2) (review of) Stat Mech

(Who remembers Stat Mech by heart? \( \rightarrow \) quick who?)

The central eq:

\[
S = -\frac{1}{\beta} P_0 \ln(p_0) = -T \ln(S_h(s))
\]  

for every ensemble (here: canonical)

Reservoir

(or; E, V, warm)

# of states = \( \Omega \)

equal prob. \( (\frac{1}{\Omega}) \) for every state

\( \Rightarrow S = -\sum_{\Omega} \frac{1}{\Omega} \ln\frac{1}{\Omega} = k_B \Omega \)

now, look at small subsystem (1 particle?) (canonical, \( \beta E \) only)

\[ \gamma = \gamma_1 + \gamma_2 + \gamma_3 + \cdots \]

\[ \gamma = \sum_{i} \gamma_i \]  

\( \beta \) of R-states, if subsystem is in state \( i \)

\[
S = k_B \gamma = \frac{k_B \gamma_i}{\beta} - \frac{k_B \gamma_i}{\beta} \ln \left( \frac{1}{\beta} \right) \equiv p_i \left( \text{for subsystem to be in state } i \right)
\]

\[ S_R(E - \gamma_i) = S_R(E) - \gamma_i \beta \partial \gamma S_R(E) \quad \text{(E}\geq0 \text{ always true)} \]

\( \Rightarrow \beta p_i = -[S - S_R(E)] - \beta \gamma_i \), where \( \beta = \partial \gamma S_\gamma \)  

\( \Rightarrow p_i = \frac{\exp(-\beta \gamma_i)}{\sum_i \exp(-\beta \gamma_i)} \rightarrow g = \frac{1}{\beta} \exp(-\beta \gamma) \quad \text{(3a)} \)

\( \beta \)'s = eigenvalues of \( g \)

normalize:

\[ \frac{1}{\beta} \sum_i p_i = \frac{1}{\beta} \sum_i \exp(-\beta \gamma_i) \rightarrow g = \text{Tr} e^{-\beta H} \quad \text{(3b)} \]
Expectation value (ensemble average)
\[
\langle \text{any} \rangle = \sum_{\rho} \rho \langle \text{any} \rangle_{\rho},
\]

\( \text{any} \) = operator \( A \) : \( \langle \text{any} \rangle_{\rho} = \text{Sdet} \, \psi_{\rho}^{\ast} A \psi_{\rho} \)

\[
\langle A \rangle = \sum_{\rho} \rho \, \text{Sdet} \, \psi_{\rho}^{\ast} A \psi_{\rho}
= \sum_{\rho} \frac{1}{2} e^{-\alpha \Phi} \text{Sdet} \, \psi_{\rho}^{\ast} A \psi_{\rho}
= \sum_{\rho} \text{Sdet} \, \psi_{\rho}^{\ast} \frac{1}{2} e^{-\alpha \Phi} A \psi_{\rho}
= \sum_{\rho} \text{Sdet} \, \psi_{\rho}^{\ast} S \Phi \psi_{\rho} \quad (= \sum \langle 0 | S A | n \rangle)
\]

\[
\langle A \rangle = \text{Tr} (S A) \quad (4)
\]

\( e^{-S} \Rightarrow \tau(\Phi) \eta(\eta) \)

note that for \( T \to 0 \) \( (\beta \to \infty) \), \( e^{-\alpha \Phi} \xrightarrow{\beta \to \infty} e^{-\beta \Theta} \)

\( \rho = 1 \), always \( = 0 \)

→ propagator \( \langle 0 | \tau(\Phi) \eta(\eta) | 1 \rangle \) is \( T \to 0 \) result of eq. (4)

\( \Rightarrow A = H \)

\[
E = \langle H \rangle = \text{Tr} \left( \frac{1}{2} e^{-\beta H} \Phi \right) = \frac{1}{2} \text{Tr} \left( e^{-\beta \Phi H} \right)
E = -\beta \Phi L Z \quad (5) \quad (\beta \Phi = -T \beta)\]

Chamody's relations

\[
\Phi = -TLZ \quad (6)
\]

\( \Rightarrow (\beta, \rho, \lambda) = (\beta, \rho, \lambda) = \Phi \quad (7)
\]

Check \( S \):

\[
\rho = \frac{1}{2} e^{-\beta \Phi} ; \quad L \Phi = -LZ - \beta \Phi ;
\]

\[
S = -\zeta \Phi ; \quad L \Phi = -\frac{1}{2} e^{-\alpha \Phi} (LZ + \beta \Phi) = \lambda L + \frac{1}{2} e^{-\alpha \Phi} \beta \Phi ;
\]

\[
L Z - \beta \Phi \lambda = \lambda L + T \beta + \lambda \beta L
\]

\[
T L Z = -2 \lambda \Phi \lambda
\]
\[ V = A \cdot \mathbf{L} \]

force \( = E \text{-deflec}, \quad A \cdot \mathbf{F}_A = -\partial E_X \quad \Delta X = \Delta t \mathbf{F}_A \)

average \( \Rightarrow A \cdot \mathbf{F}_A = A \cdot \langle \mathbf{p}_A \rangle = \frac{\hbar}{\Lambda} e^{-\beta \hbar} \langle -\hbar \mathbf{p}_A \rangle = -\hbar \frac{\beta \hbar}{\Lambda} e^{-\beta \hbar} \Delta \mathbf{F}_A \)

note that \( \partial X = -T \Delta X \partial \mathbf{2} = -T \frac{\hbar}{\Lambda} \partial \mathbf{2} = \left[ \frac{\hbar}{\Lambda} e^{-\beta \hbar} \mathbf{p}_A \right] \partial X \Rightarrow \mathbf{2} = -\mathbf{F} \mathbf{W} \)

\( \mu \) is def \( \Rightarrow \mathbf{Z} = \mathbf{Z(N)} \)

conserved quantity (in \( R \)-system)

defines \( \mu \) - 'chemical potential'

usually \( N = \text{part. #} \) (grand canonical; canonical: \( \mu = 0 \))

\[ \begin{array}{c}
\text{Specify systems (coupled to } R) \\
\hline
T \quad \hline
H, \chi
\end{array} \]

\( \text{ex.1) two-state system } \quad \beta = \frac{\hbar}{\Lambda} \quad (\text{no interaction}) \\
\mathbf{H} = -\hbar \mathbf{2}^2, \quad \mathbf{Z} = e^{\beta \hbar} + e^{-\beta \hbar} \quad (\sigma^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}) \mathbf{D} \\
\]

\( \text{ex.2) harmonic oscillator} \)

\[ \mathbf{Z} = \frac{\hbar}{\Lambda} e^{-\beta \hbar \mathbf{2}} = \frac{1}{1-e^{-\beta \hbar \mathbf{2}}} \quad (\text{geometric series}) \]

\[ \langle n \rangle = \frac{1}{\hbar \mathbf{2}} \langle \mathbf{E} \rangle = \frac{\mathbf{E}}{\hbar \mathbf{2}} \mathbf{2} \mathbf{Z} = \ldots = \frac{1}{e^{\beta \hbar \mathbf{2}} - 1} = n \text{ (level) } \]

\[ n_{\text{state}}(x) = \frac{1}{e^{\beta \hbar \mathbf{2}} - 1} \quad (8) \]

\( \text{ex.3) quark: one cosmic d.a.f.} \)

\[ \text{any } \#; \quad \langle n \rangle, \quad \langle n|n'| \rangle = \delta_{nn'}, \quad \sum_{n=0}^{\infty} |n \rangle \langle n| = 1 \]

\[ b^+ |n \rangle = \sum_{n=1}^{\infty} |n+1 \rangle \quad , \quad b|n \rangle = \sum_{n=0}^{\infty} |n \rangle \langle n-1| \]
\[ N^\pm = \hat{N}^\pm \quad (N^+ |N^-\rangle = n |n\rangle ) \]

\[ [\hat{N}^+, \hat{N}^-] = \Sigma \{ \hat{N}, \hat{N}^\pm \} = 1 \]

\[ |n\rangle = \frac{1}{n!} (\hat{b}^\dagger)^n |0\rangle \]

\[ \hat{N} = \frac{\hat{b}^\dagger \hat{b}}{2} \quad \langle \text{normal ordering} \rangle \quad \hat{b}_i^\dagger = \hat{b}^\dagger \hat{b} \]

\[ \hat{N} = \frac{\hat{b}^\dagger \hat{b}}{2} = \langle \hat{N}^+ + \hat{N}^- \rangle \]

\[ \text{no \ omission \ of \ off \ cond.} \]

\[ \text{ex. 3b) one fermion def. } \quad \langle \text{b) certain problem} \rangle \]

\[ \text{Pauli exclusion principle } \quad \text{only states } |0\rangle, |1\rangle \]

\[ f^{|0\rangle} = 1, f^{|1\rangle} = 10, \quad \text{rest } = 0 \]

\[ ff = f^{|0\rangle} f^{|0\rangle} = 0 \]

\[ \text{only states } |0\rangle, |1\rangle \quad \text{are allowed} \]

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\[ \langle \hat{N}^+ - \hat{N}^- \rangle = 0 \]
\[ E = \sum_i \frac{\hbar c \Delta_i}{2} = \frac{V}{(2\pi)^2} \int \frac{d^3k}{e^{\hbar c k / T} - 1} = \frac{V}{2\pi^2} \int_0^{\frac{\hbar c}{k}} \frac{x^2}{e^{x} - 1} \, dx \]

\[ \sqrt{E^3 - \frac{\hbar^2 c^3}{3} k^3} = \frac{2\pi}{l^3} \ll k \ll \frac{2\pi}{l} \]

(Note that for \( k \rightarrow 0 \), classical ED is wrong.)

\[ T = E - TS = E + T_{de} \mathbb{P} \]

\[ T = E - T(v, T) - T(T, T) \]

\[ T = -\frac{\hbar^2 c^2}{4\pi} \, T^3 \quad , \quad p(T, T) = \frac{\hbar^2 c^2}{4\pi} \, T^3 \]

\[ (\rho'^{1/2} \frac{du}{u^2} = \text{an pressure} \Rightarrow T = 10^{5} \, \text{K}) \]

(ex. 5) Standard Model

\[ X_{sp} = \frac{1}{2} \sum_{\text{focusing}} T \partial \phi^+ \partial \phi^+ + \frac{1}{4} \sum_{\text{4 terms}} \sum_{\text{focusing}} \frac{T^2 \partial \phi^+ \partial \phi^+}{2 \lambda_{0}^2} \]

\[ \text{90 components} \quad ; \quad 3 + (3 + 4 \cdot 3) \cdot 2 = 6 \cdot 15 \]

18 parameters: (11 mass, 3 couplings, 4 Cabibbo angles)