29 (a)

I OK

II $\det(A-\lambda\mathbb{1}) = \ldots = (1-\lambda)(4-\lambda)(7-\lambda)$, $\lambda_{1/2/3} = \{1,4,7\}$

III $Sp(A) = 3+4+5 = 12$, $\sum \lambda_i = 12$ ✓

IV $(A-\lambda_1\mathbb{1})\vec f_1 = \vec 0 = \begin{pmatrix} 2 & -2 & 0 \\ -2 & 3 & -2 \\ 0 & -2 & 4 \end{pmatrix}\vec f_1 \Rightarrow \vec f_1 = \frac{1}{3}\begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$,

$\begin{pmatrix} -1 & -2 & 0 \\ -2 & 0 & -2 \\ 0 & -2 & 1 \end{pmatrix}\vec f_2 = \vec 0 \Rightarrow \vec f_2 = \frac{1}{3}\begin{pmatrix} -2 \\ 1 \\ 2 \end{pmatrix}$, $\begin{pmatrix} -4 & -2 & 0 \\ -2 & -3 & -2 \\ 0 & -2 & -2 \end{pmatrix}\vec f_3 = 0 \Rightarrow \vec f_3 = \frac{1}{3}\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$

V Ortho: OK (geht im Kopf)

VI Rechts? z.B. $\vec f_1 \times \vec f_2 = \frac{1}{9}(3,-6,6) = \vec f_3$ ✓

VII $A' = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 7 \end{pmatrix}$, $D = \frac{1}{3}\begin{pmatrix} 2 & 2 & 1 \\ -2 & 1 & 2 \\ 1 & -2 & 2 \end{pmatrix}$

(b) $\boxed{\ddot{\vec r}\,' = -\omega^2 A'\vec r\,', \quad \dot{\vec r}\,'(0) = D\vec 0 = \vec 0, \quad \vec r\,'(0) = \frac{2}{3}R\,D\begin{pmatrix} 4 \\ 1 \\ -1 \end{pmatrix} = \frac{2R}{9}\begin{pmatrix} 9 \\ -9 \\ 0 \end{pmatrix} = 2R\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}}$

$$\left.\begin{array}{l} \ddot x' = -\omega^2 x', \quad \dot x'(0) = 0, \quad x'(0) = 2R \\ \ddot y' = -4\omega^2 y', \quad \dot y'(0) = 0, \quad y'(0) = -2R \\ \ddot z' = -7\omega^2 z', \quad \dot z'(0) = 0, \quad z'(0) = 0 \end{array}\right\} \vec r\,'(t) = \begin{pmatrix} 2R\cos(\omega t) \\ -2R\cos(2\omega t) \\ 0 \end{pmatrix}$$

30 (a) $H = \begin{pmatrix} 5 & -4 & -2 \\ -4 & 5 & -2 \\ -2 & -2 & 8 \end{pmatrix}$, $\vec a = 6\begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$

I OK

II $\det(H-\lambda\mathbb{1}) = \ldots = -\lambda(9-\lambda)^2$, $\lambda_{1/2/3} = \{0, \overbrace{9,9}^{\text{Entartung!}}\}$

III $5+5+8 = 9+9$ ✓

IV $H\vec f_1 = \vec 0 \Rightarrow \vec f_1 = \frac{1}{3}\begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$, $\vec f_{2,3}$: $\begin{pmatrix} -4 & -4 & -2 \\ -4 & -4 & -2 \\ -2 & -2 & -1 \end{pmatrix}\begin{pmatrix} r \\ s \\ t \end{pmatrix} = \vec 0 \Rightarrow$ (z.B.) $\vec f_2 = \frac{1}{3}\begin{pmatrix} -2 \\ 1 \\ 2 \end{pmatrix}$

V, VI: $\vec f_3 = \vec f_1 \times \vec f_2 = \frac{1}{3}\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$

VII $H' = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{pmatrix}$, $D = \frac{1}{3}\begin{pmatrix} 2 & 2 & 1 \\ -2 & 1 & 2 \\ 1 & -2 & 2 \end{pmatrix}$

(b) $9y'^2 + 9z'^2 = \underbrace{\vec a\,'\vec r\,'}_{=36x'}$, $\vec a\,' = D\vec a = 6D\begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} = 36\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

$x' = \frac{3}{6}(y'^2+z'^2)$ ist Rotationsparaboloid,

$\vec e$ ist $\vec f_1$, also $\vec e = \frac{1}{3}\begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$

(c) $\vec e\,'\vec r\,' = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}\vec r\,' = x' = c$, $4m = \frac{1}{100m}(y'^2+z'^2) \Rightarrow \underline{20m}$

28, **31** verrate ich nicht .. selber ..