Classical Statistical Models - Spring 2016

Bielefeld University

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Exercise Nr. 6

Discussion on June 20th, 15:00-16:00, Room E6-118

13) Low temperature expansion of the q-state potts model *(4 points)*

Write down the general expression for the low temperature expansion of the q-state potts model, following the line of argument of the low temperature expansion of the Ising model.

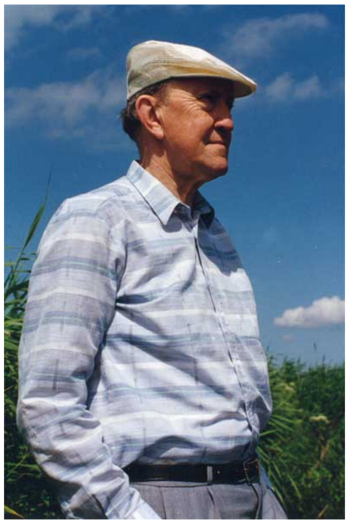
*14) Site percolation *(4 points)*

Perform a number of site percolation simulations on a square lattice and estimate the percolation threshold. Hint: the difficulty is to identify the percolating cluster, if there is any. One can easily implement that with a recursive algorithm.

15) Percolation on the Bethe lattice *(2+4 points)*

Show that both for bond and site percolation on the Bethe lattice with coordination number z, the percolation threshold is $p_c = \frac{1}{z-1}$. Hint: Compute the probability that starting from the center site, there is at least one branch which only consists of activated sites/bonds.

Pieter Willem Kasteleyn
(October 12, 1924, - January 16, 1996) was a Dutch Physicist.



After finishing high school in 1942, Piet briefly studied chemistry in Amsterdam. [...] Three months later he switched to the school for chemical analysts in Leiden where he got his diploma in 1944. After the war Leiden University reopened and he undertook the study of physics. Piet soon came under the influence of H. A. Kramers, who tutored him and took him along to attend the first postwar IUPAP meeting in Florence in 1949. This event made a lasting impression on him. Piet graduated in 1951 and was employed by the Royal Shell Laboratories in Amsterdam, a position secured for him by Kramers. In 1952 he married Laantje Boer, whom he had met in 1943.

Shell allowed Piet to pursue his studies under Kramers. He worked on lattice models of ferro- and antiferromagnetism, partly in collaboration with J. van Kranendonk. After Kramers' death in 1952 he was supervised by L.J. Oosterhof and later by S.R. de Groot, under whom he defended his thesis in 1956. [...] While at Shell, Piet did his first major piece of research: he computed the entropy of a full covering of a two-dimensional planar lattice with dimers, i.e. diatomic molecules absorbed onto a crystal surface with close packing. The ingenious combinatorial method developed by him to tackle this problem, based on Pfaffians and discovered independently by Fisher and Temperley, forever bears his name. He was able to show how this method led to a solution of the nearest neighbor Ising model on the quadratic lattice that was an alternative to those of Onsager (1944), Kaufman (1949), and Kac and Ward (1952). In addition, he showed how Pfaffians can be used to count self-avoiding walks on the quadratic lattice with the so-called "Manhattan orientation." [...]

In 1963 Piet was nominated Full Professor at the Lorentz Institute of Theoretical Physics in Leiden. [...] During this period he developed his ideas about the relation between phase transitions in different models, which culminated in a series of papers with C.M. Fortuin in 1969-1972 introducing the random-cluster model. [...] In recent years the Fortuin-Kasteleyn representation has turned out to be pivotal in the rigorous analysis of a number of interesting phenomena occurring in the Ising and Potts models. Examples are the Wulff shape of a large droplet of one phase inside another, discontinuity of the spontaneous magnetization at the critical temperature for one-dimensional long-range potentials, tunneling between phases under Glauber flip dynamics, and Swendsen-Wang algorithms for simulating Gibbs states. [...]

Throughout the 1970s Piet studied various graph-theoretic problems and had an active correspondence with W. Tutte. He tried for many years to solve the four-color problem, to find himself and others outdone by the computer. But along the way he published a paper on chromatic polynomials with L.B. Richmond. [...] From 1980 to 1985 Piet worked on random walks with the undersigned. He raised the question: "How does a random walk see and encounter random inhomogeneities in a lattice?" This work has close connections with Mark Kac's theorem on the mean recurrence time in stationary stochastic processes and with the beautiful random walk models studied by Elliott Montroll. Some problems were directly inspired by an application to photosynthesis and the trapping of light excitations by chlorophyl reaction centers. Piet greatly enjoyed seeing some of the theoretical results corroborated by experiments carried out in the Leiden biophysics group. [...]

Piet died on January 16, 1996, after an unexpected and short illness. Even in the face of death he was stronger than everyone around him and he tried to console his family and closest friends. Piet leaves behind his wife Laantje, their three sons, and their two grandchildren. For them he was the perfect companion. All who knew him will miss him dearly. With Piet a chapter of Dutch physics has closed. But his memory will continue to inspire us.

`[F. den Hollander, Journal of Statistical Physics, 85 (1996) 801-805]`