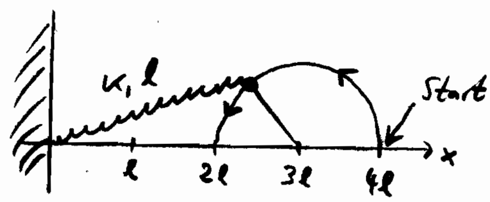
„$\int_{\mathcal{C}}$ – Fahrplan"	am bsp Kreisumfang
1. Größe, d.h. $\int_{\mathcal{C}}$-Art	$U = \int_{Kreis} ds$
2. spezif. $\mathcal{C}$	$\vec{r}(t) = R(\cos(t), \sin(t), 0)$
3. t_1, t_2	$t_1 = 0,\ t_2 = 2\pi$
4. $\dot{\vec{r}} =: \vec{v}$, ggf v bilden t-Integral	$\vec{v} = R(-s, c, 0)$ $U = \int_0^{2\pi} dt\ R$
5. $\vec{r}(t)$ in Integrand einsetzen	– entfällt –
6. ggf. Skalarprod. ausführen	– entfällt –
7. gew. Int. auswerten	$U = R \cdot 2\pi$

<u>Bsp</u>: Mausefalle



Arbeit A als Kurvenintegral!
(Fahrplan-Illustration)

((vorweg: es muß $A = V_{Start} - V_{Ende}$ herauskommen
$= \frac{\kappa}{2}(4l - l)^2 - \frac{\kappa}{2}(2l - l)^2 = \frac{\kappa}{2} l^2 (9-1) = 4\kappa l^2$))

1. $A = \int_{\mathcal{C}} d\vec{r} \cdot \vec{K}(\vec{r})$, $\vec{K}(\vec{r}) = \kappa \underbrace{\left(\frac{-\vec{r}}{r}\right)}_{\vec{e}_{m \to Haken}} (r - l)$

2. $\mathcal{C}$: $\vec{r}(t) = l(3 + \cos(t), \sin(t))$

3. $t_1 = 0,\ t_2 = \pi$

4. $\dot{\vec{r}} = \vec{v} = l(-s, c)$

$$A = \int_0^{\pi} dt\ l(-s, c) \cdot \kappa(-\vec{r})\left(1 - \frac{l}{r}\right)$$

5. $r = |\vec{r}| = l\sqrt{9 + 6c + c^2 + s^2} = l\sqrt{10 + 6c}$

$$A = l^2 \kappa \int_0^{\pi} dt\ \underbrace{(-s, c) \cdot (-3 - c, -s)}_{3s + \cancel{sc} - \cancel{sc}} \left(1 - \frac{1}{\sqrt{10 + 6c}}\right)$$

6.

7. $$= 3l^2\kappa \int_0^{\pi} dt\ \underbrace{\left(\sin(t) - \frac{\sin(t)}{\sqrt{10 + 6\cos(t)}}\right)}_{= \partial_t\left[-\cos(t) + \frac{1}{3}\sqrt{10 + 6\cos(t)}\right]}$$

$$= 3l^2\kappa\left(1 + \frac{1}{3}\sqrt{4} + 1 - \frac{1}{3}\sqrt{16}\right) = 4l^2\kappa \quad ✓$$

5 $\int_C$ – Arten: $\int_C ds \begin{Bmatrix}\phi \\ \vec A\end{Bmatrix}$ ist Skalar / Vektor, $\int_C d\vec r \begin{Bmatrix}\phi \\ \cdot\vec A \\ \times \vec A\end{Bmatrix}$ ist V. / S. / V.

manchmal geometrisch auswertbar, z.B.:

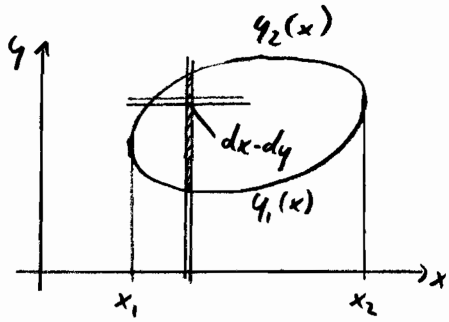
$\vec E = \alpha\, \vec e_3 \times \vec r$,

$\oint_{\text{Kreis}(R)} d\vec r\times\vec E = \vec 0$ (weil $d\vec r$ stets $\parallel \vec E$)

$\oint_{\text{Kr.}(R)} d\vec r\cdot\vec E = 2\pi R\cdot\alpha R$ (weil $d\vec r\cdot\vec E = ds\cdot|\vec E| = ds\cdot\alpha R$)

oft hilft, C geschickt zu legen!

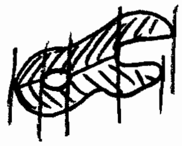
ebenes Flächenint.



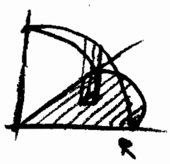
$\phi(x,y) = \frac{\text{etwas}}{\text{Fläche}}$ ← Masse, Heu, ...

gegeben, dann

$$\left.\begin{matrix}\text{gesamtes}\\ \text{etwas}\end{matrix}\right\} = \int_F d^2r\cdot\phi(x,y) \underset{\text{ausrechnen}}{=} \int_{x_1}^{x_2}dx \underbrace{\int_{y_1(x)}^{y_2(x)} dy\;\phi(x,y)}_{\text{Streifen-etwas}}$$

((Randkurve? Immer? ))

Bsp Kugelvolumen



$x_1=0,\ x_2=R,\ y_1(x)\equiv 0,\ y_2(x)=\sqrt{R^2-x^2}$, $\phi = $ Höhe $=\sqrt{R^2-(x^2+y^2)}$

$$V_R = 8\cdot\int_0^R dx\int_0^{\sqrt{R^2-x^2}} dy\, \sqrt{R^2-x^2-y^2}\,, \quad y\to\sqrt{R^2-x^2}\,y$$

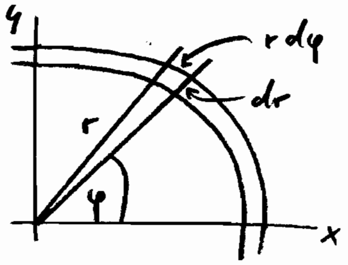
$$= 8\cdot\int_0^R dx\,(R^2-x^2)\underbrace{\int_0^1 dy\,\sqrt{1-y^2}}_{=\int_0^{\pi/2} d\varphi\,\cos^2(\varphi) = \pi/4}$$

$y=\sin(\varphi)$, $\frac{dy}{d\varphi}=\cos(\varphi)$

$$\overset{x\to Rx}{=} 2\pi R^3 \underbrace{\int_0^1 dx\,(1-x^2)}_{=1-[\frac13-0]=\frac23} = \frac{4\pi}{3}R^3$$

im letzten Bsp: kugeliges kartesisch ?!
brauchen "runde" Koordinaten!

Polarkoordinaten

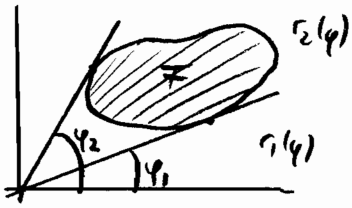


$$d^2r = dr\, r\, d\varphi$$

$$x = r\cos(\varphi), \quad y = r\sin(\varphi)$$

$$r = \sqrt{x^2+y^2}, \quad \varphi = \arctan\left(\frac{y}{x}\right) + n\cdot\pi$$

$$\Big(\big(\varphi \text{ in } (0,2\pi):\ n = 1 + \Theta(x) - 2\,\Theta(x)\,\Theta(y)\big)\Big)$$



$$\int_F d^2r\, \phi = \int_{\varphi_1}^{\varphi_2} d\varphi \int_{r_1(\varphi)}^{r_2(\varphi)} dr\, r\, \phi(r,\varphi)$$

Test an Kreisfläche ($\overset{?}{=} \pi R^2$)

$\phi = 1$,  , $4\int_0^{\pi/2} d\varphi \int_0^R dr\, r = 4\cdot\frac{\pi}{2}\cdot\frac{1}{2}R^2$ ✓✓

Bsp Kugelvolumen

$$V_R = 8 \underbrace{\int_0^{\pi/2} d\varphi}_{\pi/2} \int_0^R dr\, r\sqrt{R^2-r^2} \qquad r \to Rr$$

$$= 4\pi R^3 \int_0^1 dr\, \underbrace{r\sqrt{1-r^2}}_{=\partial_r\left[\left(-\frac{1}{3}\right)(1-r^2)^{3/2}\right]}$$

$$= 4\pi R^3\left(0 + \frac{1}{3}\right) = \frac{4\pi}{3}R^3 \quad ✓✓$$

Bsp Galaxie mit $\frac{\text{Masse}}{\text{Fläche}} =: \rho = \rho_0\, e^{-r^2/a^2}$, $M = ?$

$$M = \int_{\substack{\text{ganze}\\ \text{Ebene}}} d^2r\, \rho = \rho_0 \int_0^{2\pi} d\varphi \int_0^\infty dr\, r\, e^{-r^2/a^2}, \quad r \to ar$$

$$= \rho_0\, 2\pi\, a^2 \int_0^\infty dr\, \underbrace{r\, e^{-r^2}}_{=\partial_r\left[-\frac{1}{2}e^{-r^2}\right]}$$

$$= \rho_0\, 2\pi\, a^2\left(0 + \frac{1}{2}\right) = \rho_0\, \pi\, a^2$$