Übungen zu EMTP II	**Blatt Nr. 20**	Lösung

Aufgabe 62:

(a) $V = \int_0^R dr\, r^2 \int_0^{\pi/2} d\vartheta\, \sin(\vartheta) \int_0^{\pi/2} d\varphi \cdot 1 = \left[\frac{1}{3} r^3\right]_0^R \cdot \left[-\cos(\vartheta)\right]_0^{\pi/2} \cdot \frac{\pi}{2} = \frac{\pi R^3}{6} \quad \Rightarrow \rho = \frac{6M}{\pi R^3}$

(b) allg.: $\vec{R} = \int_0^R dr\, r^2 \int_0^{\pi/2} d\vartheta\, \sin(\vartheta) \int_0^{\pi/2} d\varphi \cdot \frac{1}{M}\, \rho(\vec{r})\vec{r}$, wobei $\frac{1}{M}\,\rho(\vec{r}) = \frac{6}{\pi R^3}$ hier $\vec{r}$-unabhängig

$R_3 = \frac{6}{\pi R^3} \int_0^R dr\, r^3 \int_0^{\pi/2} d\vartheta\, \sin(\vartheta)\cos(\vartheta) \int_0^{\pi/2} d\varphi \overset{(1)}{=} \frac{6}{\pi R^3} \cdot \frac{1}{4}R^4 \cdot \frac{1}{2} \cdot \frac{\pi}{2} = \frac{3}{8}R$

$R_1 = \frac{6}{\pi R^3} \int_0^R dr\, r^3 \int_0^{\pi/2} d\vartheta\, \sin^2(\vartheta) \int_0^{\pi/2} d\varphi\, \cos(\varphi) \overset{(2)}{=} \frac{6}{\pi R^3} \cdot \frac{1}{4}R^4 \cdot \frac{\pi}{4} \cdot 1 = \frac{3}{8}R$

$R_2 = \frac{6}{\pi R^3} \int_0^R dr\, r^3 \int_0^{\pi/2} d\vartheta\, \sin^2(\vartheta) \int_0^{\pi/2} d\varphi\, \sin(\varphi) \overset{(2)}{=} \frac{6}{\pi R^3} \cdot \frac{1}{4}R^4 \cdot \frac{\pi}{4} \cdot 1 = \frac{3}{8}R$

benutzte Stammfunktionen sind (1) $sc = \partial_\vartheta[\frac{1}{2}s^2]$ und (2) $s^2 = \partial_\vartheta[\frac{1}{2}(\vartheta - cs)]$ (oder: trig^2 $\rightarrow \frac{1}{2}$)

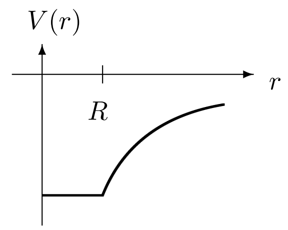
Aufgabe 63:

(a) $\sqrt{\ } - \sqrt{\ } = |r+r'| - |r-r'| = \begin{cases} r < r' : \ 2r \\ r > r' : \ 2r' \end{cases} \quad \Rightarrow V(r) = -\gamma m\, 4\pi \left(\frac{1}{r}\int_0^r dr'\, r'^2 \rho(r') + \int_r^\infty dr'\, r' \rho(r')\right)$

(b) $V_{\text{innen}} = -\gamma m\, 4\pi\rho_0 \left(\frac{1}{r}\int_0^r dr'\, r'^2 + \int_r^R dr'\, r'\right) = \gamma m\, 4\pi\rho_0 \left(\frac{r^2}{6} - \frac{R^2}{2}\right)$

(c) $\frac{1}{r}\partial_r\partial_r \left(-\gamma m\, 4\pi\right)\left(\frac{1}{r}\int_0^r dr'\, r'^2\rho(r') + \int_r^\infty dr'\, r'\rho(r')\right) = -\gamma m\, 4\pi \frac{1}{r}\partial_r \int_r^\infty dr'\, r'\rho(r') = \gamma m\, 4\pi\rho(r)$

Aufgabe 64:



Kugelschale: Vol. $= \frac{4\pi}{3}\left((r' + dr')^3 - r'^3\right) = 4\pi\, dr'\, r'^2 + \mathcal{O}(dr'^2)$

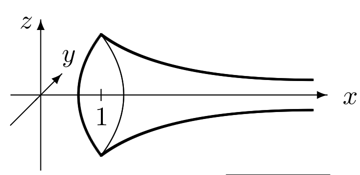
Masse $= \rho \cdot$Vol.

$dV = \{M \rightarrow \rho \cdot \text{Vol.}\ ,\ R \rightarrow r'\} = -\gamma m \rho\, 4\pi\, dr'\, r'^2 \begin{cases} 1/r' \ (\text{für } r < r') \\ 1/r \ \ (\text{für } r > r') \end{cases}$

also ist $V(r > R_E) = \int_0^{R_E} dr'\, (-\gamma m\rho\, 4\pi)\, r'^2\, \frac{1}{r} = -\gamma m\rho\, 4\pi \cdot \frac{1}{3}R_E^3 \cdot \frac{1}{r} = -\gamma m M_E / r$

und $V_{\text{innen}}(r) = V(r < R_E) = -\gamma m\rho\, 4\pi \left(\int_0^r dr'\, \frac{r'^2}{r} + \int_r^{R_E} dr'\, \frac{r'^2}{r'}\right) = \gamma m\rho\, 4\pi \left(\frac{r^2}{6} - \frac{R_E^2}{2}\right)$

Aufgabe 65:



(a) $S_{xy} = 2\int_1^\infty dx\, \frac{1}{x} = 2\left[\ln(x)\right]_1^\infty = \infty$

(b) z.B. $\vec{r} = \vec{r}(x,\varphi) = \left(x, \frac{1}{x}\sin(\varphi), \frac{1}{x}\cos(\varphi)\right)$

$\vec{df} = dx\, d\varphi\, \left(1, -\frac{s}{x^2}, -\frac{c}{x^2}\right) \times \left(0, \frac{c}{x}, \frac{s}{x}\right) = dx\, d\varphi\, \left(\frac{1}{x^3}, \frac{s}{x}, \frac{c}{x}\right) \quad , \quad df = |\vec{df}| = dx\, d\varphi \sqrt{\frac{1}{x^6} + \frac{1}{x^2}}$

$S = \int_S df = \int_0^{2\pi} d\varphi \int_1^\infty dx \sqrt{\frac{1}{x^6} + \frac{1}{x^2}} = 2\pi \int_1^\infty \frac{dx}{x}\sqrt{1 + \frac{1}{x^4}} > 2\pi \int_1^\infty \frac{dx}{x} = \pi S_{xy} = \infty$

(c) $V = \int_1^\infty dx\, \pi \frac{1}{x^2} = \pi\left[-\frac{1}{x}\right]_1^\infty = \pi$

(d) Volumen endlich, Schnitt- und Oberfläche unendlich?! Ja. Farbe? Dimensionen! [s. z.B. Wiki]