4\alpha^4}$$

Min? $0 \stackrel{!}{=} \partial_{\alpha^2} E_\varphi = \frac{\hbar^2}{4m} - \frac{3\lambda}{2\alpha^6} \quad \Rightarrow \alpha^6 = \frac{6\lambda m}{\hbar^2}$

Min oder Max? $\partial_{\alpha^2}^2 E_\varphi = 0 + \frac{9\lambda}{2\alpha^8} > 0$ ✓ Min

$$\Rightarrow E_{\varphi,\min} = \left(\frac{\hbar^4 \lambda}{4m^2} \right)^{1/3} \boxed{\frac{3^{4/3}}{4}} \quad (\approx 1.08169\ldots)$$

Abweichung (2%) nach oben.

5a

$$\langle S_x \rangle = \mathrm{Sp}(\rho S_x) = \frac{\hbar}{8}\,\mathrm{Sp}\left(\begin{pmatrix} 3 & i \\ -i & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\right)$$

$$= \frac{\hbar}{8}\,\mathrm{Sp}\begin{pmatrix} i & \cdot \\ \cdot & -i \end{pmatrix} = 0$$

$$\langle S_z \rangle = \mathrm{Sp}(\rho S_z) = \frac{\hbar}{8}\,\mathrm{Sp}\left(\begin{pmatrix} 3 & i \\ -i & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\right)$$

$$= \frac{\hbar}{8}\,\mathrm{Sp}\begin{pmatrix} 3 & \cdot \\ \cdot & -1 \end{pmatrix} = \frac{\hbar}{4}$$

$$\mathrm{Sp}(\rho^2) = \frac{1}{16}\,\mathrm{Sp}\left(\begin{pmatrix} 3 & i \\ -i & 1 \end{pmatrix}\begin{pmatrix} 3 & i \\ -i & 1 \end{pmatrix}\right) = \frac{1}{16}\,\mathrm{Sp}\begin{pmatrix} 10 & \cdot \\ \cdot & 2 \end{pmatrix} = \frac{3}{4} < 1$$

$\Rightarrow$ gemischt

5b

$$\mathrm{Sp}(\rho_t^2) = \mathrm{Sp}(U\rho_0 U^+ U \rho_0 U^+) \overset{U^+U = \mathbb{1}}{=} \mathrm{Sp}(U\rho_0^2 U^+)$$

Sp zyklisch $\searrow$ $= \mathrm{Sp}(U^+U\rho_0^2) = \mathrm{Sp}(\rho_0^2)$

$\Rightarrow$ wenn Zustand zum Zeitpkt t_0 gemischt,
bleibt er gemischt für alle t.