2.3 QCD and its symmetries

- Quantum Chromodynamics (QCD) is a Yang-Mills theory with gauge group $SU(3)$.

- Matter fields (the $q$ above) are quarks; they are in the fundamental representation of $SU(3)$, have spin $\frac{1}{2}$.

- There are six types ("flavors") of quarks: $u, d, c, s, t, b$.

- Index of gauge group is called color index $\alpha = 1, 2, 3$.

- Flavor index $A = u, d, c, s, t, b$.

- The $3^2 = 9$ vector fields (or gauge bosons) $A^\alpha$, $\alpha = 1, ..., 9$ are called gluons.

- $X_{\alpha \beta \gamma} = -i \epsilon_{\alpha \beta \gamma} \left( y^A (\partial_\mu q^{\alpha \gamma} - i g \, A^{\mu \beta} T^{\alpha \beta}) - y^A q^{\alpha \gamma} \right)$

- (Sum over color indices $\alpha, \beta$; sum over flavor index $A$.)

- Each quark flavor can have a different mass, generators of $SU(3)$ in fundamental rep.

- Sometimes, it is useful to consider the generalizations $SU(3) \rightarrow SU(N_c)$

- Colors: $\alpha, \beta = 1, ..., N_c$.

- Flavors: $A = 1, ..., N_c^2 - 1$.

- $6$ quark flavors $\rightarrow N_c$ quark flavors $\Rightarrow A = 1, ..., N_c$.

- QCD possesses not only the exact local $SU(N_c)$ color symmetry, but has also important approximate global symmetries:

  - Consider ($x$-independent) rotations in flavor space.

  - (Note: global phase redefinition for each flavor $A = u, d, ..., c$ separately $\Rightarrow X_{\alpha \beta \gamma}$ manifest $\Rightarrow$ quark number conserved)

  - Solutions between different flavors make sense if some masses are approximately degenerate.

  - (Note: in nature, $m_u \sim 3$ TeV, $m_d \sim 5$ TeV $\Rightarrow m_{u-d} \ll$ in Higgs $\sim 150$ GeV $\Rightarrow$ $X$ has increased symmetry)
\[ M = \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix} \approx m_u \Delta m^2 \]

write \[ q = \begin{pmatrix} q^u \\ q^d \end{pmatrix} \]

is invariant under \[ q \rightarrow e^{i \frac{\lambda}{2} \sigma_1} q \] \[ (\sigma_1 \sigma_2 \text{ Pauli, } \sigma \in \mathbb{C}^{2 \times 2}) \]

\[ \in U(2) = U(1) \otimes SU(2) \]

exact only if \( m_u = m_d \)

\((\text{Note: these symmetries are, via Noether's theorem, associated with vector currents } q_i = \bar{q}_i \sigma_i q, \text{ hence } \theta_{\mu \nu} \text{ in } SU(2)_L)\)

\((\text{Note: if } m_u = m_d \approx m_s \text{ is a useful approximation, then symmetry is enhanced, } SU(3)_L \text{ etc.})\)

for massless flavors, the symmetry becomes even larger: \( H = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \)

use left- and right-handed projectors

\[ \Gamma_L = \frac{1 + \gamma^5}{2} \] \( \rightarrow \gamma^5 \phi_L, \gamma^5 \phi_L, \gamma^5 \phi_L = 0 \)

decompose \( q^u = (q_L + q_R) \)

now \( q_L = \begin{pmatrix} q^u_L \\ q^d_L \end{pmatrix} \)

\( \text{and } \theta_{\mu \nu} \approx \bar{q}_L \gamma_\mu q_L + \bar{q}_R \gamma_\mu q_R \)

\((\text{Note: } t^7 \neq (33) \text{ would have coupled } L,R : \gamma^5 = -g^5 L \gamma R \text{ etc.})\)

\( \rightarrow \text{ independent transformations } q_L \rightarrow U_L q_L, q_R \rightarrow U_R q_R \) permitted!

\( \rightarrow U(1)_L \otimes U(1)_R \) symmetry

\[ = U(1)_L \otimes U(1)_R \otimes SU(2)_L \otimes SU(2)_R \]

\((\text{called chiral symmetry}) \)

\((\text{Note: the symmetry } SU(2)_L \otimes SU(2)_R \text{ is sometimes rewritten as the product } SU(4)_C \otimes SU(2)_A \text{'axial',})\)

using \( Q = \begin{pmatrix} q^u_L \\ q_R \end{pmatrix} \)

\( \text{mimic chiral } Q \rightarrow e^{i \frac{\lambda}{2} \sigma_1} Q \) and \( Q \rightarrow e^{i \frac{\lambda}{2} \sigma_3} Q \)

\((\text{Symmetry of } SU(4)_C \text{ to check this flavor symmetry invariance, see left})\)
• Similar to the above approximate global symmetries for light quarks (neglecting effects of order \( m_q \)), can also consider heavy quark symmetries (neglecting effects of order \( \frac{1}{m_q} \)).

→ Symmetries: "Heavy quark effective theories",

see e.g. [T. Nakanishi, Phys. Lett. 245 (1990) 259]

• Other important exact symmetries of QCD are the discrete global symmetries: \( C, \bar{C}, T \)

(These agree with the observed properties of the strong interactions; for tests and limits, see Particle Data Group, pdg.lbl.gov)

→ Analysis of \( \hat{Q}_{SD} \) under \( C, \bar{C}, T \) is complicated (at quantum level) due to the possible dim-4 operator we had discovered (see pg. 14)

\[
\hat{Q}_{SD} = \frac{g^2}{32\pi^2} \sum \hat{F}_{\mu\nu} \hat{F}^{\mu\nu}, \quad \text{where} \quad \hat{F}_{\mu\nu} = \frac{i}{2} [\gamma_{\mu} \gamma_{\nu} - \gamma_{\nu} \gamma_{\mu}] F_{\mu\nu}
\]

→ \( \hat{Q}_{SD} \) is the convention normalized; on pg. 14: \( \hat{Q} \)

→ \( \hat{Q}_{SD} \) would violate both \( P \) and \( T \), in contradiction to observations

⇒ set \( \Theta = 0 \), or at least \( \Theta \ll 1 \) ?!

\( \hat{Q}_{SD} \) (could be regenerated by known \( CP \) effects in weak \( \beta \)-\( \beta \)→

→ Actually, \( \hat{F}_{\mu\nu} \hat{F}^{\mu\nu} = \partial_{\alpha} \left\{ 2 \varepsilon^{\alpha\beta\gamma\delta} A_\beta (\partial_\gamma A_\delta - \frac{2}{3} g A_\gamma A_\delta) \right\} \)

is a total derivative

contributes only surface term to action \( S = \int d^4x \hat{Q} \)

therefore plays no role in perturbative QCD

→ However, \( \hat{Q}_{SD} \) can have real physical effects due to non-perturbative effects (QCD vacuum can have non-trivial topology ⇒ surface terms contribute; the \( S_{\Sigma} \) is not gauge-invariant)

[see e.g. Erice lectures by S. Coleman (1972), F. Wilczek (1983)]

→ Problem: Observations tell \( \Theta < 10^{-9} \) (neutron dipole moment)

"Naturally", \( \Theta \) should be large (coming from strong interactions)

→ "Strong CP problem"

→ Several proposed solutions; e.g. Peccei–Quinn–symmetry

→ New particleless actions