Einf. i. d. Meth. d. theor. Physik II → EMTP II

YS, E6-118 (Di 10-12:30 u.n.V.)

www.physik.uni-bielefeld.de/~yorks/emtp2

Orga Vorl Di 8.15-9.00, 9.10-9.55 (H6)
in Pause: Ü-Blatt holen
Ü-Liste eintragen (nur heute)
Übungen Do. 8-10, 10-12
Tutoren: s. Overhead
vor Vorl: Ü-Lsn in Karton
Regeln: (50%) Ü-Pkte + akt Mitarbeit ⇒ Ü-Schein
Ü-Schein + (eine) Klausur best ⇒ Schein
↳ alt OK ↳ 21.7.08, 6.10.08

EMTP I - Klausur: (Statistik: 80% bestanden; Gratu !!)
Besprechung / Fragen diese Woche in Ü → evtl. Afg.-zettel mitbringen
(neue Hörer? → als Wdh der EMTP I)
Ü-Schein: Pause
KI-Erfolg nicht EMTP II-Voraussetzung.

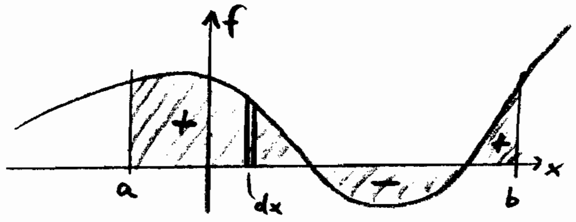
EMTP II: nicht schwerer als I. schöner! interessanter!

Integrale, krumme Koordinaten, δ
Differenzial glgn
Felder, Integralsätze
Fourier-Trafo

LIT → s. Web; Schulz PB

6. Integrale (+ deren Gebrauch i.d. Physik)

6.1. Gewöhnliche Integrale



Die so gezählte Fläche ist $\{\text{lin. Op.}\}\, f(x)$, denn (v.a.) $\{\ \}(-f) = -\{\ \}f$

$$\binom{\text{Fläche}}{\text{zw. } a,b} = \underbrace{\lim \sum}_{\int} \big(dx \cdot f(x)\big) =: \int_a^b dx\, f(x)$$

$$\int_b^a dx\, f := -\int_a^b dx\, f$$

$\int dx :=$ über alle x, d.h. $\int dx\, f := \int_{-\infty}^{\infty} dx\, f$, $\quad \int d\varphi := \int_{(2\pi)} d\varphi$

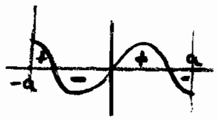
Dimension: $[\int dx\, f] = [x][f]$, $\quad [a] = [b] = [x]$

$\int$-Auswertung = Umformung, bis es trivial ist (d.h. die Fläche geometrisch erhältlich ist)

oder $f = \partial_x(\ldots\ldots)$, s.u. "Hauptsatz"

Beispiele: $\int_a^a dx\, f = 0$

$$\int_a^b dx\, \text{const}_x = (b-a)\cdot \text{const}_x \qquad \left(\int_a^{a+\varepsilon} dx\, f(x) \xrightarrow[\varepsilon\to 0]{} \varepsilon f(a)\right)$$



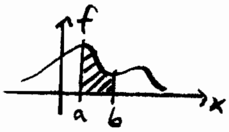
f ungerade $\Rightarrow \int_{-a}^{a} dx\, f = 0$

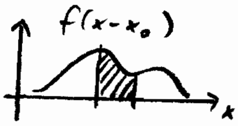
f gerade $\Rightarrow \int_{-a}^{a} dx\, f = 2\int_0^a dx\, f$

$$\int_a^b dx\, (\alpha f + \beta g) = \alpha \int_a^b dx\, f + \beta \int_a^b dx\, g$$

$$\int_a^b = \int_a^c + \int_c^b = \int_a^c - \int_b^c$$

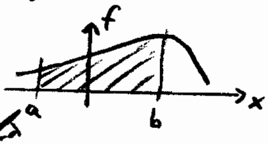
Tricks: Verschieben

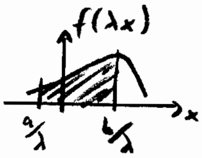




$$\int_a^b dx\, f(x) = \int_{a+x_0}^{b+x_0} dx\, f(x-x_0) \qquad \left(\left(\text{also } \begin{array}{l} f(x) \to f(x-x_0) \\ \text{Grenzen} \to \text{Grenzen} + x_0 \end{array}\right)\right)$$

Skalieren

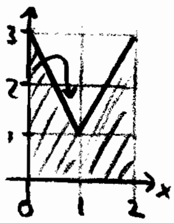




$$\int_a^b dx\, f(x) = \lambda \int_{a/\lambda}^{b/\lambda} dx\, f(\lambda x) \qquad \left(\left(\text{also } \begin{array}{l} x \to \lambda x \\ dx \to \lambda\, dx \\ \text{Grenzen} \to \text{Grenzen}/\lambda \end{array}\right)\right)$$

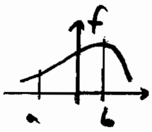
Anwendungs-Beispiel

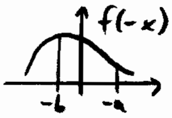
$$\begin{aligned} J &= \int_0^2 dx\, \left(2|x-1|+1\right) &&, \quad x \to x+1 \\ &= \int_{-1}^1 dx\, \left(2|x|+1\right) &&, \quad x \to \tfrac{1}{2} x \\ &= \tfrac{1}{2}\int_{-2}^2 dx\, \left(|x|+1\right) &&, \quad \text{gerade Fkt} \\ &= \int_0^2 dx\, \left(|x|+1\right) = \int_0^2 dx\, (x+1) &&, \quad x \to x+1 \\ &= \int_{-1}^1 dx\, (x+2) &&, \quad x \text{ ist ungerade Fkt} \\ &= 2\int_{-1}^1 dx = 2 \cdot 2 = 4 \end{aligned}$$

((einfacher Check hier: zeichnen 

$f \to 2,\ J = 2 \cdot 2 = 4$ ✓))

Spiegeln



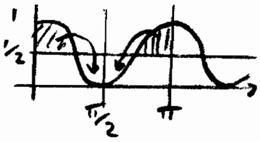


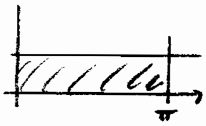
$$\int_a^b dx\, f(x) = \int_{-b}^{-a} dx\, f(-x) = -\int_{-a}^{-b} dx\, f(-x) \qquad \left(\left(\hat{=} \text{ Skalieren, } \lambda = -1\right)\right)$$

$\text{trig}^2 \to \frac{1}{2}$

$$\int_0^\pi dx \begin{Bmatrix} \cos^2(x) \\ \sin^2(x) \end{Bmatrix} = \int_0^\pi dx \cdot \tfrac{1}{2} = \frac{\pi}{2} \quad ,$$

$\overset{\shortparallel}{1-c^2}$

denn: 

$= \frac{1}{2}$ 

weil $\cos^2(x) - \frac{1}{2} = \frac{1}{2} - \sin^2(x) = \frac{1}{2} - \cos^2\left(\frac{\pi}{2} - x\right)$

$$\lim_{a\to\infty} \frac{1}{a}\int_0^a dx\, \sin^2(x) = \frac{1}{2} \quad ,$$

denn $\frac{1}{N\pi + O(1)} \cdot \left(N\frac{\pi}{2} + O(1)\right) \to \frac{1}{2}$