Short summary:

From pg. 18, 26, 27 we have now

\[
\mathcal{L}_{\text{adj}} = \frac{1}{g} \left( i D^{\mu}_{\alpha} \right) \gamma^\mu - \frac{1}{2} \left( \nabla_{\alpha} \gamma_{\beta} \right)^2 + \bar{\psi} \left( -i \gamma^\alpha \gamma^\beta \right) \psi
\]

where \( D^{\alpha}_{\beta} = \partial_{\beta} \gamma_{\alpha} + \frac{1}{2} g_{\beta \epsilon} \gamma_{\alpha}^\epsilon \) is the "covariant derivative in the adjoint representation"

→ this expression is still invariant, but not under a local gauge transformation as in § 2.2; the relevant transformation now includes the ghost fields \( \bar{\psi}, \psi \) in an essential way and is called "BRST" transformation


(a symmetry with continuous but anticommuting parameters)

(more: QFT lecture; eg. Peskin/Schroeder § 4)

→ the formal treatment of pg. 26 had implicitly assumed that the gauge condition \( \delta \phi \) (f("ll")) selects (in the Sata-lect) one unique representative \( \phi (f("ll")) \) for each "gauge orbit" \( \phi_c \).

However, [Gribov, Nucl. Phys. B 139 (1978) 1] has demonstrated that for non-Abelian theories,

this cannot always be guaranteed.

In practice, this fact has little relevance.

• set of "Feynman rules" as usual

→ see QFT lecture; Particle physics lecture; ...

draw diagrams - fix symmetry factors - insert Feynman rules for propagators + vertices - perform traces and Lorentz algebra - regularize divergent integrals - Wick rotation - evaluate deep integrals -...
3. Fundamentals

- Our QCD contains operators of dimension \( \leq 4 \)

- Theory is renormalizable: all divergences can be removed by a finite number of counterterms

Here: illustrate some divergences in QCD

Important physical consequence: asymptotic freedom

- Mini-review: renormalization [see e.g. Perlin/Schroder §10]

Loops \( \to \mathcal{S}^{d=4} \to \) ultraviolet (UV) divergences (from large \( k \) or small \( x \))

Common and natural in QED.

- Counting of UV divs (in \( \text{A}^{2} \) theory): superficial degree of divergence

- Idea: "hide" divs in \( \mathcal{Y} \) by rescaling fields + couplings

\[ \frac{1}{g} \to 2 \frac{1}{g_{R}} \text{ etc., } \mathbb{Z} = 1 + O(g^{2}) \]

Here renormalized

- Need intermediate regularization of divergent loop integrals

Many possibilities (discrete space-time; cutoff large momenta; Pauli-Villars ...)

Most elegant for us: dimensional regularization

\[ \int \frac{d^{d}k}{(2\pi)^{d}} \to \left( \frac{d^{d}k}{(2\pi)^{d}} \right) \]

\( \mu \) analytic cont. \( d \to 4 \); divs will be poles \( \sim \frac{1}{d-4} \)

Introduce artificial mass-scale \( \mu \)

\[ (\mu^{2})^{d} = \mathcal{\mu}^{2} (\mu^{2})^{\epsilon} = \mathcal{\mu}^{2} \left( 1 + \epsilon \right) \]

\( \mu = \frac{a^{2}}{\epsilon} + O(\epsilon^{3}) \)

- The renormalization group (RG) equations
describe the \( \mu \)-dependence of parameters / Green functions

- Mini-review: dimensional regularization

\[ \int d^{d}k f(h_{2}) = \int d^{d}k f(h) \]

\[ \int d^{d}k f(h_{2}) = 1/(d-1) \int d^{d}k f(h) \quad \Rightarrow \quad \int d^{d}k = 0 = \int \frac{d^{d}k}{(2\pi)^{d-1/2}} (1) \]

\[ \int d^{d}k e^{-\mathbf{k}^{2}} = [ \int_{-\epsilon}^{\epsilon} d\mathbf{k} ] e^{-\mathbf{k}^{2}} = (2\pi)^{d/2} \]

\[ \int d^{4}k f(h_{1}) = \frac{2\pi^{2}}{2(4\pi)^{d/2}} \int d^{d}k h^{d-4} f(h) \quad \text{d-dim. spherical coordinates} \]

\[ \int d^{d}k h_{\mu} f(h_{1}) = 0 \quad \text{if odd in } h \]

\[ \int d^{d}k h_{\mu} h_{\nu} f(h_{1}) = \frac{1}{2} \mu_{\nu} \int = \frac{1}{2} \int d^{d}k h_{\mu} h^{d-2} f(h_{1}) \quad \text{since } \int d^{d}k h^{d} f = \frac{1}{2} \int \]