Classical Statistical Models - Spring 2016

Bielefeld University

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Exercise Nr. 2

Discussion on May 2nd, 15:00-16:00, Room V2-121

2) Absorbing Markov Chains (from Ex. Sheet 1) *(2+4 points)*

a) Prove that for an absorbing Markov chain, the probability that the process will be absorbed in some absorbing state, starting from any transient state, is 1.

b) Consider the example of a drunkard's walk for n blocks. The absorbing states are s_0 and s_n, the transient states are s_i, $i = 1, \dots n-1$, and the non-zero transition probabilities are $p_{i,i+1} = p_{i,i-1} = \frac{1}{2}$, and $p_{00} = p_{nn} = 1$. Prove that the expected time to absorption, starting at state s_i, is equal to $i(n-i)$

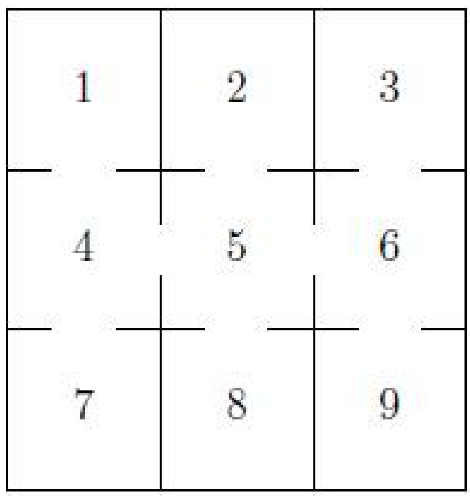
4) Ergodic Markov Chain *(2+4 points)*

a) Find the equilibrium vector for the transition matrix

$$\begin{pmatrix} p & 1-p \\ 1-q & q \end{pmatrix}.$$

Under what conditions is that matrix regular?

b) Compute the equilibrium vector of the maze problem, where a mouse moves in a 3×3 grid maze to neighboring sites with equal probability (see sektch).



5) Symmetric Random Walk in 1 dimension *(4+4 points)*

a) Use the reflection trick discussed in the lecture to prove the Ballot theorem:

$$P_0(S_1 > 0, S_2 > 0, \dots, S_n > 0 | S_n = y) = \frac{y}{n},$$

which describes the probability that the random walk never becomes zero, given that it starts at $S_0 = 0$ and ends $S_n = y$ after n time steps.

b*) Perform a simulation ot the symmetric random walk and reproduce the above probability for various $y > 0$, with $n = 100$ fixed.

Stanislaw Marcin Ulam
(April 13, 1909 - May 13, 1984)

Ulam received a doctoral degree (1933) at the Polytechnic Institute in Lvov (now Lviv). At the invitation of John von Neumann, he worked at the Institute for Advanced Study, Princeton, New Jersey, U.S., in 1936. He lectured at Harvard University in 1939-40 and taught at the University of Wisconsin at Madison from 1941 to 1943. In 1943 he became a U.S. citizen and was recruited to work at Los Alamos on the development of the atomic bomb. He remained at Los Alamos until 1965 and taught at various universities thereafter.

Ulam had a number of specialties, including set theory, mathematical logic, functions of real variables, thermonuclear reactions, topology, and the Monte Carlo theory.

Working with physicist Edward Teller, Ulam solved one major problem encountered in work on the fusion bomb by suggesting that compression was essential to explosion and that shock waves from a fission bomb could produce the compression needed. He further suggested that careful design could focus mechanical shock waves in such a way that they would promote rapid burning of the fusion fuel. Teller suggested that radiation implosion, rather than mechanical shock, be used to compress the thermonuclear fuel. This two-stage radiation implosion design, which became known as the Teller-Ulam configuration, led to the creation of modern thermonuclear weapons. Ulam's work at Los Alamos had begun with his development (in collaboration with von Neumann) of the Monte Carlo method, a technique for finding approximate solutions to problems by means of artificial sampling. Through the use of electronic computers, this method became widespread, finding applications in weapons design, mathematical economy, and operations research. Ulam also improved the flexibility and general utility of computers and wrote a number of papers and books on aspects of mathematics. The latter include A Collection of Mathematical Problems (1960), Stanislaw Ulam: Sets, Numbers, and Universes (1974), and Adventures of a Mathematician (1976).

`[from Encyclopaedia Britannica Online Academic Edition.]`