Monte Carlo Methods - Fall 2013/14

Goethe Universität, Frankfurt am Main

Lecture/Tutorial: Wolfgang Unger Office: 02.105 unger@th.physik.uni-frankfurt.de

Exercise Nr. 11

Discussion on January 29, after the lecture

31) Statistical Approach to Quantum Mechanics *(8 points)*

Read the article by M. Creutz and B. Freedman (Annals of Physics 132, 1981) on the statistical approach to quantum mechanics, in particular end of section I and section IV. Please explain in your own words

- why no mean square velocity can be defined at any time slice,
- and how the split point definition, Eq. 2.28 solves the problem.
- Explain the viral theorem used to determine E_0 in Eq. 4.13.
- Derive the classical tunneling solution

$$x(t) = \sqrt{f^2}\tanh(\lambda\sqrt{2f^2}\tau)$$

depicted in Fig. 16 for $\lambda = 1, f^2 = 2$.
Hint: Solve the classical EoM $\frac{d^2x}{d\tau^2} = \frac{\partial}{\partial x}V(x)$ with $V(x) = \lambda(x^2 - f^2)^2$.

32) Anharmonic Oscillator *(12 points)*

Try to reproduce Fig. 9 and Fig. 10 of the above article, that is measure correlation function and the first two energy levels of the anharmonic oscillator, for $\lambda = 1$ and various values of f^2.

*33) Free Scalar Propagator *(6 points)*

The Fourier transform of a free scalar massive propagator

$$G(p) = 1/(p^2 + m^2)$$

in d spatial dimensions is given by

$$G(r) \sim \frac{e^{-mr}}{r^{(d-1)/2}}$$

in the large distance limit.

Show this for $d = 4$. Hint: You may expand the correlation function around the saddle point $t_s = \frac{r}{2m}$ with $r = |x - y|$ and x, y four-vectors.

Satyendra Nath Bose
(1 January 1894 - 4 February 1974) was an Indian physicist.

[..] He began his studies at Presidency College, Calcutta, in 1909 where he had a brilliant academic record. He was awarded a B.Sc. in 1913 and an M.Sc. in 1915 proving himself to be by far the best student of mathematics. In the year he was awarded his Master's degree, Bose married Ushabala Ghosh. They had five children, three daughters and two sons. Had Indians been allowed to take administrative posts in the government service, Bose would almost certainly have followed that route. As it was, he continued to study physics and mathematics and was appointed to the newly opened University College of Science in Calcutta in 1917. This university was a research institution for postgraduate studies and here Bose was able to study recent European texts on quantum theory and relativity which, before the opening of the new institution, had not been readily available in India. Gibbs book on statistical mechanics stimulated Bose's interest in this topic. He also studied Einstein's papers on relativity and obtained Einstein's permission to translate them for publication in India.

Bose was appointed as a Reader in physics at the University of Dacca in 1921 and taught there until 1945, being a professor and head of the physics department from 1927. In 1945 he returned to Calcutta University when he was appointed as Guprasad Sing Professor of Physics, a position he held until he retired in 1956 when he was made Professor Emeritus.

He did important work in quantum theory, in particular on Planck's black body radiation law. Bose sent his paper "Planck's Law and the Hypothesis of Light Quanta" (1924) to Einstein. [...] This paper was only four pages long but it was highly significant. The derivation of Planck's formula had not been to Planck's satisfaction, and Einstein too was unhappy with it. Bose was able to derive the formula for radiation from Boltzmann's statistics. The paper, and his method of deriving Planck's radiation formula, was enthusiastically endorsed by Einstein who saw at once that Bose had removed a major objection against light quanta. The paper was translated into German by Einstein and submitted with a strong recommendation to the Zeitschrift für Physik. Einstein extended Bose's treatment to material particles whose number is conserved and published several papers on this extension.

An important consequence of Einstein's response to Bose's article was that his application to the University of Dacca for two years research leave beginning in 1924 was approved. He now had the chance of meeting European scientists and travelled first to Paris where he met Langevin and de Broglie. In October 1925 Bose travelled from Paris to Berlin where he met Einstein. Much progress had been made by Einstein following his receipt of Bose's paper for he was able to see how the ideas could be taken forward. While he was in Berlin Bose attended a course on quantum theory given by Born.

Bose published on statistical mechanics leading to the Einstein-Bose statistics. Dirac coined the term boson for particles obeying these statistics. Through these terms his name is rightly known and remembered, for indeed his contributions are remarkable, especially given the fact that he made his important discoveries working in isolation from the mainstream developments in Europe. [...] After Bose retired from Calcutta University in 1956 he was appointed as vice-chancellor of Viswa-Bharati University, Santiniketan. Two years later he was honoured with the post of national professor.

[http://www-history.mcs.st-and.ac.uk/Biographies/Bose.html]