6.2 Axial current non-conservation

→ in §6.1, learned how to properly define $\Delta^{\lambda\mu\nu}$;

now, check axial current conservation by computing $q_\lambda \Delta^{\lambda\mu\nu}(\overset{\text{(fixed by above)}}{a}, b_1, b_2)$

$((= -\frac{1}{2}(b_1-b_2)))$

- $$q_\lambda \Delta^{\lambda\mu\nu}(a, b_1, b_2) = \underbrace{q_\lambda \Delta^{\lambda\mu\nu}(b_1, b_2)} - \underbrace{\frac{i}{8\pi^2}\,\varepsilon^{\lambda\mu\nu\sigma}(b_1+b_2)_\lambda (b_1-b_2)_\sigma}_{=+\frac{i}{4\pi^2}\varepsilon^{\mu\nu\lambda\sigma} b_{1\lambda} b_{2\sigma}}$$

$$= i\int \frac{d^4p}{(2\pi)^4}\, \mathrm{tr}\left(\not{q}\,\gamma^5 \frac{1}{\not{p}-\not{q}} \gamma^\nu \frac{1}{\not{p}-\not{b}_1} \gamma^\mu \frac{1}{\not{p}} + \begin{pmatrix} \mu\leftrightarrow\nu \\ b_1\leftrightarrow b_2 \end{pmatrix} \right)$$

$\not{q} = \not{p} - (\not{p}-\not{q})$; use cyclicity of trace; $\{\gamma^\mu, \gamma^5\} = 0$

$$= i\int \frac{d^4p}{(2\pi)^4}\, \mathrm{tr}\left(\gamma^\mu\gamma^5 \frac{1}{\not{p}-\not{q}} \gamma^\nu \frac{1}{\not{p}-\not{b}_1} - \gamma^\nu\gamma^5 \frac{1}{\not{p}-\not{b}_1}\gamma^\mu \frac{1}{\not{p}} + \begin{pmatrix} \mu\leftrightarrow\nu \\ b_1\leftrightarrow b_2 \end{pmatrix} - \begin{pmatrix} \mu\leftrightarrow\nu \\ b_1\leftrightarrow b_2 \end{pmatrix} \right)$$

(pg. 73)

$$= b_{1\sigma}\Delta^{\mu\sigma\nu}(b_1, b_2) + \begin{pmatrix} \mu\leftrightarrow\nu \\ b_1\leftrightarrow b_2 \end{pmatrix} = \frac{i}{8\pi^2}\varepsilon^{\mu\nu\tau\sigma} b_{1\tau} b_{2\sigma} \cdot 2$$

$$= \frac{i}{2\pi^2}\varepsilon^{\mu\nu\lambda\sigma} b_{1\lambda} b_{2\sigma}$$

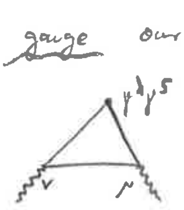
⇒ axial current is <u>not</u> conserved !

this is known as <u>(axial/chiral) anomaly</u> :

quantum fluctuations destroyed the (classical) axial current conservation.

<u>consequences / remarks</u> (w/o derivations)

- <u>gauge</u> our theory $\mathcal{L}_1$: $\boxed{\mathcal{L}_2 \equiv \bar{\psi}\, i\gamma^\mu(\partial_\mu - ie\overset{\text{"photon"}}{A_\mu})\psi}$



$$\rightsquigarrow \partial_\mu j_5^\mu = \begin{cases} 0 & \text{classically} \\ \frac{e^2}{(4\pi)^2}\varepsilon^{\mu\nu\lambda\sigma}F_{\mu\nu}F_{\lambda\sigma} & \text{quantum} \end{cases}$$

((check ?! : $b \to \partial$ in $F = \partial A - \partial A$))

historically important ! decay $\pi^0 \to \overset{\text{(massless)}}{\gamma+\gamma}$ <u>forbidden</u> via (wrong) $\partial_\mu j_5^\mu = 0$,

but decay is <u>observed</u> experimentally, as predicted by (correct) $\partial_\mu j_5^\mu \neq 0$.

($\Gamma_i/\Gamma \approx 98.8\,\%$, see PDG

- re-write $\mathcal{L}_2$ with $\psi_{R/L} \equiv \frac{1 \pm \gamma^5}{2} \psi$,

  introduce left- and right-handed currents $J_{R/L}^{\mu} \equiv \bar{\psi}_{R/L} \gamma^{\mu} \psi_{R/L}$

  $\Rightarrow \partial_{\mu} J_{R/L}^{\mu} = \pm \frac{1}{2} \frac{e^2}{(4\pi)^2} \varepsilon^{\mu\nu\lambda\sigma} F_{\mu\nu} F_{\lambda\sigma}$

  ((that's why the anomaly is called "chiral"))

- add a fermion mass to $\mathcal{L}_2$: $\boxed{\mathcal{L}_3 \equiv \bar{\psi} \left[i\gamma^{\mu} (\partial_{\mu} - ieA_{\mu}) - m \right] \psi}$

  $\Rightarrow$ invariance under $\psi \to e^{i\theta\gamma^5} \psi$ broken by $m \neq 0$.

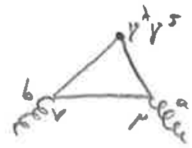
  classically, $\partial_{\mu} J_5^{\mu} = 2m \bar{\psi} i\gamma^5 \psi$, axial current not conserved.

  $\leadsto$ anomaly dictates an additional term (generated by quantum fluct.),

  $$\partial_{\mu} J_5^{\mu} = 2m \bar{\psi} i\gamma^5 \psi + \frac{e^2}{(4\pi)^2} \varepsilon^{\mu\nu\lambda\sigma} F_{\mu\nu} F_{\lambda\sigma}$$

- generalize to non-abelian case : $\boxed{\mathcal{L}_4 = \bar{\psi} i\gamma^{\mu} (\partial_{\mu} - ig A_{\mu}^a T^a) \psi}$

  calculation as before; vertex "μ" gets T^a
  vertex "ν" gets T^b

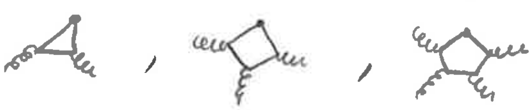
  

  summing over fermions in the loop gives $\mathrm{Tr}(T^a T^b)$

  $$\Rightarrow \partial_{\mu} J_5^{\mu} = \frac{g^2}{(4\pi)^2} \varepsilon^{\mu\nu\lambda\sigma} \mathrm{Tr} \left(F_{\mu\nu} F_{\lambda\sigma} \right)$$

  $\quad F_{\mu\nu}^a T^a$, see § 2.2, pg. 17

  $\leadsto$ since $FF \sim A^2, A^3, A^4$, non-abelian symmetry immediately tells us there are triangle / square / pentagon anomalies in QCD :

- higher orders? e.g. 3 loop  etc.

  expect correction $\sim * \left[1 + \mathrm{fct}(e, g, \ldots) \right]$ ← all couplings of theory

  anomaly nonrenormalization theorem : $\mathrm{fct}(e, g, \ldots) = 0$ (!)

  for a proof see [Adler / Bardeen, Phys. Rev. 182 (1969) 1517]
  [Collins, Renormalization, pg. 352]

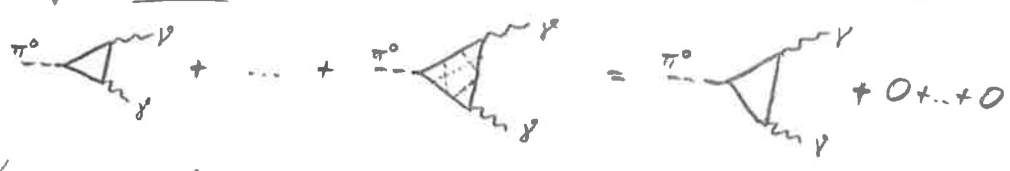
→ we can heuristically <u>understand</u> this:

before integrating over momenta of internal propagators,

the integrand has ≥ 5 fermion propags

⇝ sufficiently <u>convergent</u>, so we can <u>shift</u> momenta naively (cf. p. 73)

→ historically, nonrenormalization of the anomaly important for developing concept of <u>color</u>:

$\pi^0 \to \gamma + \gamma$ ~ 

process could be computed with confidence from <u>one</u> diagram,

decay amplitude does <u>not</u> depend on details of strong interactions;

result was factor of 3 too small ⇒ 3 types of quarks!

- beyond the Standard Model (BSM) – considerations:

  are quarks / leptons composed of more fundamental fermions (preons)?

  ⇝ nonrenormalization theorem severely constrains possible preon models/theories (as long as they are formulated via QFT as we know it):

  anomaly at preon level must be the <u>same</u> as at quark/lepton level.

  → <u>anomaly matching conditions</u> ((e.g. el. charges $Q_\nu + Q_{e^-} + N_c Q_u + N_c Q_d \overset{!}{=} 0$))

  see e.g. ['t Hooft, Recent developments in gauge theories, Plenum Press 1980]

  [Zee, Phys. Lett. B 95 (1980) 290]

- a last historic note:

  after discovery of chiral anomaly, there were claims that path integral is wrong!

  → is $\int \mathcal{D}\bar\psi\, \mathcal{D}\psi\; e^{i\int d^4x\, \bar\psi i\gamma^\mu(\partial_\mu - ieA_\mu)\psi}$ unable to tell us

  that it is <u>not</u> invariant under chiral trafo $\psi \to e^{i\theta\gamma^5}\psi$?!

  → it <u>does</u> tell us: action invariant, measure changes ("Jacobian")

  see [Fujikawa, Phys. Rev. Lett. 42 (1979) 1195]