12) Outlook: some stories...

((we now have the tools: many directions to go...))

compute higher orders

4) tech. problem: technology for particular cases in kernels FT

problem #1) Yang-Yang invariance

soln. 1990 [Brandt/Pomorski, NPB 337 (90) 561]

application:
1) $\nu$ [BP, PRD 42 (90) 2156]
2) $\nu \to NCO [H. Schulz, NPB 418 (94) 353]

problem #2) massless (semi-) integrals

$g^3$ terms in free $E$

$\phi^4$ [Finkel/Sam/Taylor] 1992

QED [Corini/Parrini] 1994

QCD [Amlot/Zhu] 1994

$g^5$ terms in free $E$

$\phi^6$ [Parrini/Singh] [Brandt/Nich] 1995

QED [Parrini/Corini] 1995

[Abreu] 1996

QCD [Kastening/Zhao] 1995

[Brandt/Nich] 1996

(( $g^6$, QCD, KK et al. 200X ?? ))

new problem: convergence (of weak coupling expansion) [B]

problem #3) massive (semi-) integrals

state of art: $\phi^4, g^4$ [Abreu/Brand/Streetland] 2000

$\phi^{\infty}$ (hep-ph/0002048) [for ref. 4]

3-loop $O+\infty+\infty+\infty$
B) conceptual problem: convergence of weak coupling expansion

- screened pert. theory [Kanzaki et al.] 1997
  - massless → massive theory: $\pm w^2$; $\partial^4$ only
  - $\mathcal{L}$ optimal? gap? min. scale?
  - done to $O(g^2)$

$\mathcal{A}^2 \rightarrow 2000$, [Aharon/Bruckner/Strickland J] ???

- HTL pert. th. (screened p.t. $g^2$ for gauge theories (QCD),
  since local mass term not p.t. o.k.)
  - unlocal mass term
    - $\mathcal{A}^2$? local et cetera unclear...

  → off-props. are complicated fields of momenta
  - done to $O(1)$ [Aharon/Bruckner/Strickland]

PRD 61 (00) 014017: $\bigcirc$
PRD 61 (00) 074016: $\bigcirc$

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**Dimensional Reduction**

Note: high $T$ QCD governs a hierarchy of length scales

- $(T)^{-2}$ → inverse of typical momenta of plasma particles
- $(gT)^{-1}$ → electric screening length; first scale of collective excitation
- $(g^2T, \frac{1}{2})^{-1}$ → damping length of color exc.
- $(g^2T)^{-1}$ → non-pert. magnetic fluctuations

We have not seen yet

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**Scales**

\[
T, \frac{1}{T}
\]

\[
\frac{\sum_{\nu=m+1}^{\nu=m}}{m = 0, \frac{1}{e^2 + e^{-2}}}
\]

\[
\begin{cases} 
  1 \text{ massive field (QCD)} \\
  \infty \text{ many massive fields; one per Fourier mode}
\end{cases}
\]

\[
\text{truncated sum \to more fields; close to } S^4 \text{ dec. \to 3D!!}
\]

Integrated and scale $T$ (heavy modes \( \to \) loops only; light exctual
\( \to \) give effective couplings)
before: \( X : g^3, T \rightarrow S_u(n) 3\times D \), fields \( A^\mu \)

after: \( X : g^3, m^3, \lambda_3 \rightarrow S_u(n) + u_i, H_{ij}, 3D \), fields \( A_i, A_\mu \)

\( f_{\gamma\gamma} \) as result of integration out

\[
\begin{array}{c}
\text{QCD} \\
\text{SU(1)} \\
\text{SU(2)} \\
\text{SU(3)}
\end{array}
\]

\( \rightarrow \) unrenormalizability of 3D theories

two big advantages: 3D is super-renormalizable!

(finite # of diver. diagrams)

\( \rightarrow \) it's known exactly to 2-loop

\( \bullet \) 3D much easier on lattice!

[Leutwyler et al.]

things done: derive mass, free en. ... [Brambilla/Nolo]

\( \bullet \) integrate out more scales of fermions:

recently \( \gamma T \rightarrow \gamma H_{ij} \) [Brodsky, co. 1998]

\( \gamma T \rightarrow \gamma H_{ij} \) unresolved

e.g. [Gurwin, hep-ph/0004046]

\[ \text{Linel: IR problem} \]

we have seen: \( \Phi \) (in our L) appears can vanish \( \rightarrow \) unscreened propagator

\( \text{consider } \Phi \)-loop graph for \( \Phi \)
Leading IR behaviour?

$$F(T) = \frac{1}{a^3 + b^2 a^2} \quad \omega_n = 2\pi nT$$

Leading IR: $n=0$

$$\Rightarrow (T^2 \frac{\sqrt{d^2 \nu}}{\nu})^{\frac{3}{2}} \frac{g \nu^2}{\nu^2} \left( \frac{1}{\nu^2} \right)^{\frac{3}{2}} \mu_0 = \left( \frac{d^2 x}{x^2} \right)^{\frac{3}{2}}$$

$$\Rightarrow \begin{cases} \text{ok for } \nu < 3; \text{ big dev for } \nu \geq 3 \left( g^{\nu^2} x^{\nu} \right) \\ \text{dev for } \nu > 3 \left( g^{\nu^2} \left( \frac{x^{\nu}}{x^{\nu}} \right)^{\nu} \right) \end{cases}$$

Long : Plams, screening $\omega \gg T \Rightarrow$ end. of IR. $T > g^2 \gg$ here: $g \gg 3$

Trans. plams, earliest $\omega \gg T \Rightarrow$ all $\sim g^2 T$.

Complete failure of part. 1.1.

2.1 D Yang Mills progress! hot 900

[ Kamalabahi / Kim / Nasri] several of papers 1998 (badda hepp!; extract gii daaf's)

→ solvable? (well, so far only coarse fun - intractable?)

→ three miss-ap

via doc. red. → this is 'magical mass'!!??

Lots to do $\Rightarrow$ FTT $\Rightarrow$ invitation! (not many open fields or them physics...)

(re.d.) young field (1990) $\Rightarrow$ (bg) discoveries

+ Breakthroughs possible!

God's games (ch.) are done $\Rightarrow$ well stuff to be attacked.