Lösung 14

42 (a) $\ddot x^{(0)} + \ddot x^{(1)} = 2a\omega^2 e^{-x^{(0)}/a} - \frac{1}{a}\left(\dot x^{(0)2} + 2\dot x^{(0)}\dot x^{(1)}\right)$

$\boxed{\ddot x^{(0)} = -\frac{\dot x^{(0)2}}{a},\ \dot x^{(0)}=0,\ x^{(0)}=0}$ $\left(\left(\dot v = -\frac{v^2}{a},\ v(0)=0\right)\right) \Rightarrow \underline{x^{(0)} \equiv 0}$

$\boxed{\ddot x^{(1)} = 2a\omega^2,\ \dot x^{(1)}(0)=0,\ x^{(1)}(0)=0} \Rightarrow x^{(1)} = a\omega^2 t^2$ ged.

(b) $\dot v = v_0(-\lambda - \lambda\omega t)e^{-\cdots} = -\lambda(1+\omega t)v$ ✓

$v^{(0)} + v^{(1)} \overset{?!}{=} v_0 e^{-\lambda t}\left(1 - \frac{\lambda\omega}{2}t^2\right)$

$\boxed{\dot v^{(0)} = -\lambda v^{(0)},\ v^{(0)} = v_0} \Rightarrow v^{(0)} = v_0 e^{-\lambda t}$

$\boxed{\dot v^{(1)} = -\lambda v^{(1)} - \lambda\omega t\, v_0 e^{-\lambda t},\ v^{(1)}(0)=0}$, Subst. $v^{(1)} = e^{-\lambda t}u$

$\Rightarrow \boxed{\dot u = -\lambda v_0\omega t,\ u(0)=0} \Rightarrow u = -\frac{\lambda}{2}v_0\omega t^2,\ v^{(1)} = -v_0 e^{-\lambda t}\frac{\lambda\omega}{2}t^2$

$\rightsquigarrow v^{(0)} + v^{(1)} = v_0 e^{-\lambda t}\left(1 - \frac{\lambda\omega}{2}t^2\right)$, toll!

43 $\ddot x^{(0)} + \ddot x^{(1)} + \ddot x^{(2)} = \frac{-\gamma M}{(x^{(0)} + x^{(1)})^2} = \frac{-\gamma M}{x^{(0)2}} + \frac{2\gamma M x^{(1)}}{x^{(0)3}}$

$\Rightarrow \boxed{\ddot x^{(0)} = 0,\ \dot x^{(0)}(t_1) = v_0,\ x^{(0)}(t_1) = a}$

$\boxed{\ddot x^{(1)} = \frac{-\gamma M}{x^{(0)2}},\ \dot x^{(1)}(t_1)=0,\ x^{(1)}(t_1)=0}$ $\boxed{\ddot x^{(2)} = 2\gamma M\frac{x^{(1)}}{x^{(0)3}},\ \dot x^{(2)}(t_1)=0,\ x^{(2)}(t_1)=0}$

$x^{(0)} = A + Bt$, $B = v_0$, $A + v_0 t_1 = a$, $A = 0$, $\underline{x^{(0)} = v_0 t}$

$\ddot x^{(1)} = -\frac{\gamma M}{v_0^2}\frac{1}{t^2}$, ⇝ Ansatz $x^{(1)} = C + Dt + E\ln(t)$

$\Rightarrow -E = -\frac{\gamma M}{v_0^2}$; $\dot x^{(1)}(t_1) = D + \frac{E}{t_1} \overset{!}{=} 0 \Rightarrow D = -\frac{v_0}{a}E = -\frac{\gamma M}{a v_0}$;

$x^{(1)}(t_1) = C - \frac{\gamma M}{a v_0}t_1 + \frac{\gamma M}{v_0^2}\ln(t_1) \overset{!}{=} 0 \Rightarrow C = \frac{\gamma M}{v_0^2} - \frac{\gamma M}{v_0^2}\ln\left(\frac{a}{v_0}\right)$;

$\rightsquigarrow x^{(1)}(t) = \frac{\gamma M}{v_0^2} - \frac{\gamma M}{a v_0}t + \frac{\gamma M}{v_0^2}\ln\left(\frac{v_0}{a}t\right)$

$\Rightarrow t_0$ aus $x(t_0) \overset{!}{=} 0$, d.h. $0 \overset{!}{=} v_0 t_0 + \underbrace{\frac{\gamma M}{v_0^2} - \frac{\gamma M}{a v_0}t_0}_{\text{klein gegen dominierenden ln}} + \frac{\gamma M}{v_0^2}\ln\left(\frac{v_0}{a}t_0\right)$

((Zusatz: $\frac{v_0}{a}t_0 =: \tau$, $\frac{\gamma M}{a v_0^2} =: \varepsilon$, $\tau = \varepsilon\ln\left(\frac{1}{\tau}\right)$, $\ln\left(\frac{1}{\tau}\right) = \ln\left(\frac{1}{\varepsilon}\right) - \ln\left(\ln\left(\frac{1}{\tau}\right)\right)$ und 1. Iteration: $\tau \approx \varepsilon$ im kleinen $\ln(\ln)$ $\Rightarrow \tau = \varepsilon\ln\left(\frac{1}{\varepsilon}\right)$, $t_0 = \frac{\gamma M}{v_0^3}\ln\left(\frac{a v_0^2}{\gamma M}\right)$))

44 (a) $\boxed{\dot v = -\alpha v + \beta\frac{1}{t}\ln\left(\frac{v}{v_0}\right),\ v(0) = v_0}$

(b) OK, weil $v < v_0$ wird, d.h. ln negativ

(c) $[\beta] = [\dot v t] = [v] \Rightarrow \beta \ll v_0$

(d) $\boxed{\dot v^{(0)} = -\alpha v^{(0)},\ v^{(0)}(0) = v_0} \Rightarrow v^{(0)}(t) = v_0 e^{-\alpha t}$

$\dot v^{(1)} = -\alpha v^{(1)} + \beta\frac{1}{t}\ln\left(\frac{v^{(0)}}{v_0}\right)$

$\boxed{\dot v^{(1)} = -\alpha v^{(1)} - \alpha\beta,\ v^{(1)}(0) = 0} \Rightarrow v^{(1)}(t) = -\beta + \beta e^{-\alpha t}$