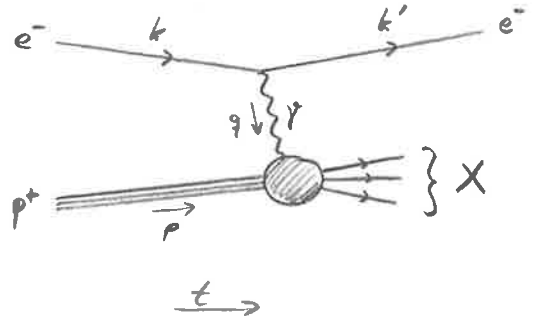
5. Deep inelastic scattering (DIS)

We now (temporarily) go back to tree-level phenomenology.

Q.: are quarks real physical constituents of hadrons, or just a mathematical convenience for describing the hadron's wavefct?

⇝ DIS gives information on internal proton structure

⇝ structure functions, Bjorken scaling, parton distribution fcts



photon virtuality $Q^2 = -q^2$ $\left(\left(Q^2 \sim \frac{1}{\lambda^2}\right)\right)$

controls resolving power of photon

$Q^2 \ll \frac{1}{R^2}$ → elastic e-p scattering

← proton radius

$Q^2 \gg \frac{1}{R^2}$ → resolve p constituents, elastic e-q scattering

we are interested in deep ($Q^2 \gg M_p^2$) inelastic ($(p+q)^2 \gg M_p^2$) scattering

5.1 Structure functions

- want to describe process with Lorentz-invariant variables

  center-of-mass energy $s \equiv (k+p)^2$

  at fixed s, scattered e^- has 2 non-trivial variables (E, θ);

  use $Q^2 = -q^2$, $x \equiv \frac{Q^2}{2p\cdot q}$

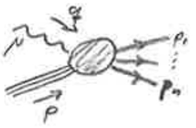
  ((other choices: $W^2 \equiv (p+q)^2 = Q^2\,\frac{1-x}{x}$ (invariant mass of γp-system)

  $y \equiv \frac{p\cdot q}{p\cdot k} = \frac{Q^2}{xs}$))

  kinematic limits: $Q^2 < s$, $x > Q^2/s$

  ((of course $\underbrace{k^2 = m_e^2}_{\approx 0}$, $\underbrace{p^2 = M_p^2}_{\approx 0}$; for $Q^2 \gtrsim M_Z^2$, also Z^0 exchange))

- know nothing about detailed structure of proton

$\rightarrow$ parametrize $\quad \mathcal{M} =$  $= e\, T_\mu(p, q; \{p_X\})$

$\Rightarrow \frac{1}{4} \left|\right.$  $\left.\right|^2 = \frac{1}{4} \frac{e^4}{Q^4} \underbrace{tr(\not k \gamma^\mu \not k' \gamma^\nu)}_{\equiv L^{\mu\nu} = 4(k^\mu k'^\nu + k^\nu k'^\mu - k\cdot k' g^{\mu\nu})} T_\mu(p, q, \{p_X\})\, T_\nu^*(p, q, \{p_X\})$ (see pg. 49)

in complete analogy to $e^+e^- \to \mu^+\mu^-$, $q\bar{q}$ etc.

- for total cross section, need to integrate over phase space

$$\int d\Phi_{X+1} = \underbrace{\int dQ^2\, dx\, \frac{Q^2}{16\pi^2 s x^2}}_{\text{electron kinematics}} \underbrace{\int d\Phi_X}_{\text{n-body phase space for } X}$$

for an inclusive process (don't measure X $\rightarrow$ sum over them),

$$\frac{d^2\sigma}{dQ^2\, dx} = \frac{1}{2s} \frac{Q^2}{16\pi^2 s x^2} \underbrace{\sum_X \int d\Phi_X \frac{1}{4}|\mathcal{M}|^2}_{= \frac{1}{4}\frac{e^4}{Q^4} L^{\mu\nu} \underbrace{\sum_X \int d\Phi_X\, T_\mu(p, q, \{p_X\})\, T_\nu^*(p, q, \{p_X\})}_{\equiv H_{\mu\nu}}}$$

- consider $H_{\mu\nu}$: have summed and integrated all X dependence,

so $H_{\mu\nu}(p, q)$, must be symm. in μ, ν ((parity cons. in QED, QCD))

$$\Rightarrow H_{\mu\nu} = -H_1 g_{\mu\nu} + H_2 \frac{p_\mu p_\nu}{Q^2} + H_3 \frac{q_\mu q_\nu}{Q^2} + H_4 \frac{p_\mu q_\nu + q_\mu p_\nu}{Q^2}$$

where H_i are scalar fcts, $H_i(q^2 = -Q^2,\ p\cdot q = \frac{Q^2}{2x},\ \underbrace{\cancel{p^2 = M_p^2}}_{\text{neglect in DIS}})$

((including Z^0 exchange, ... $+ H_5\, \varepsilon_{\mu\nu\varrho\sigma} \frac{p^\varrho q^\sigma}{Q^2}$))

(check ?!)

$$\Rightarrow L^{\mu\nu} H_{\mu\nu} = 8(kk') H_1 + 8 \frac{(pk)(pk')}{Q^2} H_2 + 0 \cdot H_3 + 0 \cdot H_4$$

used $d = 4$, neglected M_p^2

now $Q^2 = -q^2 = -(k-k')^2 = 2kk' - \cancel{2m_e^2}$

$s = (p+k)^2 = 2pk + \cancel{M_p^2} + \cancel{m_e^2}$

$pk' = p(k-q) = pk\left(1 - \frac{pq}{pk}\right) = pk(1-y)$

$$= 4Q^2 H_1 + 2 \frac{s^2}{Q^2}(1-y) H_2$$

- $$\frac{d^2\sigma}{dQ^2\,dx} = \frac{1}{2s}\,\frac{Q^2}{16\pi^2 s x^2}\,\frac{1}{4}\,\frac{\alpha^2 16\pi^2}{Q^4}\left(4Q^2 H_1 + 2\,\frac{s^2}{Q^2}(1-y)H_2\right)\ ;\ \text{def}\ \begin{aligned}H_1 &= 8\pi F_1\\ H_2 &= 16\pi x F_2\end{aligned}$$
$$\equiv \frac{4\pi\alpha^2}{xQ^4}\left\{ x y^2 F_1(x,Q^2) + (1-y)\,F_2(x,Q^2)\right\}$$

((recall $y = Q^2/xs$))

→ amazing: w/o knowing ep interaction, derived s-dependence of σ !

→ the F's are called "structure functions" of the proton

sometimes, see $F_T(x,Q^2) = 2x F_1(x,Q^2)$ "transverse"
$F_L(x,Q^2) = F_2(x,Q^2) - 2x F_1(x,Q^2)$ "longitudinal"

so $$\frac{d^2\sigma}{d^2Q\,dx} = \frac{2\pi\alpha^2}{xQ^4}\left\{\left(1+(1-y)^2\right)F_T(x,Q^2) + 2(1-y)F_L(x,Q^2)\right\}$$
$$= \frac{2\pi\alpha^2}{xQ^4}\left\{\left(1+(1-y)^2\right)F_2(x,Q^2) - y^2 F_L(x,Q^2)\right\}$$

useful since (for most current data) $y^2 \ll 1$

- have isolated all non-trivial x, Q^2 dependence into F_2, F_L ;
but still don't know anything about these fcts.

Assumption: interaction of γ with innards of proton does not involve any dimensionful scale

⇒ dim-less F's can not depend on dimensionful parameter Q^2

$$\Rightarrow \frac{d^2\sigma}{dQ^2\,dx} = \frac{2\pi\alpha^2}{xQ^4}\left\{\left(1+(1-y)^2\right)F_2(x) - y^2 F_L(x)\right\}$$ Bjorken scaling

→ experimentally, this is true (to a pretty good approximation);
but proton consists of quarks, bound at distance scale $\sim 1/M_p$,
so how can the interaction possibly be M_p-indep. ?!

⇝ answer via parton model