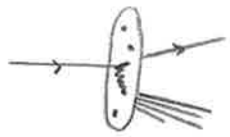
5.2 Parton distribution functions

- description of process in Lorentz-invariant;
  parton model most easily formulated in "Breit-frame": $E_\gamma = 0$, $E_p = \frac{Q}{2x}$
  proton in its rest frame: ◯ ↕2R ⇝ Breit frame: ▯ ↕2R L-contraction
  $\frac{4 R x M_p}{Q} \ll 2R$

  DIS in Breit frame: 

  transverse size of photon $\sim \frac{1}{Q} \ll 2R$
  ⇒ photon interacts with tiny fraction of disk
  → if quarks sufficiently dilute in p, photon does *not* resolve q interactions; incoherent γq collisions!

  ⇝ since quarks act as if they don't interact, their interaction does *not* introduce a dimensionful scale ⇒ Bjorken scaling

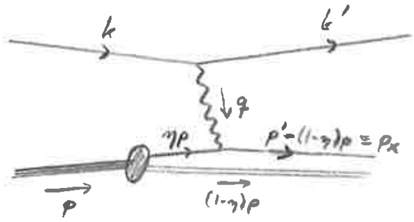
- more precisely:
  proton ≡ bundle of comoving partons, carrying the proton's momentum p.
  parton of type q carries fraction between $(\eta, \eta + d\eta)$ of p
  during a fraction $d\eta \cdot f_q(\eta)$ of the time
  ↖ pdf, probability / parton distrib. fct.

  assume partons pointlike ($r^2 \ll \frac{1}{Q^2}$) and dilute ($f_q(\eta) \ll Q^2 R^2$)
  → incoherent γ-parton-scattering
  with
  $$\frac{d^2\sigma(e + p(p))}{dQ^2\, dx} = \sum_q \int_0^1 d\eta\, f_q(\eta) \frac{d^2\sigma(e + q(\eta p))}{dQ^2\, dx}$$
  ↖ partonic cross section

  note: In partonic cross section, elastic scattering
  → outgoing parton is on mass-shell.
  2→2 scattering → only 1 non-triv. kinematical variable (see pg. 45),
  so $\frac{d^2\sigma}{dQ^2 dx} \sim \delta$, where δ fct fixes one of the variables x, Q^2.
  For massless partons, $(q + \eta p)^2 = 2\eta(pq) - Q^2 \overset{!}{=} 0 \Leftrightarrow \eta = x$

  (partons = scalars ⇒ $F_T = 0$)
  note: partons = quarks = fermions ⇒ (helicity cons.) $F_L = 0$ Callan-Gross relation

- to obtain the parton model prediction for structure fcts, need to calculate partonic cross section.



→ need matrix el. for $eq \to eq$

get by "crossing symmetry" from $e^+e^- \to q\bar{q}$

$$\langle |\mathcal{M}|^2 \rangle = \frac{1}{4}\frac{1}{N_c} N_c \sum_s \left| \times \right|^2$$

↑ from sum over outgoing color; from average over incoming color

(see pg. 44, $\mu \to q$, $m=0$)

$$= \frac{8 e^2 (Q_q e)^2}{(q^2)^2} \left[(\eta p \cdot k)^2 + ((p' - (1-\eta)p)\cdot k)^2 \right]$$

convert to our kinematic invariants

$Q^2 = -q^2$, $s = (p+k)^2 = 2p\cdot k + \cancel{m_p^2} + \cancel{m_e^2}$, $p'k = (p+q)\cdot k = \frac{s}{2} + q\cdot k$

$q\cdot k = (k-k')\cdot k = \frac{1}{2}(k-k')^2 + \frac{1}{2}\cancel{m_e^2} - \frac{1}{2}\cancel{m_e^2} = -Q^2/2$

$\Rightarrow [\ldots] = \left[\left(\frac{\eta s}{2}\right)^2 + \left(\cancel{\frac{s}{2}} - \frac{Q^2}{2} - (\cancel{1}-\eta)\frac{s}{2}\right)^2 \right] = \left(\frac{\eta s}{2}\right)^2 \left[1 + \left(1 - \frac{Q^2}{\eta s}\right)^2 \right]$

$$= 8 (4\pi\alpha)^2 Q_q^2 \frac{1 + \left(1 - \frac{Q^2}{\eta s}\right)^2}{4\left(\frac{Q^2}{\eta s}\right)^2}$$

→ phase space integration, see pg. 60

$$\int d\Phi_{X+1} = \int dQ^2 dx \frac{Q^2}{16\pi^2 s x^2} \int d\Phi_X$$

(electron kinematics)

here: X consists of <u>one</u> massless parton

$$\int d\Phi_X = \int \frac{d^4 p_X}{(2\pi)^3} \delta(p_X^2) (2\pi)^4 \delta^4(\eta p + q - p_X)$$

$$= (2\pi)\, \delta((\eta p + q)^2) = (2\pi)\, \delta(\eta^2 \cancel{p^2} + 2\eta p q + q^2)$$

$$= (2\pi) \frac{1}{|2pq|} \delta\left(\eta - \frac{Q^2}{2pq}\right) = (2\pi)\frac{x}{Q^2}\delta(\eta - x)$$

→ partonic cross section

$$\sigma(e + q(\eta p)) = \frac{1}{2\eta s} \int d\Phi_{1+1} \langle |\mathcal{M}|^2 \rangle$$

$$y = \frac{Q^2}{xs}$$

$$\frac{d^2\sigma(e+q(\eta p))}{dQ^2 dx} = \frac{1}{2\eta s}\frac{Q^2}{16\pi^2 s x^2}\frac{2\pi x}{Q^2}\delta(\eta - x)\langle |\mathcal{M}|^2 \rangle = \frac{y^2}{16\pi Q^4}\delta(\eta - x)\, 2(4\pi\alpha)^2 Q_q^2 \frac{1 + (1-y)^2}{y^2}$$

$$= \frac{2\pi\alpha^2 Q_q^2}{Q^4}\delta(\eta - x)\left[1 + (1-y)^2\right]$$

- finally get cross section for e+p (see pg. 62)

$$\frac{d\sigma(e+p(p))}{dQ^2\,dx} = \sum_q \int_0^1 d\eta\, f_q(\eta)\cdot \frac{2\pi\alpha^2 Q_q^2}{Q^4}\,\delta(\eta-x)\left[1+(1-y)^2\right]$$

$$= \frac{2\pi\alpha^2}{xQ^4}\left[1+(1-y)^2\right]\sum_q Q_q^2\, x\, f_q(x)$$

→ comparing with §5.1 ((result in terms of structure fcts F_2, F_L (pg. 61)))

$$\Rightarrow \quad F_2(x,Q^2) = \sum_q Q_q^2\, x\, f_q(x) \quad , \quad F_L(x,Q^2) = 0$$

note: F_2 is Q^2-independent: Bjorken scaling!

$F_L = 0$ was the Callan-Gross relation.

→ we will see that QCD corrections do violate Bjorken scaling;
in experimental data, however, it is satisfied pretty well → Figure 1

- in practice, measure F_2 from different data sets and extract the f_q's.

$$F_2^{ep} = x\left[\tfrac{1}{9}(f_d+f_{\bar d}+f_s+f_{\bar s}) + \tfrac{4}{9}(f_u+f_{\bar u}+f_c+f_{\bar c})\right]$$

since the pdf's contribute differently in different experiments
(e.g. F_2^{en}, $F_2^{\nu p}$, $F_2^{\bar\nu p}$, ...)
can do global fits to extract them. Typical results → Figure 2

→ useful checks via sum rules: $\int_0^1 dx\, f_{u_v}(x) = 2$, $\int_0^1 dx\, f_{d_v}(x) = 1$, etc.

5.3 QCD corrections in DIS

→ α_s is not small, so our above LO treatment of DIS might get important corrections.

→ how does the parton model emerge from QCD?

→ structure fcts will (slowly: logarithmically) depend on Q^2, leading to violation of Bjorken scaling

→ have to compute NLO corrections to DIS;
divergences, splitting fcts, factorization, (DGLAP) evolution eqs, data