

# Classical Statistical Models - Spring 2016

Bielefeld University

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## Exercise Nr. 5

Discussion on June 13th, 15:00-16:00, Room E6-118

### 10) High Temperature Expansion (*4 points*)

Compute the first four orders of the high-temperature expansion of the Ising model on a square lattice with isotropic couplings ( $J = J'$ ), i.e. determine the volume-dependent prefactors.

### 11) Pfaffian (*4 points*)

Show explicitly that the Pfaffian of a  $4 \times 4$  antisymmetric matrix is the square root of the determinant of the same matrix.

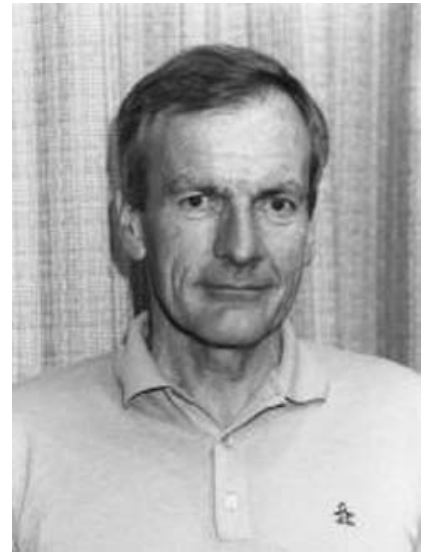
### 12) Potts Model Simulation (*2+4 points*)

- a) Replace the interaction in your code for the Ising model in 2 dimensions to the 2-state Potts model by computing the action (and action difference) in the Metropolis update via the Kronecker delta instead of the product of spins.
- b) Extend your simulation to the 3-state Potts model and check that the transition temperature is  $T_c = \frac{J}{\ln(1+\sqrt{q})}$

## Renfrey Potts

(October 4, 1925, - August 9, 2005) was an Australian Mathematician.

In 1943 he began an accelerated engineering course at the University of Adelaide but because of the end of war and the greater delights of mathematics, he switched his field of study and obtained a First Class Honours in Mathematics from Adelaide in 1947. After a brief stint as a junior lecturer at Adelaide, he went to Oxford as a Rhodes Scholar in 1948. His interests turned to mathematical physics and he received his Ph.D. from Oxford in 1951 with a dissertation on Ising models under the supervision of Cyril Domb. The 'Potts model', which he developed in his thesis, is a generalization of the Ising model for interacting spins on a crystalline lattice. His paper on this Some Generalized Order-Disorder Transformations, in the Proceedings of the Cambridge Philosophical Society (1952) is his most cited paper. He carried out post-doctoral work on cosmic rays with Harry Messel at the University of Sydney and also had a post-doctoral position at the University of Maryland 1955-56. He was again in Australia at the University of Adelaide in 1957. As an Associate Professor at the University of Toronto 1958-59 he became interested in operations research, the beginning of a life-long interest in the applications of mathematics to real-world problems. While there, he acted as a consultant to General Motors in Detroit and worked on 'car-following' models, studying the interactions between pairs of vehicles on a busy highway.



Potts was appointed to a newly created chair in applied mathematics at the University of Adelaide in 1959 where he remained until his retirement in 1990. Potts served as head of the department and was a leader both in his department and in Australian mathematics in general, especially in applied mathematics. Along with Eric Barnes he oversaw the formation at Adelaide of a Faculty of Mathematical Sciences including Pure Mathematics, Applied Mathematics, Statistics, Mathematical Physics, and Computer Science. [...] In addition to research on Ising-type models in mathematical physics and on road traffic analysis, Potts contributed to three other areas of research: operations research, especially networks; difference equations; and robotics. [...] He was interested in computing from the early days of the computing revolution and oversaw the first computer purchases at the University of Adelaide. Potts was an outstanding lecturer who drew large audiences to his talks. [...] He married his wife Barbara Kidman (a computer scientist) in Oxford on 1 July 1950. They had two daughters, Linda and Rebecca. Among the honours which were given to Potts for his outstanding contributions we mention election to the Australian Academy of Science in 1975, election to the Australian Academy of Technological Sciences and Engineering in 1983. He became an Officer of the Order of Australia in 1991 and was awarded the ANZIAM Medal in 1995.

[from <http://www-history.mcs.st-and.ac.uk/Biographies/Potts.html>]