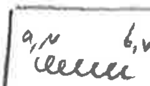
- **gluon propagator**

collection from above, $O(\phi)\, B(f(\phi)) = e^{i\int d^4x\, \mathcal{L}_{QCD}}\; e^{i\int d^4x \left(-\frac{1}{2\xi}\right)\left(\partial^\mu A^a_\mu\right)^2}$

such that $S_0 \ni \int d^4x\; \frac{1}{2} A^a_\mu \delta^{ab} \left(\partial^2 g^{\mu\nu} - \partial^\mu \partial^\nu + \frac{1}{\xi}\partial^\mu\partial^\nu\right) A^b_\nu$

$$\overset{!}{=} \int \frac{d^4k}{(2\pi)^4}\; \frac{1}{2} \tilde{A}^a_\mu(k)\, \delta^{ab} \left(-k^2 g^{\mu\nu} + \left(1-\frac{1}{\xi}\right) k^\mu k^\nu\right) A^b_\nu(-k)$$



$\hat{=} \frac{-i}{k^2+i\epsilon}\left(g_{\mu\nu} - (1-\xi)\frac{k_\mu k_\nu}{k^2}\right)\delta^{ab}$ ((since $(\text{gluon})^{ab}_{\mu\nu} \cdot (\sim)^{bc,\nu\rho} \overset{!}{=} i\delta^{ac} g_\mu^{\ \rho}$)) ((check?!))

↑ gauge parameter; often, use $\xi = 1$ Feynman gauge

as in QED, physics is ξ-independent

- **Faddeev-Popov ghost fields**

have to take care of $\det \mathcal{F}(\phi)$ factor (on bottom of pg 26)

using Grassmann numbers again (see Gauss integral on pg 25), rewrite

$$\det \mathcal{F}(A) \overset{!}{=} \det\left(\frac{1}{g}\partial^\mu\left[g f^{abc} A^b_\mu + \delta^{ac}\partial_\mu\right]\right)$$

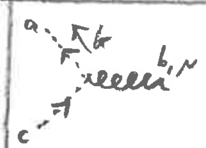
$$\overset{!}{=} \int \mathcal{D}c\, \mathcal{D}\bar{c}\; e^{i\int d^4x\, \bar{c}^a\left(-\partial^\mu\left[\partial_\mu \delta^{ac} + g f^{abc} A^b_\mu\right]\right)c^c} \qquad \left[\partial_\mu \delta^{ac} + g f^{abc} A^b_\mu\right] \equiv D_\mu^{ac}$$

where the "FP ghosts" are anticommuting fields

(but there are no γ-matrices $\Rightarrow$ spin 0!)

$\Rightarrow$ ghost propagator


$\hat{=} \frac{i}{k^2+i\epsilon}\delta^{ab}$

$\Rightarrow$ ghost-gluon-vertex


$\hat{=} -g f^{abc} k_\mu$

((for physical interpretation of ghosts, see e.g. Peskin/Schroeder §16.3))

→ now, know all propagators and vertices of QCD, so we can again (as in ϕ^4 theory, see §2.4) do perturbative expansions via generating functional.

- short summary:

  from pg. 18, 26, 27 we have now

$$\mathcal{L}_{QCD} = \sum_f \bar{\psi}_f \left(i\not{D} - m_f \right) \psi_f - \frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} - \frac{1}{2\xi} \left(\partial^\mu A^a_\mu \right)^2 + \bar{c}^a \left(-\partial^\mu D^{ac}_\mu \right) c^c$$

  where $D^{ac}_\mu = \partial_\mu \delta^{ac} + g f^{abc} A^b_\mu$ is the "covariant derivative in the adjoint representation"

  → this expression is still invariant, but not under a local gauge ~~transformation~~ as in § 2.2; the relevant transformation now includes the ghost fields $\bar{c}, c$ in an essential way and is called "BRST" transformation

  [Becchi/Rouet/Stora, Ann. Phys. 98 (1976) 287;
  Tyutin, Theor. Math. Phys. 27 (1976) 316]

  (a symmetry with continuous but anticommuting parameters)
  ((more: QFT lecture; e.g. Peskin/Schroeder §))

  → the formal treatment of pg. 26 had implicitly assumed that the gauge condition $f(\phi)$ (($f^a(A^a_\mu)$)) selects (via the Delta-fct) one unique representative ϕ ((A^a_μ)) for each "gauge orbit" ϕ_Ω.

  However, Gribov [Gribov, Nucl. Phys. B 139 (1978) 1] has demonstrated that for non-Abelian theories, this cannot always be guaranteed.

  In practice, this fact has little relevance.

- set of "Feynman rules" as usual

  → see QFT lecture; Particle physics lecture; ...

  draw diagrams – fix symmetry factors – insert Feynman rules for propagators + vertices – perform traces and Lorentz algebra – regularize divergent integrals – Wick rotation – evaluate loop integrals – ...