(1) $9y^2+5a^2+x^2=\text{const.}$, $(x/3)^2+y^2=C^2$, Ellipsen

(2) $I=-\partial_\alpha J|_{\alpha=1}$ mit $J=\int_0^\infty du\, e^{-\alpha u^2} = \frac{\sqrt\pi}{2}\alpha^{-1/2}$ ⇒ $I=\frac{\sqrt\pi}{4}$

(3) $A = \int_0^\pi d\alpha\,(\partial_\alpha \vec r)\cdot(-\vec K(\vec r)) = \int_0^\pi d\alpha\, R\alpha\,(s,c)\cdot\left(\frac{\gamma m M}{r^3}\vec r\right)$ mit $s\equiv\sin(\alpha)$, $c\equiv\cos(\alpha)$

$r^2=R^2(1+\alpha^2)$, $(s,c)\cdot\vec r = R$

$= \frac{\gamma m M}{R}\int_0^\pi d\alpha\left(\frac{\alpha}{(1+\alpha^2)^{3/2}} = \partial_\alpha\left[-(1+\alpha^2)^{-1/2}\right]\right) = \frac{\gamma m M}{R}\left(-\frac{1}{\sqrt{1+\pi^2}}+1\right)$

oder $\vec K = \vec\nabla\frac{\gamma m M}{r} = -\vec\nabla V$, $A = V(r(\pi))-V(r(0)) = V(R\sqrt{1+\pi^2})-V(R) = -\frac{\gamma mM}{R}\left(\frac{1}{\sqrt{1+\pi^2}}+1\right)$

(4) $M=\rho_0 V_{HK} = \rho_0\frac{2\pi R^3}{3}$, $\vec R=(0,0,R_3)$, $MR_3=\rho_0\int_{(HK)} d^3r\, z$

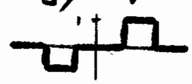
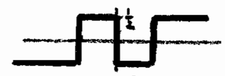
Zylinderko.: $MR_3=\rho_0 2\pi\int_0^R d\rho\,\rho\int_0^{\sqrt{R^2-\rho^2}}dz\,z = \rho_0\pi\int_0^R d\rho\,\rho(R^2-\rho^2) = \rho_0\pi\left(\frac{R^4}{2}-\frac{R^4}{4}\right)$

oder Ku.: $MR_3=\rho_0\int_0^R dr\,r^2\int_{(2\pi)}d\varphi\int_0^{\pi/2}d\vartheta\,\sin(\vartheta)\,r\cos(\vartheta) = \rho_0\frac{R^4}{4}2\pi\int_0^{\pi/2}d\vartheta\left(sc=\partial_\vartheta\tfrac12 s^2\right)=\rho_0\pi\frac{R^4}{4}$

⇒ $R_3=\frac38 R$

(5) (a) $1\stackrel{!}{=}\alpha\int_0^\infty dx\,e^{-x^2/\varepsilon^2}=\alpha\varepsilon\frac{\sqrt\pi}{2}$, $\alpha=\frac{2}{\varepsilon\sqrt\pi}$

(b) $1\stackrel{!}{=}\gamma 2\pi\int_0^\infty dr\left(re^{-r^2/\varepsilon^2}=\partial_r\left[-\frac{\varepsilon^2}{2}e^{-r^2/\varepsilon^2}\right]\right)=\gamma\pi\varepsilon^2$, $\gamma=\frac{1}{\pi\varepsilon^2}$

(c) $\partial_x\theta=\delta$ ⇒   

(6) $\dot v+Pv=Q$ mit $P=\alpha$, $Q(t)=b_0e^{\beta t}$. PQ ⇒ $v_{allg.}(t)=e^{-\int_{t_0}^t dt' P}\left(C+\int_{t_1}^t dt'\,Q(t')\,e^{\int_{t_0}^{t'}dt''P}\right)$

⇒ $v_{allg}(t)=e^{-\alpha(t-t_0)}\left(C+\int_{t_1}^t dt'\,b_0e^{\beta t'}e^{\alpha(t'-t_0)}\right)=\tilde C e^{-\alpha t}+\frac{b_0}{\alpha+\beta}e^{\beta t}$

(7) $\sin(\varphi)=b/r$ ⇒ $r(\varphi)=b/\sin(\varphi)$

(8) (a) $\vec\nabla\vec a\cdot(\vec b\times\vec r)=\partial_i a_j\varepsilon_{jkl}b_k x_l = a_j\varepsilon_{jki}b_k=\varepsilon_{ijk}a_jb_k=\vec a\times\vec b$

(b) $(\vec r\cdot\vec\nabla)r^2=\vec r\cdot 2r\vec\nabla r=\vec r\cdot 2r\frac{\vec r}{r}=2r^2$, $e^{\vec r\cdot\vec\nabla}r^2=e^2r^2$

(9) $(\vec r\cdot\vec\nabla)\vec\omega\times\vec r=\vec\omega\times(\vec r\cdot\vec\nabla)\vec r=1\cdot\vec\omega\times\vec r$, $\vec A=-\frac13\vec r\times(\vec\omega\times\vec r)$

$\vec\nabla\times\vec A=-\frac13\vec\nabla\times(\vec\omega r^2-\vec r(\vec\omega\cdot\vec r))=-\frac13\left(-\vec\omega\times\underbrace{\vec\nabla r^2}_{=2\vec r}-\underbrace{(\vec\nabla\times\vec r)}_{=0}(\vec\omega\cdot\vec r)+\vec r\times\underbrace{\vec\nabla(\vec\omega\cdot\vec r)}_{=\vec\omega}\right)=\vec\omega\times\vec r=\vec B$ ✓

(10) $n=N\frac{3}{4\pi R^3}$, Cont.: $\vec\nabla\cdot\vec j=-\dot n=\frac{9N v}{4\pi R^4}$, Ansatz: $\vec j=\vec r f(r,t)$

$3f+\vec r(\vec\nabla\vec r)\partial_r f = \boxed{3f+rf'=\frac{9Nv}{4\pi R^4}}$, $f_{hom}=\frac{c}{r^3}$ (z.B. via Potenzansatz), $f_{spez}=\frac{3Nv}{4\pi R^4}$

$c\stackrel{!}{=}0$, damit $\vec j=\vec 0$ am Ursprung ⇒ $\vec j=\vec r\frac{3Nv}{4\pi R^4}$

(11) $\Delta_r\chi=\frac1r\partial_r\partial_r\frac{r}{r+\varepsilon}=\frac1r\partial_r\frac{\varepsilon}{(r+\varepsilon)^2}=\frac{-2\varepsilon}{r(r+\varepsilon)^3}$

$\int d^3r\left(\frac{-2\varepsilon}{r(\,)^3}\right)=-2\varepsilon\,4\pi\int_0^\infty dr\,\frac{r+\varepsilon-\varepsilon}{(\,)^3}=-2\varepsilon\,4\pi\left[-\frac{1}{(\,)}+\frac{\varepsilon}{2(\,)^2}\right]_0^\infty=-4\pi$ ✓

(12) $c_n=\frac1L\int_0^a dx\,e^{-in\frac{2\pi}{L}x}$, $c_0=\frac aL$, $c_{n\neq0}=\frac{i}{2\pi n}\left(e^{-in\frac{2\pi}{L}a}-1\right)$

(13) $\tilde\rho(\vec k)=\int d^3r\,\gamma\,\delta(r-R)e^{i\vec k\vec r}=\gamma 2\pi\int_0^\infty dr\,r^2\delta(r-R)\underbrace{\int_{-1}^1du\,e^{ikru}}_{=\frac{2}{kr}\sin(kr)}=\gamma 4\pi R\frac{\sin(kR)}{k}$