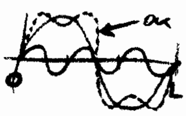
88 (a) $c_n = \frac{1}{L}\int_0^{L/2} dx\, e^{-i\frac{2\pi}{L}x} - \frac{1}{L}\int_{-L/2}^{0}\ldots = \ldots - \frac{1}{L}\int_0^{L/2} dx\, e^{+in\frac{2\pi}{L}x}$

$= \frac{1}{L}\frac{1}{-in\frac{2\pi}{L}}\left(e^{-in\frac{2\pi}{L}\cdot\frac{L}{2}} - 1\right) - \text{dito}_{-n} = \frac{1-(-1)^n}{in2\pi} - \text{dito}_{-n}$

$\Rightarrow c_{n\,\text{gerade}} = 0, \quad c_{n\,\text{ungerade}} = \frac{2}{in\pi}$

(b) $f(x) = \sum_{n\,\text{ung.}} \frac{2}{in\pi} \underbrace{e^{in\frac{2\pi}{L}x}}_{\text{ung. Anteil} = i\sin(\ldots)} = \frac{4}{\pi}\sum_{n=1,3,\ldots}\frac{1}{n}\sin\left(n\frac{2\pi}{L}x\right)$

$= \frac{4}{\pi}\sin\left(\frac{2\pi}{L}x\right) + \frac{4}{3\pi}\sin\left(3\cdot\frac{2\pi}{L}x\right) + \ldots$



(c) $1 = f\left(\frac{L}{4}\right) = \frac{4}{\pi}\left(1 - \frac{1}{3} + \frac{1}{5} - \ldots\right)$ ✓

89 (a) $c_n = \frac{h}{L}\int_0^L dx\, e^{-in\frac{2\pi}{L}x}\,\frac{1}{2}e^{\frac{2\alpha}{L}\left(x-\frac{L}{2}\right)} + \text{dito}_{-\alpha}$

$= \frac{h}{2L}e^{-\alpha}\left[\frac{e\,e}{-in\frac{2\pi}{L}+\frac{2\alpha}{L}}\right]_0^L + \text{dito}_{-\alpha} = \frac{he^{-\alpha}}{2L}\,\frac{L\left(e^{2\alpha}-1\right)}{2(\alpha - in\pi)} + \text{dito}_{-\alpha}$

$= \frac{h}{2}\sinh(\alpha)\left(\frac{1}{\alpha - in\pi} + \frac{1}{\alpha + in\pi}\right) = \frac{h\alpha\sinh(\alpha)}{\alpha^2 + (n\pi)^2}$

$\rightarrow f_0 = c_0 = \frac{h}{\alpha}\sinh(\alpha), \quad a_n = 2c_n, \quad b_n = 0$

für $\alpha \to 0$, erwarte $f_0 \to h$, $a_n, b_n \to 0$ – und so ist es auch ✓

(b) $f(x) = \sum_n \frac{h\alpha\sinh(\alpha)}{\alpha^2 + (n\pi)^2}\, e^{in\frac{2\pi}{L}x}$: setze $h=1$, $L = 2\alpha$, $\alpha = 1$

damit ist $\cosh(x-1) = \sum_n \frac{\sinh(1)}{1+(n\pi)^2}\, e^{in\pi x}$

$x \to x+1$ gibt nun $\cosh(x) = \sum_n \frac{\sinh(1)\,(-1)^n}{1+(n\pi)^2}\, e^{in\pi x}$

90 (a) $\tilde{f}(k) = \int dx\, e^{-ikx}e^{-\gamma|x|} = \int dx\, \cos(kx)\, e^{-\gamma|x|}, \quad \int = 2\int_0^\infty$

$= \int_0^\infty dx\, e^{ikx - \gamma x} + \text{dito}_{-k} = \frac{1}{\gamma - ik} + \frac{1}{\gamma + ik} = \frac{2\gamma}{\gamma^2 + k^2}$

(b) also ist $e^{-\gamma|x|} = \frac{1}{2\pi}\int dk\, e^{ikx}\frac{2\gamma}{\gamma^2+k^2} = \frac{1}{2\pi}\int dk\, \cos(kx)\frac{2\gamma}{\gamma^2+k^2}$, qed.

(c) ∂_x auf (b) $\rightsquigarrow -\int dk\, \frac{k\sin(kx)}{\gamma^2+k^2} = -\pi\,\text{sign}(x)\, e^{-\gamma|x|}$

$\gamma \to 0 \Rightarrow \int_0^\infty dk\, \frac{\sin(kx)}{k} = \frac{\pi}{2}\,\text{sign}(x)$

91 (a) $\tilde{T}(\vec{k},0) = \int d^3r\, e^{-i\vec{k}\vec{r}}\, T_0\,\Theta(R-r) = 2\pi\int_0^R dr\, r^2\, T_0 \underbrace{\int_{-1}^{1} du\, e^{-ikru}}_{= \frac{2\sin(kr)}{kr}}$

$r \to \frac{1}{k}r$: $= T_0\frac{4\pi}{k^3}\int_0^{kR} dr\left(r\sin(r) = \partial_r\left[\sin(r) - r\cos(r)\right]\right)$

$= 4\pi T_0\,\frac{\sin(kR) - kR\cos(kR)}{k^3}$

$T(\vec{r},t) = \left(\frac{1}{2\pi}\right)^3\int d^3k\, e^{i\vec{k}\vec{r}}\, e^{-tDk^2}\, 4\pi T_0\, \frac{s - kRc}{k^3}$

$= \left(\frac{1}{2\pi}\right)^2\int_0^\infty dk\, k^2\, \frac{2\sin(kr)}{kr}\, e^{-tDk^2}\, 4\pi T_0\, \frac{s - kRc}{k^3}$

$r \to 0$: $\frac{\sin(kr)}{kr} \to 1 \Rightarrow$ das angegebene $T(\vec{0},t)$

(b) $t \to \infty$: kleine k, $(s - kRc) \to -\frac{1}{6}(kR)^3 + \frac{1}{2}(kR)^3 = \frac{1}{3}k^3R^3$, $k \to \frac{k}{\sqrt{tD}}$

$T(\vec{0}, t\to\infty) = \frac{2T_0}{\pi}\cdot\frac{1}{3}R^3\cdot\left(\frac{1}{\sqrt{Dt}}\right)^3\cdot\underbrace{\int_0^\infty dk\, k^2 e^{-k^2}}_{= \frac{1}{4}\sqrt{\pi}} = \frac{T_0}{6\sqrt{\pi}}\left(\frac{R}{\sqrt{Dt}}\right)^3$