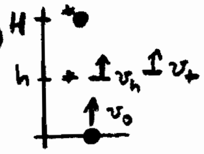
Klausur zu EMTP I (11. Feb. 2008) — Lösung

① $\vec R = \frac{1}{3m}\left[m(a,0,0)+m(-a,0,0)+m(0,0,h)\right] = (0,0,h/3)$

② (a) $\cos(x)\,e^{\sin(x)}$ (b) $\partial_x e^{x\ln a} = \ln(a)\,a^x$

(c) $-1/\sin(\arccos(x)) = -1/\sqrt{1-\cos^2(\arccos(x))} = -1/\sqrt{1-x^2}$ (d) $\frac{1}{1+1+\frac{x^2}{2}+\dots} = \frac{1}{2}-\frac{x^2}{8}+O(x^4)$

③ 

E-Erh: $\frac{M}{2}v_0^2 = \frac{M}{2}v_h^2 + Mgh \Rightarrow v_h = \sqrt{v_0^2-2gh}$

Imp-Erh: $Mv_h = (M+m)v_+ \Rightarrow v_+ = \frac{M}{M+m}v_h$

E-Erh: $\frac{M+m}{2}v_+^2 + (M+m)gh = (M+m)gH \Rightarrow H = h+\frac{1}{2g}v_+^2 = h+\frac{M^2}{(M+m)^2}\,\frac{v_0^2-2gh}{2g}$

④ egal, ob DS kreist oder fest $\to (0,0,R),\ (0,0,-R)$

$\vec k = -\gamma m M\left(\frac{(x,0,R)}{|\ \ |^3}+\frac{(x,0,-R)}{|\ \ |^3}\right) \Rightarrow k_1(x) = -\frac{\gamma m M 2x}{\sqrt{R^2+x^2}^3}$

⑤ $m\dot{\vec v} = \vec 0 = q\left[(0,0,E)+(v_0,0,0)\times(B_1,B_2,B_3)\right] = q(0,-v_0B_3,\ E+v_0B_2)$

$\Rightarrow \vec B = (\text{beliebig},\ -E/v_0,\ 0)$

⑥ Ans $x(t) \overset{?}{=} A+Bt+\frac{C}{1+\omega t}$, $\dot x = B-\frac{C\omega}{(1+\omega t)^2}$, $\ddot x = +\frac{2C\omega^2}{(1+\omega t)^3} \overset{!}{=} -\frac{2a\omega^2}{(1+\omega t)^3}$,

$\Rightarrow C=-a$; $\dot x(0) \overset{!}{=} a\omega \Rightarrow B=0$; $x(0)\overset{!}{=}0 \Rightarrow A=a$; $x(t) = \frac{a\omega t}{1+\omega t}$

⑦ (a) Ans $V(x) = C\sqrt{a^2+x^2}$ in $-\lambda\frac{x}{\sqrt{\ }} = -\partial_x V = -\frac{C\,2x}{2\sqrt{\ }} \Rightarrow C=\lambda$ 

(b) Ans $V(x) \approx A+Bx^2+\dots = \lambda\sqrt{a^2+x^2}$, beide Seiten quadrieren:

$A^2+2ABx^2+\dots = \lambda^2a^2+\lambda^2x^2 \Rightarrow A=\lambda a,\ B=\frac{\lambda}{2a}$, $V(x) = \lambda a+\frac{\lambda}{2a}x^2+\dots$;

$m\ddot x = -\partial_x V$, $\ddot x = -\frac{\lambda}{am}x \Rightarrow \omega = \sqrt{\frac{\lambda}{am}}$

⑧ $DD^T = \dots = \mathbb{1}$ ✓✓ $\det D = +1$ ✓✓ $\Rightarrow$ D ist Drehmatrix.

$\mathrm{Sp}(D) = 1 \overset{!}{=} 1+2\cos\varphi \Rightarrow \cos\varphi \overset{!}{=} 0$, $\varphi = \pi/2$

Achse: $\vec 0 = (D-\mathbb{1})\vec b \Rightarrow \vec b = (b_1,b_1,0) \Rightarrow \vec e = \vec b/|\vec b| = \frac{1}{\sqrt2}(1,1,0)$

⑨ I ok. II $(7-\lambda)(1-\lambda)-16 = \lambda^2-8\lambda-9 = (\lambda+1)(\lambda-9) \Rightarrow \lambda_1=-1,\ \lambda_2=9$

III $7+1 = -1+9$ ✓✓ IV $\begin{pmatrix}8&-4\\-4&2\end{pmatrix}\begin{pmatrix}r\\s\end{pmatrix} \overset{!}{=} \begin{pmatrix}0\\0\end{pmatrix} \to \vec f_1 = \frac{1}{\sqrt5}\begin{pmatrix}1\\2\end{pmatrix}$; dito $\vec f_2 = \frac{1}{\sqrt5}\begin{pmatrix}-2\\1\end{pmatrix}$

V $\vec f_1\cdot\vec f_2 = \frac{1}{5}(-2+2) = 0$ ✓✓ VII $H' = \begin{pmatrix}-1&0\\0&9\end{pmatrix}$, $D = \frac{1}{\sqrt5}\begin{pmatrix}1&2\\-2&1\end{pmatrix}$

⑩ (a) $\dot v^{(0)}+\dot v^{(1)}+\dot v^{(2)} = -\lambda\left(v^{(0)3}+3v^{(0)2}v^{(1)}\right)$,

$\boxed{\dot v^{(0)} = 0,\ v^{(0)}(0) = v_0} \Rightarrow v^{(0)} = v_0$

$\boxed{\dot v^{(1)} = -\lambda v_0^3,\ v^{(1)}(0) = 0} \Rightarrow v^{(1)} = -\lambda v_0^3 t$

$\boxed{\dot v^{(2)} = -3\lambda v_0^2 v^{(1)} = 3\lambda^2 v_0^5 t,\ v^{(2)}(0) = 0} \Rightarrow v^{(2)} = \frac{3}{2}\lambda^2 v_0^5 t^2$

(b) $-\frac{2}{v^3}\dot v = \left(\frac{1}{v^2}\right)^\bullet = 2\lambda$, $v^2 = \frac{1}{A+2\lambda t}$, $v(t) = \frac{v_0}{\sqrt{1+2\lambda v_0^2 t}}$

use $(1+x)^\lambda = 1+\lambda x+\frac{\lambda(\lambda-1)}{2}x^2+\dots$, $\lambda = -\frac{1}{2}$: $= v_0\left[1-\frac{1}{2}2\lambda v_0^2 t+\frac{3}{8}(2\lambda v_0^2 t)^2+\dots\right]$

Ergebnisse: ab 25. Feb; online + Aushang (E6)