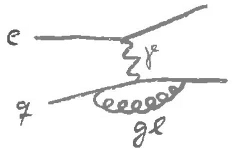
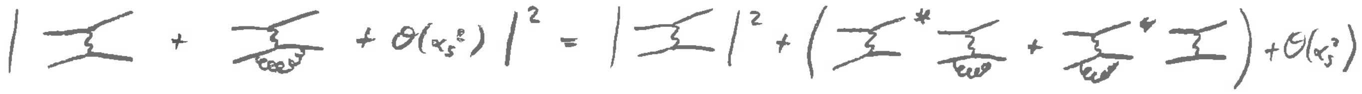
5.3.2 DIS @ NLO: 1-loop $eq \to eq$

diagram:  $\sim \alpha_s$

in $|\mathcal{M}|^2$, only need interference term with tree level diagram



LO   NLO, virtual correction

can again take everything from $e^+e^- \to q\bar{q}$ (§ 4.3.2) via crossing

$\to$ here, recall only some of the features of that calculation, to illustrate the physics; interested in divergences.

- external particles same as at LO $\Rightarrow$ kinematics is the same
  $\to \int d\Phi \sim \delta(y-x)$ (see § 5.2, pg. 63)
- as in $e^+e^- \to q\bar{q}$, interference term is divergent (and negative); divergence comes from same kinematic region as in § 5.3.1: if gluon is soft, or collinear with either of the quarks.
- result: same form as $F_2(x, Q^2)$ above, with (unregularized) splitting fct $\hat{P}(\bar{x})$ replaced by $\underbrace{\tilde{P}(\bar{x})}_{\sim \delta(1-\bar{x}) \text{ from } \delta(y-x)}$ ((and different $\tilde{R}(\bar{x})$))

$\Rightarrow$ real + virtual $\sim P(x) \equiv \hat{P}(x) + \tilde{P}(x)$

[mathematical trick: plus-distribution

given $f(x)$, well-defined for $0 \le x < 1$

define distribution $f(x)_+$ on $0 \le x \le 1$ as

$$f(x)_+ \equiv f(x) - \delta(1-x) \int_0^1 dx' f(x')$$

((most useful for $f(x)$ which diverge at $x \to 1$))

$\Rightarrow$ for any test function $g(x)$ which is smooth at $x=1$,

$$\int_0^1 dx\, f(x)_+ g(x) = \int_0^1 dx\, f(x) \left[g(x) - g(1) \right]$$

((the latter integral being finite if $g(x) \to g(1)$ sufficiently quickly))]

- splitting fct $P(x) = C_F \left[\frac{1+x^2}{(1-x)_+} + \frac{3}{2}\,\delta(1-x) \right]$

  note: – inserting $P(z)$ into $F_2(x, Q^2)$, the divergence at $z \to 1$ cancels; but the divergence at $z \to 1$, parametrized by $\ln \frac{Q^2}{\mu^2}$, remains.

  – in fact, $P(x)$ is the first correction to a fct that describes the momentum distribution of quarks within quarks,

  $$\mathcal{P}(x) \equiv \delta(1-x) + \frac{\alpha_s}{2\pi} \ln \frac{Q^2}{Q_0^2}\, P(x) + \mathcal{O}\left(\alpha_s^2 \ln^2\left(\frac{Q^2}{Q_0^2}\right)\right)$$

  ((see pg. 66: $\xrightarrow{p} \; xp$ etc ; "pure quark" at scale Q_0^2))

5.3.3 Factorization, evolution

⤳ understand why results are still divergent!

compare to $e^+e^- \to q\bar{q}$ case: there, real + virtual = finite

here, main difference is: introduced pdf's f_q

- $z \to 1$ divergence comes from internal quark propagator $\sim \frac{1}{1-z}$ (see pg. 66); roughly, vanishingly small virtuality $\hat{=}$ arbitrarily long time scales (uncertainty princ.)

  ⤳ contradiction to assumptions of parton model: rapid snapshot of proton

- the actual problem is overcounting:

  pdf's $\hat{=}$ internal proton structure $\hat{=}$ result of QCD interactions

  our calculation: QCD corrections to q scattering integrated over all final states (hence all E scales)

  ⤳ but are these QCD corr. already somehow in the pdf's?

⇒ to resolve overcounting: separate ("factorize") different types of physics at different energy scales!

introduce factorization scale μ.

physics at scales $< \mu$ ≡ part of hadron wave fct ≡ pdf's

$> \mu$ ≡ part of partonic scattering cross section ≡ coefficient fcts

note: the cutoff introduced in § 5.3.2, $eq \to eqg$, was hence correct.

((• kinematic limits: note that $\int_x^\infty d\bar{x} \to \int_x^1 d\bar{x}$ (cf pg. 66);

outgoing quark (p_1) and gluon (p_2) are <u>real</u> particles

$$0 \le (p_1+p_2)^2 = (\eta p+q)^2 = \cancel{\eta^2 p^2} + 2\eta p\cdot q + q^2 = 2p\cdot q\left(\eta - \frac{Q^2}{2p\cdot q}\right) = 2p\cdot q\,(\eta - x) = 2p\cdot q\,\eta(1-\bar{x})$$))

• since pdf's contain physics below μ only, they <u>depend</u> on μ

$$F_2(x,Q^2) = \sum_q Q_q^2 \int_x^1 d\bar{x}\, f_q\!\left(\frac{x}{\bar{x}},\mu^2\right)\frac{x}{\bar{x}}\left\{\delta(1-\bar{x}) + \frac{\alpha_s}{2\pi}\left(P(\bar{x})\ln\frac{Q^2}{\mu^2} + R(\bar{x})\right) + O(\alpha_s^2)\right\}$$

⇒ structure fct depends on Q^2 now, <u>violates</u> Bjorken scaling!

→ μ^2-dependence ?!↯ ad-hoc theoretical construct

• physical cross sections cannot depend on μ^2

$$\Rightarrow \partial_{\mu^2} F_2(x,Q^2) \overset{!}{=} 0$$

or, at least, in our calculation $\mu^2\partial_{\mu^2} F_2(x,Q^2) = O(\alpha_s^2)$

$$\Leftrightarrow \mu^2\partial_{\mu^2} f_q(x,\mu^2) = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\bar{x}}{\bar{x}}\, f_q\!\left(\frac{x}{\bar{x}},\mu^2\right)P(\bar{x}) + O(\alpha_s^2)$$

<u>Dokshitzer – Gribov – Lipatov – Altarelli – Parisi</u> eqn (DGLAP, GLAP, AP)

• try to understand <u>physical content</u> of DGLAP eqn

$$\left(\frac{1+x^2}{1-x}\right)_+ \overset{(pg\,67)}{=} \frac{1+x^2}{1-x} - \delta(1-x)\int_0^1 dx'\,\frac{1+x'^2}{1-x'}$$

$1+x'^2 = 2-(1-x')(1+x')$ ← $= 1+x^2$ due to δ fct

$$= (1+x^2)\left\{\frac{1}{1-x} - \delta(1-x)\int_0^1 dx'\,\frac{1}{1-x'}\right\} + \delta(1-x)\underbrace{\int_0^1 dx'\,(1+x')}_{3/2}$$

$$= \frac{1+x^2}{(1-x)_+} + \frac{3}{2}\delta(1-x)$$

$$\Rightarrow P(x) = C_F\left(\frac{1+x^2}{1-x}\right)_+$$, (cf. pg 68)

now, $$\mu^2\partial_{\mu^2} f_q(x,\mu^2) = \underbrace{\frac{\alpha_s}{2\pi}\int_x^1 \frac{d\bar{x}}{\bar{x}}\, f_q\!\left(\frac{x}{\bar{x}},\mu^2\right) C_F\,\frac{1+\bar{x}^2}{1-\bar{x}}}_{\text{pdf increase}} \underbrace{- \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\bar{x}}{\bar{x}}\, f_q\!\left(\frac{x}{\bar{x}},\mu^2\right) C_F\,\delta(1-\bar{x})\int_0^1 dx'\,\frac{1+x'^2}{1-x'}}_{\overset{!}{=} -\frac{\alpha_s}{2\pi} f_q(x,\mu^2)\, C_F \int_0^1 d\bar{x}\,\frac{1+\bar{x}^2}{1-\bar{x}}\ \text{(pdf decrease)}}$$

pdf <u>increase</u> pdf <u>decrease</u>

so at given x, pdf $\left\{\begin{array}{l}\text{increase}\\ \text{decrease}\end{array}\right\}$ from $\left\{\begin{array}{l}\text{higher-}x\text{ quarks}\\ \text{quarks at }x\end{array}\right\}$ reducing their momentum fraction by radiating off gluons.