Monte Carlo Methods - Fall 2013/14

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Exercise Nr. 1

Discussion on October 21st, 14:00-15:00

1) Stochastic determination of π *(4 points)*

Implement the "Hit and Miss" Monte Carlo method for the computation of π, given the pseudo-code on the right.

Explanation of the Pseudo-Code:

- $variable \leftarrow value$: assignment
- $\texttt{ran}(a, b)$: uniformly distributed real random number in interval [a,b[

```
procedure direct-pi
N_hits ← 0
for i = 1, ..., N do
    x ← ran(−1, 1)
    y ← ran(−1, 1)
    if (x^2 + y^2 < 1) N_hits ← N_hits + 1
end
output N_hits
```

2) Buffon's needle experiment *(2+5 points)*

Consider the following pseudo-code for Buffon's needle experiment to compute π. Here, a is the needle length and b is the distance between the cracks.

a) Why is this code a cheat? How can you patch it?

b) Implement the patched code.

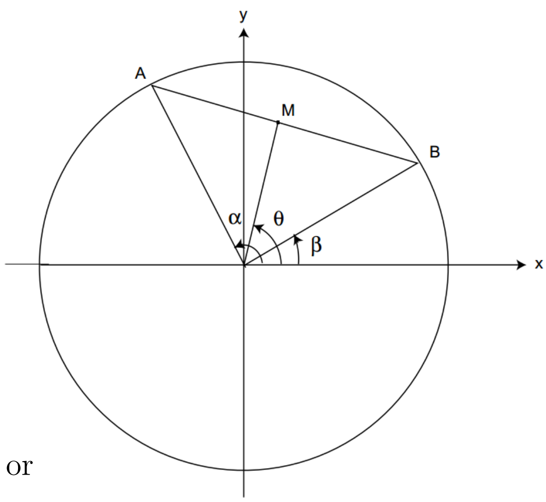
Hint: find a way to get a uniform distribution in ϕ.

```
procedure direct-needle
N_hits ← 0
for i = 1, ..., N do
    x_center ← ran(0, b/2)
    φ ← ran(0, π/2)
    x_tip ← x_center − a/2cos(φ)
    if (x_tip < 0) N_hits ← N_hits + 1
end
output N_hits
```

3) Bertrand's Paradox *(3+3+3 points)*

A chord of a circle is a line segment both of whose endpoints lie on the circle. Suppose that a chord is drawn at random in a unit circle. What is the probability that its length exceeds $\sqrt{3}$?

It is not clear in the above question what is meant by "random". The answer will depend on the choice of coordinates which are assumed to be uniformly distributed. We can describe each cord by one of the following choices:



a) The rectangular coordinates (x, y) of the midpoint M, or

b) The polar coordinates (r, θ) of the midpoint M, or

c) The polar coordinates $(1, \alpha)$ and $(1, \beta)$ of the endpoints A and B

Estimate in each case the probability with a computer program, and show that the answers will differ. Which choice yields the largest probability? Can you explain the outcome and obtain theoretical values for each choice?

Georges-Louis Leclerc, Comte de Buffon
(September 7, 1707 - April 16, 1788) was a French naturalist, mathematician, cosmologist, and encyclopedic author.

French naturalist, remembered for his comprehensive work on natural history [...] The name Buffon came from an estate that he inherited from his mother at about the age of 25. Beginning his studies at the College of Godrans in Dijon, which was run by the Jesuits, he seems now to have been only an average student, but one with a marked taste for mathematics. His father wanted him to have a legal career, and in 1723 he began the study of law. In 1728, however, he went to Angers, where he seems to have studied medicine and botany as well as mathematics. [...] The death of his mother called him back to France. He settled down on the family estate at Montbard, where he undertook his first research in the calculus of probability and in the physical sciences. Buffon at that time was particularly interested in questions of plant physiology. [...] Maintaining an interest in mathematics, he published a translation of Sir Isaac Newton's Fluxions in 1740. In his preface to this work he discussed the history of the differences between Newton and Gottfried Wilhelm Leibniz over the discovery of the infinitesimal calculus. He also made researches on the properties of timbers and their improvement in his forests in Burgundy.

In 1739, at the age of 32, he was appointed keeper of the Jardin du Roi (the royal botanical garden, now the Jardin des Plantes) [...] Maurepas also charged Buffon to undertake a catalog of the royal collections in natural history, which the ambitious Buffon transformed into an undertaking to produce an account of the whole of nature. This became his great work, Histoire naturelle, générale et particulière (1749-1804), which was the first modern attempt to systematically present all existing knowledge in the fields of natural history, geology, and anthropology in a single publication. Buffon's Histoire naturelle was translated into various languages and widely read throughout Europe. [...] He was elected to the French Academy, where, on August 25, 1753, he delivered his celebrated Discours sur le style ("Discourse on Style"), containing the line, "Le style c'est l'homme même" ("The style is the man himself"). [...] Although he was a friend of Denis Diderot and Jean Le Rond d'Alembert, he did not collaborate on their Encyclopédie. [...] Buffon's position among his contemporaries was by no means assured. Though the public was nearly unanimous in its admiration of him, he met with numerous detractors among the learned. The theologians were aroused by his conceptions of geological history; others criticized his views on biological classification; [...] Voltaire did not appreciate his style, and d'Alembert called him "the great phrasemonger." [...] In some areas of natural science Buffon had a lasting influence. He was the first to reconstruct geological history in a series of stages, in Époques de la nature (1778). With his notion of lost species he opened the way to the development of paleontology. He was the first to propose the theory that the planets had been created in a collision between the Sun and a comet. While his great project opened up vast areas of knowledge that were beyond his powers to encompass, his Histoire naturelle was the first work to present the previously isolated and apparently disconnected facts of natural history in a generally intelligible form. [...]

`[from Encyclopaedia Britannica Online Academic Edition.]`