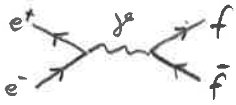
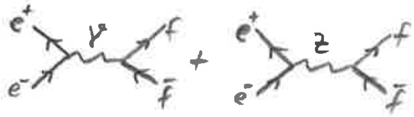
4.2 The Z-peak in $R(s)$

- in §4.1, studied tree level  , $f \neq e$

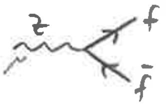
$$\frac{d\sigma_{e\bar{e}\to f\bar{f}}}{d\cos\theta} \overset{\text{pg. 45}}{=} \frac{2\pi}{(8\pi)^2}\frac{\boxed{\langle|\mathcal{M}|^2\rangle}}{(2E)^2}\,\Theta(E-m_f) \qquad = \sum_{\substack{\text{colors}\\ f}} Q_f^2 e^4 \left\{1+\frac{m_e^2+m_f^2}{E^2}+\left(1-\frac{m_e^2}{E^2}\right)\left(1-\frac{m_f^2}{E^2}\right)\cos^2\theta\right\}$$

use $\alpha=\frac{e^2}{4\pi}$, $(2E)^2=s$ in CMS

$$= \frac{\pi\alpha^2}{2s}\sum_{c,f} Q_f^2\left\{1+4\,\frac{m_e^2+m_f^2}{s}+\left(1-\frac{4m_e^2}{s}\right)\left(1-\frac{4m_f^2}{s}\right)\cos^2\theta\right\}\Theta(\sqrt{s}-2m_f)$$

- generalization in Standard Model : 

[see, e.g., Ellis/Stirling/Webber §3.1]

use SM Feynman rules :  $= -\dfrac{ie}{2\sin\theta_W\cos\theta_W}\gamma^\mu\left(V_f - A_f\gamma^5\right)$

where $\sin^2\theta_W \approx 0.23$ is the weak mixing angle

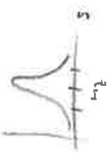
$A_f = \pm\frac{1}{2}$ for $f \in \left\{\begin{matrix} \nu_e, \nu_\mu, \nu_\tau, u, c, t \\ e, \mu, \tau, d, s, b\end{matrix}\right\}$ is the axial $fZ\bar{f}$ coupling

$V_f = A_f - 2Q_f\sin^2\theta_W$ vector $fZ\bar{f}$ coupling

now, taking $|\mathcal{M}|^2$, get interference term as well

take high E limit, $\sqrt{s} \gg m_f \gg m_e$

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2}{2s}\sum_{c,f}\left\{\underbrace{(1+\cos^2\theta)}_{\to 2+\frac{2}{3}}\left(\overbrace{Q_f^2}^{\gamma^2} - \overbrace{2Q_f V_e V_f\,\kappa\,\frac{s(s-m_Z^2)}{(s-m_Z^2)^2+\Gamma_Z^2 m_Z^2}}^{\gamma Z \text{ interference}}\right.\right.$$
$$\left. + \overbrace{(A_e^2+V_e^2)(A_f^2+V_f^2)\,\kappa^2\,\frac{s^2}{(s-m_Z^2)^2+\Gamma_Z^2 m_Z^2}}^{Z^2}\right)$$
$$\left. + \frac{\overbrace{\cos\theta}^{\int_{-1}^{1} d\cos\theta \to 0}}{(s-m_Z^2)^2+\Gamma_Z^2 m_Z^2}\left(-4Q_f A_e A_f\,\kappa\, s(s-m_Z^2) + 8A_eV_eA_fV_f\,\kappa^2 s^2\right)\right\}$$

(s-channel Z propagator $\sim \dfrac{1}{s-(m_Z-i\Gamma_Z)^2}$) 

where $\kappa \equiv \dfrac{\sqrt{2}\,G_F\,m_Z^2}{16\pi\alpha}$, Fermi const $G_F = \dfrac{1}{\sqrt{2}v^2} \approx 1.166\cdot 10^{-5}\,\text{GeV}^{-2}$

$m_Z \approx 91.1876$ GeV , Z decay width $\Gamma_Z \approx 2.5$ GeV

- at small s/m_Z^2, the additional weak effects are small

  $\Rightarrow$ neglecting them, get back result of pg. 45: $\sigma_{s \ll m_Z^2} = \frac{4\pi\alpha^2}{3s} \sum_{c,f} Q_f^2$

- on Z pole, $s = m_Z^2$, 2^{nd} line dominates

  $$\Rightarrow \quad \sigma_{s=m_Z^2} = \frac{4\pi\alpha^2}{3\cancel{m_Z^2}} \sum_{c,f} (A_e^2+V_e^2)(A_f^2+V_f^2)\,\kappa^2 \frac{\cancel{m_Z^2}\cancel{m_Z^2}}{\Gamma_Z^2 \cancel{m_Z^2}}$$

  $$\Rightarrow \quad R_{Z\,peak} = \frac{\sigma(e^+e^-\to Z \to hadrons)}{\sigma(e^+e^- \to Z \to \mu^+\mu^-)} = \frac{N_c \sum_q (A_q^2+V_q^2)}{A_\mu^2+V_\mu^2}$$

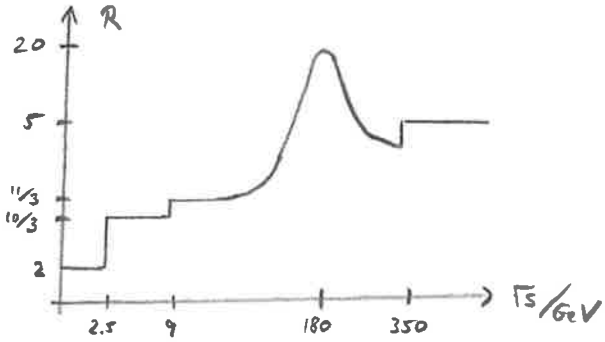
  only 5 quarks lighter than Z (($m_{top} \approx 172$ GeV))

  $\rightsquigarrow \sum_q$ goes over $q \in \left\{ \begin{matrix} u, c \\ d, s, b \end{matrix} \right\}$

  $$= \frac{3\left[2\left(\frac{1}{4}+\left(\frac{1}{2}-2\left(\frac{2}{3}\right)\sin^2\theta_W\right)^2\right)+3\left(\frac{1}{4}+\left(-\frac{1}{2}-2\left(-\frac{1}{3}\right)\sin^2\theta_W\right)^2\right)\right]}{\frac{1}{4}+\left(-\frac{1}{2}-2(-1)\sin^2\theta_W\right)^2} \overset{N_c=3,\ \sin^2\theta_W=0.23}{\approx} 20.095$$

((adding the γ channel, value changes to 19.984))

- so our result would look like this: step functions from γ exchange + broad peak from Z exchange



- comparison with experiment:

  ((note that PDG-plot was for $\frac{\sigma(e^+e^-\to hadrons)}{\frac{4\pi\alpha^2}{3s}}$, not $\frac{\sigma(e^+e^-\to hadrons)}{\sigma(e^+e^-\to\mu^+\mu^-)}$))

  LEP measured $R_{Z\,peak} = 20.767 \pm 0.025$

  which is $\sim 3.5\%$ higher than the above lowest-order prediction

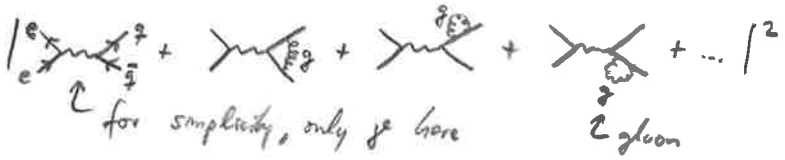
  $\Rightarrow$ discrepancy is (mostly) due to higher-order QCD-corrections!

  $\Rightarrow$ compute those (($\sigma \to \sigma(1+\alpha_s+\ldots)$)),

  then use experiment to determine α_s.

4.3 QCD corrections to $R(s)$

$\rightsquigarrow$ goal: compute $\sigma(e^+e^- \to \text{hadrons}) = \sigma_0 \left(1 + c\,\alpha_s + \mathcal{O}(\alpha_s^2)\right)$, $c = ?$

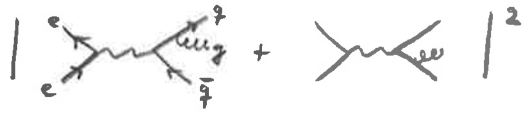
- $\sigma_{e^+e^- \to q\bar{q}} \sim |\mathcal{M}|^2 =$ 

  for simplicity, only γ here

  gluon

  $\Rightarrow$ the $c_1 \alpha_s$ term gets contributions from interference of tree + 1loop amplitudes . "virtual correction" c_V

- note that there is another class of diagrams, contributing to the same order:

  $\sigma_{e^+e^- \to q\bar{q}g} \sim$ 

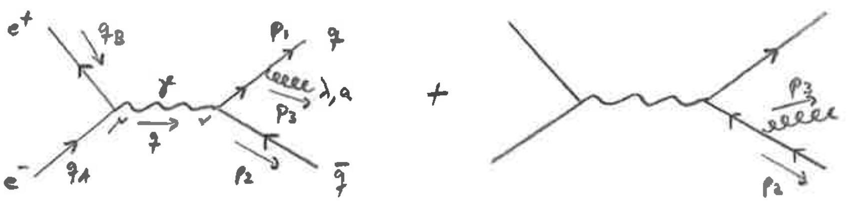
  $\Rightarrow$ "real correction" c_R

$\rightsquigarrow$ total cross section to produce (any number of) partons ($\to$ hadrons) is sum of $\sigma_{e^+e^-\to q\bar{q}} + \sigma_{e^+e^-\to q\bar{q}g} + \dots$

$\rightsquigarrow$ in fact, both will turn out to be (infrared) divergent, only their sum is finite, hence physical.

$\rightarrow$ in practice, pick a regularization scheme (again $d = 4-2\varepsilon$) and remove regulator in the end ($\varepsilon \to 0$)

4.3.1 real corrections : $\sigma_{e^+e^-\to q\bar{q}g}$



(see pg. 44, plus Fy rules from pp 24,25)

spin $= \pm\frac{1}{2}$ — Dirac spinor for incoming particle — polarization vector for outgoing gluon

$$= \bar{v}(q_B, s_B)(-ie\gamma^\mu)u(q_A, s_A)\left(\frac{-ig_{\mu\nu}}{q^2}\right)\varepsilon_\lambda^{*a} \,*$$

$$* \;\bar{u}(p_1,s_1)\left\{(ig_s\gamma^\lambda T^a)\frac{i(\not p_1+\not p_3)}{(p_1+p_3)^2}(-ieQ_q\gamma^\nu) + (-ieQ_q\gamma^\nu)\frac{i(-\not p_2-\not p_3)}{(p_2+p_3)^2}(ig_s\gamma^\lambda T^a)\right\}v(p_2,s_2)$$

note: have used Feynman gauge here, $\xi = 1$

have used massless quarks, $m_q = 0$