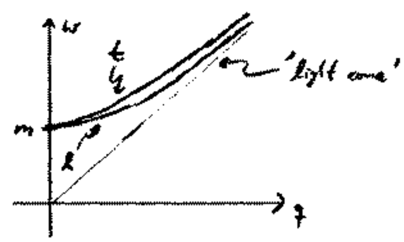
$$\rightsquigarrow \quad \omega_t^2 = q^2 + \tfrac{3}{2} m^2 g\left(\tfrac{\omega}{q}\right)$$
$$\omega_\ell^2 = q^2 + 3m^2 \left(1 - g\left(\tfrac{\omega}{q}\right)\right)$$

transcendental eqns for $\omega_{t/\ell}$! cannot solve explicitly.

- look at limits:
  a) $q \to 0$: $\omega_t^2(0) = \frac{3}{2} m^2 g(\infty) = m^2$
  $\omega_\ell^2(0) = 3m^2(1-g(\infty)) = m^2$

  b) $q \to \infty$: $\omega_t^2(q \gg m) = q^2 + 0 = q^2$
  $\omega_\ell^2(q \gg m) = q^2 + 3m^2 \to q^2$

- solve numerically



(126)

  c) $q \ll m$: $\omega_t^2 = q^2 + \frac{3}{2} m^2 g\left(\frac{\omega}{q} \gg 1\right) = m^2 + \frac{6}{5} q^2 + \dots$
  $\omega_\ell^2 = q^2 + 3m^2\left(1 - g\left(\frac{\omega}{q} \gg 1\right)\right) = m^2 + \frac{3}{5} q^2 + \dots$

((• the fct. $g(z)$, Eq. (122), has a richer analytic structure. cuts in z-plane. here, we only considered soft Q_0's. full treatment ...

- there is no damping ($=$ imag. part of Π) in leading order, since $\ln\left(\frac{z+1}{z-1}\right)$ does not have an imag. part at $z = 1 + i\varepsilon$.

- 2-loop-calculation gives
  - next-to leading order for ω
  - leading order for damping γ :

$$\Rightarrow \quad \omega^2(q) = \omega_0^2(q) \left[1 + (\eta + i\eta_{im})\, g\sqrt{C_A} + \dots \right] \qquad (\vec{q} = 0)$$

see above

LO-damping ; g.i. ; [Braaten/Pisarski, PRD42 (1990) 2156]

NLO-pl.freq ; g.i. ; cf. [Schulz, NPB 413 (1994) 353]

...))

<u>damping</u>

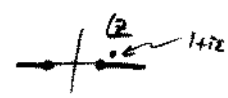
((from lin resp., $Q_0 \to \underset{\text{pl.fr.}}{\omega} + i\underset{\text{damping}}{\gamma}$))

$\gamma \ll \omega$ (so waves <u>do</u> propagate)

pole of propag.: $0 = Q^2 + \Pi(Q)$

$$\leadsto \omega^2 + 2i\gamma\omega + O(\gamma^2) = \vec{q}^{\,2} - \Pi(\omega + i\gamma, q)$$

$$\left.\begin{aligned} &\Rightarrow \text{Real part gives} \quad \omega^2 = \vec{q}^{\,2} - \mathrm{Re}\,\Pi(\omega+i\gamma, q) \\ &\quad \text{Imag. part gives} \quad \gamma = -\frac{1}{2\omega}\,\mathrm{Im}\,\Pi(\omega+i\gamma, q) \end{aligned}\right\} \quad (125a)$$

$\omega > q \leadsto z = \frac{\omega}{q} > 1 \leadsto$ 

$\ln\left(\frac{z+1}{z-1}\right)$ does <u>not</u> have an imag. part @ $z = 1 + i\varepsilon$

$\Rightarrow$ <u>no damping</u> at leading order, $\gamma^{(LO)} = 0$ (126)

NLO calc. required

<u>10) Linear response theory @ finite T</u>

$\leadsto$ Literature:

- Kapusta, '89 $\leftarrow$ gauge problems; HTL's not discovered yet.

  <u>§6</u>, <u>§6.4</u> : dispersion relations (QED)

  <u>§8.5</u> : QCD, but 4 gauge

- Le Bellac, '96

  <u>§6</u> : different notation, but easy to adapt to ours.

  $\leadsto$ <u>long.</u> excitation $\hat{=}$ "plasmons"

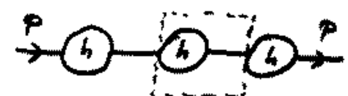
((we only needed dispersion relation, $0 = Q^2 + \Pi(Q)\big|_{Q_0 \to \omega + i\gamma}$))

11) hard thermal loops (HTL's); eff. action

know from chapter 9): leading $\Pi \sim T^2$
(obtained in 'hard' approx.)

can be generalized to N-point fct's!
some (not all) are $\sim T^2$ also.

systematics?



$$\sim g^2 \sum_K^{\text{hard}} \frac{K\cdot K}{K^2(K-P)^2}\cdot\frac{1}{P^2} \sim g^2\cdot T^2\cdot\frac{1}{P^2}$$

$= 1$ if $P \sim gT$ ('soft') ???

⇒ hard 1-loop must be included to all orders!

⇒ for soft-scale physics, $G = \frac{A}{Q^2+\Pi_t^{\text{hard}}(Q)} + \dots$

is leading order (tree-level) propagator!

(for $Q^2 \to T^2$, $G \to G_0$ automatically)

more 1's?

⇝ need 'hard' power counting rules [Braaten/Pisarski, NPB 337(1990)569] BP

(i) $\sum_K \frac{1}{K^2} \sim T^2$ for first propagator
(ii) $\frac{1}{PT}$ for each additional propag.
(iii) K's in numerator $\sim T$
(iv) if ≥ 2 bosonic (ghosts, too) propags $\to P/T$

(127)

((actual computation of sum-integral might have cancellations ⇝ can have less T-powers. power-counting gives max. T^n))

test:  $\sim g^2 \sum_K \frac{1}{K^2} = -g^2\frac{T^2}{12}$; rules : $g^2\cdot \overset{(i)}{T^2}$ ✓✓

test: $\sum_K^{\text{hard}} \frac{\underline{k}^2}{K^2(K-P)^2} = \frac{T^2}{24}$; rules $\overset{(i)}{T^2}\cdot\overset{(ii)}{\frac{1}{PT}}\cdot\overset{(iii)}{T^2}\cdot\overset{(iv)}{\frac{P}{T}} = T^2$ ✓✓

now:  $\sim g^3 \sum \frac{kkk}{k^2(\ )^2(\ )^2} \sim g^3 \cdot T^2 \cdot \frac{1}{(PT)^2} \cdot T^3 \cdot \frac{P}{T} = gP\left(\frac{gT}{P}\right)^2$

tree-level vertex $\lambda \sim gP$

$\Rightarrow$ if $P \sim gT$ (soft), then 3-vertex has to be resummed !

now:  $\sim g^n \cdot T^n \cdot \frac{1}{(PT)^{n-1}} \cdot T^n \cdot \frac{P}{T} = \underbrace{g^{n-2} P^{4-n}}_{} \left(\frac{gT}{P}\right)^2$

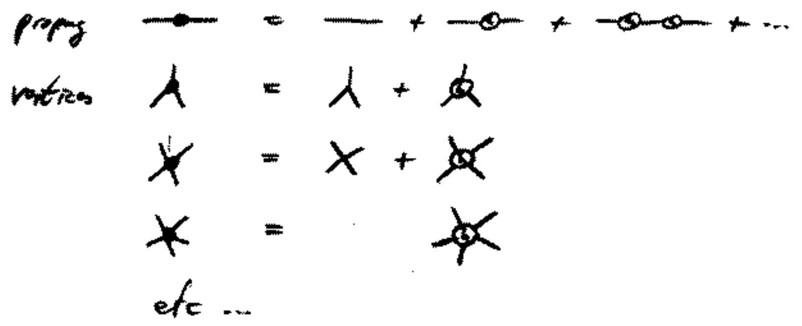
same

or  $\sim$

tree-level n-vertex (dim. reasons)
(if existent)

$\Rightarrow$ if $P \sim gT$ (soft), then 4-vertex has to be resummed,
new 5,6,7,... -vertices appear @ tree-level!

$\Rightarrow$ new 'effective' expansion has leading elements



effective action

... try to formulate systematics in a compact way ...

note: had seen (§9) that HTL in Π is gauge independent.

more general feature: can generate all HTL's by a
manifestly gauge invariant eff. action

$\rightsquigarrow$ systematic pert. th., g.i. order by order !!!
with new tree-level propag and vertices etc

write $\quad S = \underbrace{S + S_{eff}}_{LO} \underbrace{- S_{eff}}_{\text{higher order}}$

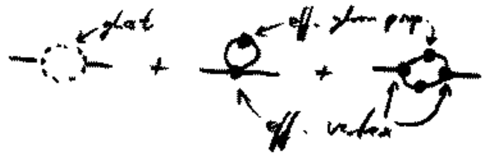
$$\left.\begin{aligned} \mathcal{L}_{eff} &= \frac{g^2 C_A T^2}{6} \,\mathrm{Tr}\left(F_{\mu\rho} \int_\Omega \frac{Y^\rho Y^\lambda}{(Y \cdot D)^2} F_\lambda{}^\mu\right) \\ Y^\mu &= (1, \vec{e}) \ , \quad \int_\Omega = \frac{1}{4\pi}\int_0^{2\pi} d\varphi \int_{-1}^{1} d(\cos\theta) \ , \quad D_\mu^{ab} = \delta^{ab}\partial_\mu - gf^{abc}A_\mu^c \end{aligned}\right\} \quad (128)$$

e.g. $\int^\beta \mathcal{L}_{eff}^{(g^2)} = \ldots = \frac{1}{2}\sum_K \tilde{A}_\mu^a(-K)\, \Pi_{hard}^{\mu\nu}(K)\, \tilde{A}_\nu^a(K)$

⇝ naive perturbation theory fails for soft ($\sim gT$) external momenta.

→ correct by resummed (or effective) pert. exp. for consistent (and g.i. !) calc. of higher orders.

ex.: 1^{st} correction to Π has contributions from



and gives as result (at $\vec{q} \to 0$)

$$\omega^2(q) = \omega_0^2(q) \left[1 + (\eta + i\eta_{im})\, g\sqrt{C_A} + \dots \right]$$

↑ LO damping
NLO pl. freq.
} Lit: cf. FT²/57

⇝ the eff. action (g.i., true zeroth order) makes it conceptually clear how to do pert. calculations.

But: technically extremely hard.

slow convergence of pert. series (⇝ see comments in chapter 11)

Lit for eff. action: [BP, PRD 45 (1992) R1827] (rapid comm.)

[Frenkel/Taylor, NPB 374 (1992) 156]

EXAM: (take-home)

pick up Mon, 15.5., 14^{00} (room 510)

return Wed, 17.5., 16^{00} (latest)