Classical Statistical Models - Spring 2016

Bielefeld University

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Exercise Nr. 3

Discussion on May 9th, 15:00-16:00, Room V2-121

5) Symmetric Random Walk in 1 dimension ***(4+4 points)***

a) Use the reflection trick discussed in the lecture to prove the Ballot theorem:

$$P_0(S_1 > 0, S_2 > 0, \ldots, S_n > 0 | S_n = y) = \frac{y}{n},$$

which describes the probability that the random walk never becomes zero, given that it starts at $S_0 = 0$ and ends $S_n = y$ after n time steps.

b*) Perform a simulation ot the symmetric random walk and reproduce the above probability for various $y > 0$, with $n = 100$ fixed.

6) Connective Constant and End-to-End Distance ***(3+3 points)***

a) Find the number of nearest neighbor self-avoiding walks on a d-dimensional hypercubic lattice $Z_n^{(d)}(\lambda = 1)$ for $n = 1, 2, 3, 4$ steps and estimate from this the connective constant μ

*b) Perform a number of random walk simulations to estimate the value of ν for a simple random walk on a square lattice. Give error bars and compare your result with the exact answer.

7) Critical Exponents ***(2+4 points)***

Consider the free energy density in the vicinity of a 2nd order phase transition:

$$f(t, h) = t^{2-\alpha} g(t/h^{1/\phi}),$$

where h is a (reduced) external field and $t = \frac{T-T_c}{T_c}$ the reduced temperature.

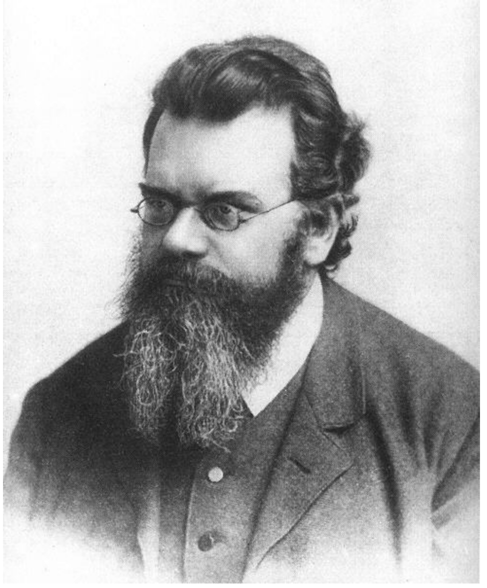
a) Show that α is the critical exponent of the specific heat.

b) Express the coefficients β and δ by α and ϕ and show that the following scaling relation holds:

$$2 - \alpha = \beta(\delta + 1).$$

Ludwig Eduardo Boltzmann
(February 20, 1844 - September 5, 1906) was an Austrian physicist and philosopher.

His greatest achievement was in the development of statistical mechanics, which explains and predicts how the properties of atoms (such as mass, charge, and structure) determine the visible properties of matter (such as viscosity, thermal conductivity, and diffusion). After receiving his doctorate from the University of Vienna in 1866, Boltzmann held professorships in mathematics and physics at Vienna, Graz, Munich, and Leipzig. In the 1870s Boltzmann published a series of papers in which he showed that the second law of thermodynamics, which concerns energy exchange, could be explained by applying the laws of mechanics and the theory of probability to the motions of the atoms. In so doing, he made clear that the second law is essentially statistical and that a system approaches a state of thermodynamic equilibrium (uniform energy distribution throughout) because equilibrium is overwhelmingly the most probable state of a material system. During these investigations Boltzmann worked out the general law for the distribution of energy among the various parts of a system at a specific temperature and derived the theorem of equipartition of energy (Maxwell-Boltzmann distribution law).



This law states that the average amount of energy involved in each different direction of motion of an atom is the same. He derived an equation for the change of the distribution of energy among atoms due to atomic collisions and laid the foundations of statistical mechanics.
Boltzmann was also one of the first continental scientists to recognize the importance of the electromagnetic theory proposed by James Clerk Maxwell of England. Though his work on statistical mechanics was strongly attacked and long-misunderstood, his conclusions were finally supported by the discoveries in atomic physics that began shortly before 1900 and by recognition that fluctuation phenomena, such as Brownian motion (random movement of microscopic particles suspended in a fluid), could be explained only by statistical mechanics.

`[from Encyclopaedia Britannica Online Academic Edition]`