EMTP II – Klausur 1.10.08

Lösungsvorschläge

① $x^2 - a^2 + 4y^2 = const$, $\left(\frac{x}{2}\right)^2 + y^2 = C^2$, Ellipsen

② (a) $\partial_x[x\sin(x) + \cos(x)]$ (b)  $= 1$ (c) $\int_{-2}^{2} dx \; \overset{u}{\sinh}(3\,\overset{u}{\operatorname{atn}}(\overset{\text{ungerade}}{x})) = 0$

③ nach $\int_{\mathcal{C}}$ – Fahrplan ; $\mathcal{C}$: $\vec{r} = R(\tau - \sin\tau, 1 - \cos\tau)$, $\tau = 0..\pi$; $v = |R(1-c, s)| = R\sqrt{2}\sqrt{1-c}$

$$L = \int_{\mathcal{C}} ds = R\sqrt{2}\int_0^{\pi} d\tau \sqrt{1-\cos(\tau)} \overset{\tau \to 2\tau}{=} R\sqrt{2}\, 2\int_0^{\pi/2} d\tau \sqrt{2\sin^2(\tau)} = 4R\int_0^{\pi/2} d\tau\,(-\partial_\tau \cos\tau) = 4R$$

④ (a) $= (0,1,0)$

(b) $= \vec{e}_r S s + \vec{e}_\vartheta \cancel{\frac{1}{r}} \cancel{r}\, C s + \vec{e}_\varphi \frac{1}{\cancel{rS}} \cancel{rS} c = \ldots = (sc - cs, s^2 + c^2, 0) = (0,1,0)$

[mit $S \equiv \sin\vartheta$, $c = \cos(\varphi)$ etc.] → (musste rauskommen, da $y = rSs$ in Kugelkoord.)

⑤ $\rho(\vec{r}) = \frac{Q}{F}\delta(x)$; Ans $\vec{E} = f(x)\,\vec{e}_x$, $\vec{\nabla}\vec{E} = f' \overset{!}{=} \rho$, $f = \frac{Q}{F}\Theta(x) + const_x$,

$\vec{E}(x) \overset{!}{=} -\vec{E}(-x)$ ⇒ $const_x = -\frac{1}{2}\frac{Q}{F}$; $\vec{E} = \frac{Q}{F}\left(\Theta(x) - \frac{1}{2}\right)\vec{e}_x$

⑥ (a) $x^2\partial_x^2 x^3 = x^2\partial_x 3x^2 = 6x^3$; $e^{-6\alpha}x^3$

(b) $(\vec{r}\vec{\nabla})r^2 = (x\partial_x + y\partial_y + z\partial_z)(x^2+y^2+z^2) = 2r^2$; $e^2 r^2$

(c) geom. Reihe, $= \frac{1}{2}\sum_{n=0}^{\infty}\left(-\frac{1}{2}\partial_x\right)^n \frac{1}{2}(e^x + e^{-x}) = \frac{1}{4}\left(e^x\sum\left(-\frac{1}{2}\right)^n + e^{-x}\sum\left(\frac{1}{2}\right)^n\right) = \frac{1}{2}\left(\frac{1}{3}e^x + e^{-x}\right)$

⑦ $\vec{\nabla}\vec{B} = 2\underbrace{\vec{e}_3\vec{\nabla}r^2}_{=\partial_z r^2 = 2z} - \vec{r}\underbrace{(\vec{\nabla}z)}_{z} - z\underbrace{(\vec{\nabla}\vec{r})}_{3} = 4z - z - 3z = 0$. $\vec{B}$ quellenfrei ✓

$(\vec{r}\vec{\nabla})\vec{B} = 2\vec{e}_3\underbrace{(\vec{r}\vec{\nabla})r^2}_{2r^2} - \vec{r}\underbrace{(\vec{r}\vec{\nabla})z}_{z} - z\underbrace{(\vec{r}\vec{\nabla})\vec{r}}_{\vec{r}} = 2[2r^2\vec{e}_3 - z\vec{r}] = 2\vec{B}$. ist EF ✓

$\vec{A} = -\vec{r}\times\frac{1}{4}\vec{B} = \frac{1}{4}\vec{B}\times\vec{r}$

$4\vec{\nabla}\times\vec{A} = \vec{\nabla}\times(\vec{B}\times\vec{r}) \overset{\text{bac-cab}}{=} \vec{B}\underbrace{(\vec{\nabla}\vec{r})}_{3} + \underbrace{(\vec{r}\vec{\nabla})\vec{B}}_{2\vec{B}} - \vec{r}\underbrace{(\vec{\nabla}\vec{B})}_{0} - \underbrace{(\vec{B}\vec{\nabla})\vec{r}}_{\vec{B}} = 4\vec{B}$ ✓

⑧ $n(t) = \frac{N}{F(R_0 - vt)}$; Ans $\vec{j} = j(x,t)\,\vec{e}_x$; Conti: $\vec{\nabla}\vec{j} = j' \overset{!}{=} -\dot{n}$ ⇒ $j = -\dot{n}x + const(t)$

$\vec{j}(x=0,t) \overset{!}{=} \vec{0}$ ⇒ $const(t) \equiv 0$ ⇒ $\vec{j}(x,t) = -\dot{n}x\,\vec{e}_x = -\frac{vNx}{F(R_0 - vt)^2}\vec{e}_x$

⑨ $\dot{G} + 3t^2 G \overset{!}{=} \delta(t-a)$; $G_{hom} = e^{-t^3}$; z.B. VdK: $G = e^{-t^3}u(t)$, $e^{-t^3}\dot{u} = \delta(t-a)$,

$\dot{u} = e^{t^3}\delta(t-a) = e^{a^3}\delta(t-a)$, $u = e^{a^3}\Theta(t-a) + const_t$ ⇒ $G(t,a) = e^{-t^3}\left(C + e^{a^3}\Theta(t-a)\right)$

⑩ $c_n = \frac{1}{L}\int_0^L dx\,\beta\,\delta\left(x - \frac{L}{2}\right)e^{-in\frac{2\pi}{L}x} = \frac{1}{L}\beta e^{-in\pi} = \frac{\beta}{L}(-)^n$

$T(x,t) = e^{tD\partial_x^2}\sum_{n=-\infty}^{\infty}\frac{\beta}{L}(-)^n e^{in\frac{2\pi}{L}x} = \sum e^{-tDn^2\left(\frac{2\pi}{L}\right)^2}\frac{\beta}{L}(-)^n e^{in\frac{2\pi}{L}x}$

$T\left(\frac{L}{2},t\right) = \sum e^{-tDn^2\left(\frac{2\pi}{L}\right)^2}\frac{\beta}{L}\cancel{(-)^n(-)^n} \xrightarrow{t\to 0} \frac{\beta}{L}\int_{-\infty}^{\infty}dn\; e^{-tDn^2\left(\frac{2\pi}{L}\right)^2} = \frac{\beta}{\sqrt{4\pi^2 tD}}\int_{-\infty}^{\infty}dn\,e^{-n^2} = \frac{\beta}{\sqrt{4\pi tD}}$

⑪ $\tilde{f}(\vec{k}) = \int d^3r\; e^{-i\vec{k}\vec{r}} f_0\frac{a}{r}e^{-r/a} = 2\pi f_0\int_0^{\infty}dr\; r^2\frac{a}{r}e^{-r/a}\underbrace{\int_{-1}^{1}du\; e^{-ikru}}_{= \frac{1}{-ikr}\left(e^{-ikr} - e^{ikr}\right)}$

$= 2\pi f_0\frac{a}{-ik}\int_0^{\infty}dr\left(e^{-r/a - ikr} - e^{-r/a + ikr}\right)$

$= 2\pi f_0\frac{a}{-ik}\left(-\frac{1}{-\frac{1}{a} - ik} + \frac{1}{-\frac{1}{a} + ik}\right) = 4\pi f_0\frac{a^3}{1 + k^2a^2}$

Ergebnisse: ab 6. Okt. ; online + Aushang (E6)

Einsicht im Prüfungsamt (D3-155)