Klausur zu EMTP I (9. Juli. 2008) **Lösung**

① $\lambda(-b,h)+\eta(a,h)=(0,mg)$; $\lambda=\frac{a}{b}\eta$; $\eta\left(\frac{a}{b}+1\right)h=mg$ $\Rightarrow$ $\vec{K}=\frac{mgb}{(a+b)h}(a,h)$

② $\vec{r}_0=R(1+c,s)$; $\vec{r}_{\text{Quelle-Spalt}}=\vec{r}_0-(a,0)=(R(1+c)-a,\,Rs)$; $\tan(\varphi)=\frac{Rs}{R+Rc-a}\equiv\frac{Rs}{N}$

$\varphi=\arctan\left(\frac{Rs}{N}\right)$, $\dot\varphi=\dfrac{\frac{R\omega c}{N}-\frac{Rs}{N^2}(-R\omega s)}{1+(Rs/N)^2}=\omega\,\dfrac{R^2(c+1)-Rac}{2R^2(c+1)-2aR(c+1)+a^2}$

$a=0$ $\Rightarrow$ $\dot\varphi=\frac{\omega}{2}$ ((hier: $c\equiv\cos(\omega t)$, $s\equiv\sin(\omega t)$))

③ $-\frac{\gamma mM}{2R}=-\frac{\gamma mM}{R}+\frac{k}{2}R^2+\frac{m}{2}v^2$; $v^2=\frac{2}{m}\frac{\gamma mM}{R}\left(1-\frac{1}{2}-\frac{1}{4}\right)$ $\Rightarrow$ $v=\sqrt{\frac{\gamma M}{2R}}$

④ $\vec{K}=-(\partial_x V,\partial_y V,\partial_z V)$; $\partial_x V=-K_1=-\alpha xz$ $\Rightarrow$ $V=-\frac{1}{2}\alpha x^2 z+f(y,z)$;

$\partial_y V=-K_2=-\beta yz$ $\Rightarrow$ $V=-\frac{1}{2}\alpha x^2 z-\frac{1}{2}\beta y^2 z+f(z)$; $\partial_z V=-K_3=-x^2-\gamma y^2=-\frac{1}{2}\alpha x^2-\frac{1}{2}\beta y^2+f'(z)$

$\Rightarrow$ $\alpha=2$, $\beta=2\gamma$, $V=-z(x^2+\gamma y^2)$

⑤ Ans $x=As+Bsh+Dt$; $\dot x=A\omega c+B\omega\,ch+D$; $\ddot x=-A\omega^2 s+B\omega^2 sh\overset{!}{=}-\omega^2 As-$

$-\omega^2 Bsh-\omega^2 Dt+\alpha\omega t-2\alpha\,sh$ $\Rightarrow$ $D=\frac{\alpha}{\omega}$, $B=-\frac{\alpha}{\omega^2}$;

$\dot x(0)=A\omega+B\omega+D\overset{!}{=}v_0$ $\Rightarrow$ $A=\frac{v_0}{\omega}$ $\Rightarrow$ $x(t)=\frac{v_0}{\omega}\sin(\omega t)+\frac{\alpha}{\omega^2}(\omega t-\sinh(\omega t))$

⑥ (a) $\vec{v}=a\omega(1+\omega t)(2,1,0)$; $m\dot{\vec{v}}=ma\omega^2(2,1,0)\overset{!}{=}q(0,E,0)+qB(t)(v_2,-v_1,0)$
$=(qBa\omega(1+\omega t),\,qE-qBa\omega 2(1+\omega t))$

$\Rightarrow$ $qB=\frac{2m\omega}{1+\omega t}$, $qE=5ma\omega^2$

(b) Bewgl.$\cdot\vec{v}$: $\left(\frac{m}{2}v^2\right)^\cdot=q\vec{E}\vec{v}=qEv_2=5ma^2\omega^3(1+\omega t)$

Kontrolle: $\frac{m}{2}v^2=\frac{m}{2}a^2\omega^2(1+\omega t)^2\cdot 5$ $\Rightarrow$ $\dot T=ma^2\omega^3(1+\omega t)\cdot 5$ ✓

⑦ $f(s)\approx[1+\ln(1+s^2+\ldots)][1-3s^2+\ldots]^{-\frac{1}{2}}=(1+s^2+\ldots)(1+\frac{3}{2}s^2\ldots)=1+\frac{5}{2}s^2+\ldots$ $\Rightarrow$ $\omega=\sqrt{5\frac{k}{m}}$

⑧ $H=\begin{pmatrix}1&0&0\\0&3&2\\0&2&6\end{pmatrix}$; $\lambda_1=1$, $0\overset{!}{=}(3-\lambda)(6-\lambda)-4=\lambda^2-9\lambda+14=(\lambda-2)(\lambda-7)$, $\lambda_2=2$, $\lambda_3=7$

$\vec{f}_1=\begin{pmatrix}1\\0\\0\end{pmatrix}$; $\begin{pmatrix}1&2\\2&4\end{pmatrix}\begin{pmatrix}\ \end{pmatrix}=\vec{0}$ $\Rightarrow$ $\vec{f}_2=\frac{1}{\sqrt5}\begin{pmatrix}0\\2\\-1\end{pmatrix}$; $\vec{f}_3=\vec{f}_1\times\vec{f}_2=\frac{1}{\sqrt5}\begin{pmatrix}0\\1\\2\end{pmatrix}$

$\Rightarrow$ $H'=\begin{pmatrix}1&0&0\\0&2&0\\0&0&7\end{pmatrix}$, $D=\frac{1}{\sqrt5}\begin{pmatrix}\sqrt5&0&0\\0&2&-1\\0&1&2\end{pmatrix}$.

$\vec{r}\,'(0)=D\vec{r}(0)=5a\begin{pmatrix}0\\0\\1\end{pmatrix}$ $\Rightarrow$ m bleibt auf z'-Achse.

$V'=\alpha(x'^2+2y'^2+7z'^2)$ $\Rightarrow$ $\omega=\sqrt{14\frac{\alpha}{m}}$

⑨ (a) $\dot v^{(0)}+\dot v^{(1)}+\dot v^{(2)}+\ldots=\alpha e^{-(v^{(0)}+v^{(1)}+\ldots)}=\alpha e^{-v^{(0)}}(1-v^{(1)}+\ldots)$

$ER^{(0)}$: $\boxed{\dot v^{(0)}=0,\ v^{(0)}(0)=0}$ $\Rightarrow$ $v^{(0)}(t)=0$

$ER^{(1)}$: $\boxed{\dot v^{(1)}=\alpha e^{-v^{(0)}}=\alpha,\ v^{(1)}(0)=0}$ $\Rightarrow$ $v^{(1)}(t)=\alpha t$

$ER^{(2)}$: $\boxed{\dot v^{(2)}=-\alpha e^{-v^{(0)}}v^{(1)}=-\alpha^2 t,\ v^{(2)}(0)=0}$ $\Rightarrow$ $v^{(2)}(t)=-\frac{1}{2}\alpha^2 t^2$

(b) $\dot v e^v=(e^v)^\cdot=\alpha$; $e^v=\text{const}+\alpha t$, $v=\ln(\text{const}+\alpha t)$

$v(0)=\ln(\text{const})\overset{!}{=}0$ $\Rightarrow$ $\text{const}=1$; $v(t)=\ln(1+\alpha t)\approx\alpha t-\frac{1}{2}(\alpha t)^2$ ✓(a)