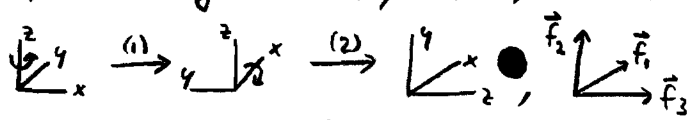
Lösung

(1) $v = \frac{ds}{dt} = \frac{n \cdot d\varphi \cdot R}{dt} = \frac{n \cdot \frac{2\pi}{17} \cdot \frac{0.8\,m}{2}}{\frac{1}{50}\,s} = n \cdot \frac{40\pi}{17} \frac{m}{s} \left(\left(\approx n \cdot 7.39 \frac{m}{s}\right)\right), \quad n = 0, 1, 2, \dots$

(2) (a) $E_{z=h} = E_{z=\ell}, \quad mgh + \frac{k}{2}(\ell-h)^2 = \frac{m}{2}v^2 + mg\ell \Rightarrow v = \sqrt{\ell-h}\sqrt{\frac{k}{m}(\ell-h) - 2g}$

$k_{min} = 2gm/(\ell-h)$, damit $\sqrt{\ } \geq 0$

(b) $m\ddot{z} = -mg + k(\ell - z), \quad \dot{z}(0) = 0, \quad z(0) = h$

(3) 

$D = D_{x,\frac{\pi}{2}} D_{z,\frac{\pi}{2}} = \begin{pmatrix} 1&0&0\\0&0&1\\0&-1&0 \end{pmatrix}\begin{pmatrix} 0&1&0\\-1&0&0\\0&0&1 \end{pmatrix} = \begin{pmatrix} 0&1&0\\0&0&1\\1&0&0 \end{pmatrix}, \quad D\begin{pmatrix}1\\1\\1\end{pmatrix} = \begin{pmatrix}1\\1\\1\end{pmatrix}$

(4) (a) $\dot{f} = -\lambda(1+\omega t), \quad f = const_t - \lambda(t + \frac{1}{2}\omega t^2), \quad f(0) = const_t = \ln(v_0) \Rightarrow v(t) = v_0 e^{-\lambda(t+\frac{1}{2}\omega t^2)}$

(b) $\dot{f} = -\lambda/(1+\omega t), \quad f = const_t - \frac{\lambda}{\omega}\ln(1+\omega t), \quad f(0) = const_t = \ln(v_0) \Rightarrow v(t) = v_0 (1+\omega t)^{-\lambda/\omega}$

(5) (a) $v\dot{v} = (\frac{1}{2}v^2)^\bullet = -\lambda = (const_t - \lambda t)^\bullet \Rightarrow v(t) = \sqrt{2(const_t - \lambda t)} = \sqrt{v_0^2 - 2\lambda t}$.

(b) $v^n \dot{v} = (\frac{1}{n+1}v^{n+1})^\bullet = -\lambda = (const_t - \lambda t)^\bullet \Rightarrow v(t) = [(n+1)(const_t - \lambda t)]^{\frac{1}{n+1}} = [v_0^{n+1} - (n+1)\lambda t]^{\frac{1}{n+1}}$

(6) $\vec{K} = -(\partial_x V, \partial_y V, \partial_z V), \quad K_1 = -\alpha x^2 \overset{!}{=} -\partial_x V = -\partial_x\left(\frac{\alpha}{3}x^3 + f(y,z)\right), \quad K_2 = \beta z \overset{!}{=} -\partial_y V = -\partial_y\left(\frac{\alpha}{3}x^3 - \beta yz + f(z)\right),$

$K_3 = \gamma y \overset{!}{=} -\partial_z V = -\partial_z\left(\frac{\alpha}{3}x^3 - \beta yz + f(z)\right) = \beta y + f'(z) \Rightarrow \beta \overset{!}{=} \gamma, \quad V = \frac{\alpha}{3}x^3 - \beta yz$

(7) (a) $0 \overset{!}{=} (1-\lambda)^2 - 4 = \lambda^2 - 2\lambda - 3 = (\lambda-3)(\lambda+1), \quad \lambda_1 = 3, \lambda_2 = -1$.

$\begin{pmatrix} -2&2\\2&-2 \end{pmatrix}\begin{pmatrix} r\\s \end{pmatrix} = 2\begin{pmatrix} s-r\\r-s \end{pmatrix} \overset{!}{=} \begin{pmatrix} 0\\0 \end{pmatrix} \Rightarrow \vec{f}_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1\\1 \end{pmatrix}, \ldots \vec{f}_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1\\-1 \end{pmatrix}, \quad \vec{f}_1 \cdot \vec{f}_2 = 0 ✓, \quad D = \frac{1}{\sqrt{2}}\begin{pmatrix} 1&1\\1&-1 \end{pmatrix}$

(b) $0 \overset{!}{=} (3-\lambda)(4-\lambda)(5-\lambda) - (3-\lambda)4 - (5-\lambda)4 = -\lambda^3 + 12\lambda^2 - 39\lambda + 28, \quad \lambda_1 = 1$ (raten)

$= (1-\lambda)(28 - 11\lambda + \lambda^2), \quad \lambda_{2/3} = \frac{11}{2} \pm \sqrt{\frac{121}{4} - 28} \Rightarrow \lambda_2 = 4, \lambda_3 = 7, \quad 3+4+5 = 1+4+7$ ✓

(8) (a) $\ln(\sqrt{4+\varepsilon^2}) = \ln(2\sqrt{1+\varepsilon^2/4}) = \ln 2 + \frac{1}{2}\ln(1+\varepsilon^2/4) = \ln 2 + \varepsilon^2/8 + O(\varepsilon^4)$

(b) $\ln\left(2\varepsilon + (2\varepsilon)^2/2! + (2\varepsilon)^3/3! + \dots\right) = \ln(2\varepsilon) + \ln\left(1 + [\varepsilon + \frac{2}{3}\varepsilon^2 + \dots]\right) = \ln(2\varepsilon) + [\ ] - \frac{1}{2}[\ ]^2 + \dots$

$= \ln(2\varepsilon) + \varepsilon + \frac{1}{6}\varepsilon^2 + O(\varepsilon^3)$

(c) $= $ (b), oder: $\varepsilon + \ln(2\sinh(\varepsilon)) = \varepsilon + \ln\left(2\varepsilon + 2\varepsilon^3/3! + \dots\right) = \varepsilon + \ln(2\varepsilon) + \frac{1}{6}\varepsilon^2 + O(\varepsilon^3)$

(9) $f = \ln(x), \quad f' = \frac{1}{x}, \quad f_u = e^x; \quad f_u' = \partial_x e^x = \frac{1}{f'(f_u(x))} = \frac{1}{1/e^x} = e^x$

(10) $ER^{(0)}: \quad \ddot{x}^{(0)} = 0, \quad \dot{x}^{(0)}(0) = 0, \quad x^{(0)}(0) = a \Rightarrow x^{(0)} = a$

$ER^{(1)}: \quad \ddot{x}^{(1)} = -\omega^2 a, \quad \dot{x}^{(1)}(0) = 0, \quad x^{(1)}(0) = 0 \Rightarrow x^{(1)} = -\frac{1}{2}\omega^2 a t^2$

$ER^{(2)}: \quad \ddot{x}^{(2)} = -\omega^2 x^{(1)} = +\omega^4 \frac{a}{2} t^2, \quad \dot{x}^{(2)}(0) = 0, \quad x^{(2)}(0) = 0 \Rightarrow x^{(2)} = \omega^4 \frac{a}{2} \frac{t^4}{3 \cdot 4}$

$\Rightarrow x(t) \approx a\left[1 - \frac{1}{2}(\omega t)^2 + \frac{1}{4!}(\omega t)^4 + \dots\right] \overset{!}{=} a \cdot \cos(\omega t)$ ✓