Five lectures on

INTRODUCTION TO COSMOLOGY

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Lecture 1: The large picture

observations, cosmological principle, Friedmann model, Hubble diagram, thermal history

Lecture 2: From quantum to classical

cosmological inflation, isotropy & homogeneity, causality, flatness, metric & matter fluctuations

Lecture 3: Hot big bang

radiation domination, hot phase transitions, relics, nucleosythesis, cosmic microwave radiation

Lecture 4: Cosmic structure

primary and secondary cmb fluctuations, large scale structure, gravitational instability

Lecture 5: Cosmic substratum

evidence and candidates for dark matter and dark energy, direct and indirect dm searches

Inflationary ΛCDM model

this is the current "standard model", it is the "minimal model"

topology: trivial

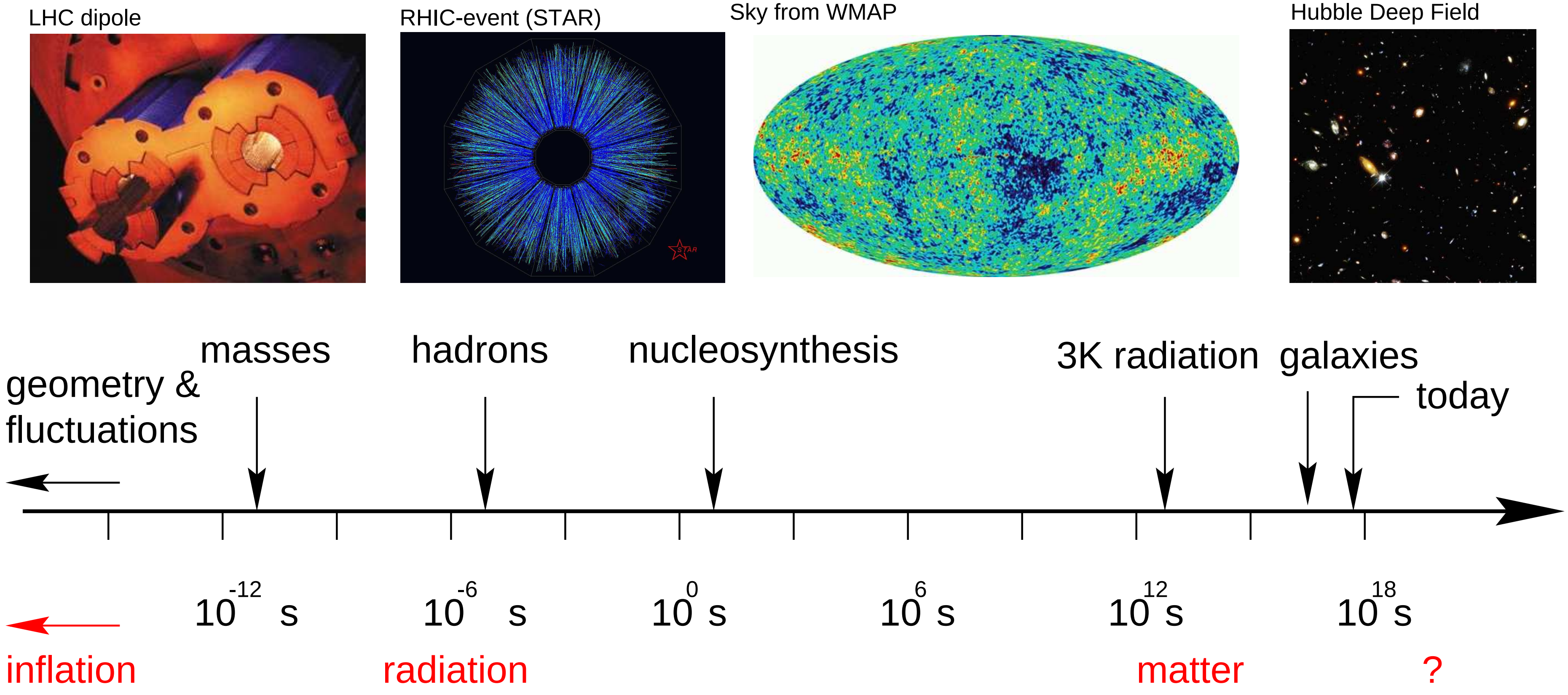
geometry: flat Friedmann model

components: $\Lambda > 0$, cold dark matter, baryons, γ, ν (massless)

small fluctuations of matter and metric: slow-roll inflation

minimal set of parameters necessary to study structure formation:
$h, T_0, \omega_\mathrm{b}, \omega_\mathrm{m}, A, n-1$ plus some astrophysical parameters $(\tau, b, Q_\mathrm{nl}, \sigma_v, \ldots)$

History of the Universe



Cosmological perturbations

small fluctuations of Friedmann cosmology

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \delta g_{\mu\nu}, \quad T_{\mu\nu} = \bar{T}_{\mu\nu} + \delta T_{\mu\nu}$$

split up into scalar, vector and tensor perturbations (analogous for $\delta T_{\mu\nu}$)

$$\delta g_{00} = A, \quad \delta g_{0i} = \bar{\nabla}_i B + B_i^{\perp},$$

$$\delta g_{ij} = \bar{g}_{ij} C_1 + \bar{D}_{ij} C_2 + \bar{\nabla}_{(i} C_{j)}^{\perp} + C_{ij}^{\mathsf{TT}}, \quad \bar{D}_{ij} \equiv \bar{\nabla}_{(i}\bar{\nabla}_{j)} - \frac{1}{3}\bar{g}_{ij}\bar{\nabla}^2$$

$$\bar{\nabla}^i B_i^{\perp} = \bar{\nabla}^i C_i^{\perp} = 0, \quad \bar{\nabla}^j C_{ij}^{\mathsf{TT}} = 0, \quad \bar{g}^{ij} C_{ij}^{\mathsf{TT}} = 0$$

gauge fix two scalar and two vector degrees of freedom

linear regime: scalar, vector & tensor perturbations decouple
(2 dof per each type)

restrict for lecture to $K = 0$ and neglect anisotropic pressure (important for νs)

Linearised Einstein equations: scalars I

several perfect fluids $a =$ b, cdm, rad, ...

$$\Delta_a \equiv \frac{\delta\epsilon_a}{(\epsilon+p)_a}, \ \Delta = \sum_a \frac{(\epsilon+p)_a}{\epsilon+p}\Delta_a, \quad v_a \equiv -i\hat{\mathbf{k}}\mathbf{v}_a, \ v = \sum_a \frac{(\epsilon+p)_a}{\epsilon+p} v_a$$

sum of perfect fluids makes up one imperfect fluid:

entropy (isocurvature) perturbations

$$\mathcal{S} \equiv \frac{\delta p - c_s^2 \delta\epsilon}{\epsilon+p} = \sum_a c_a^2 \frac{(\epsilon+p)_a}{\epsilon+p}(\Delta_a - \Delta)$$

isentropic initial conditions: $\mathcal{S} = \mathcal{S}' = 0 \Rightarrow \Delta_a = \Delta$ and $v_a = v$

Newtonian longitudinal gauge: $A = -2a^2\phi, B = 0, C_1 = -2a^2\psi, C_2 = 0$

Linearised Einstein equations: scalars II

continuity and Euler equations ($\mathcal{H} = a'/a, \eta$ conformal time)

$$\Delta_a' = kv_a + 3\psi', \quad v_a' + (1 - 3c_a^2)\mathcal{H}v_a = -c_a^2 k\Delta_a - k\phi$$

$\zeta_a \equiv \Delta_a/3 - \psi$ is constant on large scales ($k \ll \mathcal{H}$) Bardeen 1989

Poisson equation

$$-k^2\psi - 3\mathcal{H}\psi' - 3\mathcal{H}^2\phi = (\mathcal{H}' - \mathcal{H}^2)\Delta$$

vanishing of anisotropic pressure: $\phi = \psi$

dominant mode on superhorizon scales:
$\zeta \simeq -(5 + 3w)/[3(1 + w)]\phi$, with $w \equiv p/\epsilon$

$\zeta \simeq -\frac{5}{3}\phi(t > t_{\rm eq}) = -\frac{3}{2}\phi(t < t_{\rm eq})$ ϕ decreases by factor of 9/10 at equality

Linearised Einstein equations: scalars III

superhorizon scales: $\zeta_a \simeq \mathrm{const}$

subhorizon scales:

$$\Delta_\mathrm{r}'' + c_\mathrm{r}^2 k^2 \Delta_\mathrm{r} \simeq 0, \quad \Delta_\mathrm{m}'' + \mathcal{H}\Delta_\mathrm{m}' \simeq \frac{3}{2}(1+w)\mathcal{H}^2\Delta$$

until decoupling: $\Delta_\mathrm{r} \propto \cos(c_\mathrm{r} k \eta)$ — acoustic oscillations

radiation era: $\Delta \approx \Delta_\mathrm{r}$; $\Delta_\mathrm{m} \propto b_1 + b_2 \log\eta$ — supression of growth

matter era: $\Delta \approx \Delta_\mathrm{m}$; $\Delta_\mathrm{m} \propto \eta^2 \propto a$ — growth of structure

Λ era: $\Delta \approx \Delta_\mathrm{m}$; $\Delta_\mathrm{m} \simeq \mathrm{const}$ — stop of structure formation

Anisotropy of cosmic microwave background (CMB)

photon decoupling at $t \sim 350\,000$ years

temperature fluctuations $\delta T/T$ Sachs & Wolfe 1967

$$\frac{\delta T^{\mathsf{S}}}{T}(\vec{e}) = [\frac{\delta T_\gamma}{T_\gamma} + \phi - e^i v^{\mathsf{S}}_{\gamma i}]_{\mathsf{dec}} + \int_{\eta_{\mathsf{dec}}}^{\eta_0} \mathsf{d}\bar{\eta} \frac{\partial}{\partial \bar{\eta}}(\phi + \psi)$$
$$\frac{\delta T^{\mathsf{T}}}{T}(\vec{e}) = -\frac{1}{2} e^i e^j \int_{\eta_{\mathsf{dec}}}^{\eta_0} \mathsf{d}\bar{\eta} \frac{\partial}{\partial \bar{\eta}} h_{ij}$$

scalar: temperature fluctuation, graviational red-shift, Doppler effect at decoupling;

scalar & tensor: integrated SW effect

extra astrophysical parameter:

optical depth τ or redshift of reionization

Predictions for cosmic microwave background

$\delta T(\mathbf{e}) = \sum a_{\ell m} Y_{\ell m}(\mathbf{e})$ with $a^*_{\ell m} = (-1)^m a_{\ell -m}$ (reality condition)
$\Rightarrow 2\ell + 1$ degrees of freedom for ℓth moment

statistical isotropy:

$\langle \delta T(\mathsf{R}\mathbf{e}_1) \ldots \delta T(\mathsf{R}\mathbf{e}_n) \rangle = \langle \delta T(\mathbf{e}_1) \ldots \delta T(\mathbf{e}_n) \rangle, \quad \forall \mathsf{R} \in \mathsf{SO}(3),\ \forall n > 0$

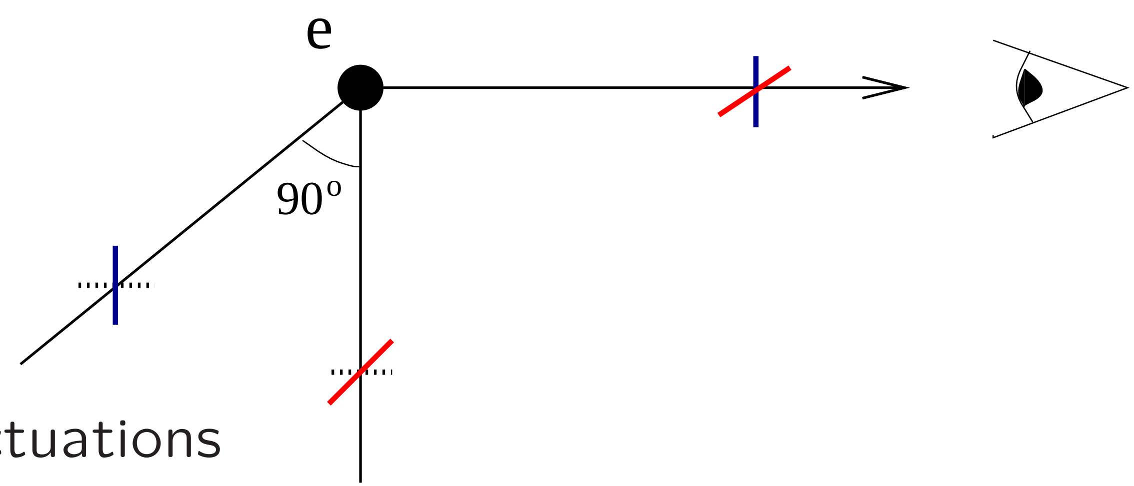
- $\langle \delta T(\mathbf{e}) \rangle = 0$ and $\langle a_{\ell m} \rangle = 0$
- $\langle \delta T(\mathbf{e}_1) \delta T(\mathbf{e}_2) \rangle = f(\mathbf{e}_1 \cdot \mathbf{e}_2) = \frac{1}{4\pi} \sum_\ell (2\ell + 1) C_\ell P_\ell(\cos\theta), \quad \cos\theta \equiv \mathbf{e}_1 \cdot \mathbf{e}_2$ with $\langle a_{\ell m} a^*_{\ell' m'} \rangle = C_\ell \delta_{\ell\ell'} \delta_{mm'}$, C_ℓ multipole moments

gaussianity: no extra information in higher correlation functions

(best) estimator: $\widehat{C}_\ell = 1/(2\ell + 1) \sum_m |a_{\ell m}|^2$ (assumes statistical isotropy)
cosmic variance: $\mathrm{Var}(\widehat{C}_\ell) = 2C_\ell^2/(2\ell + 1)$ (assumes gaussianity)

Polarisation of CMB

quadrupole at
photon decoupling induces
linear polarisation

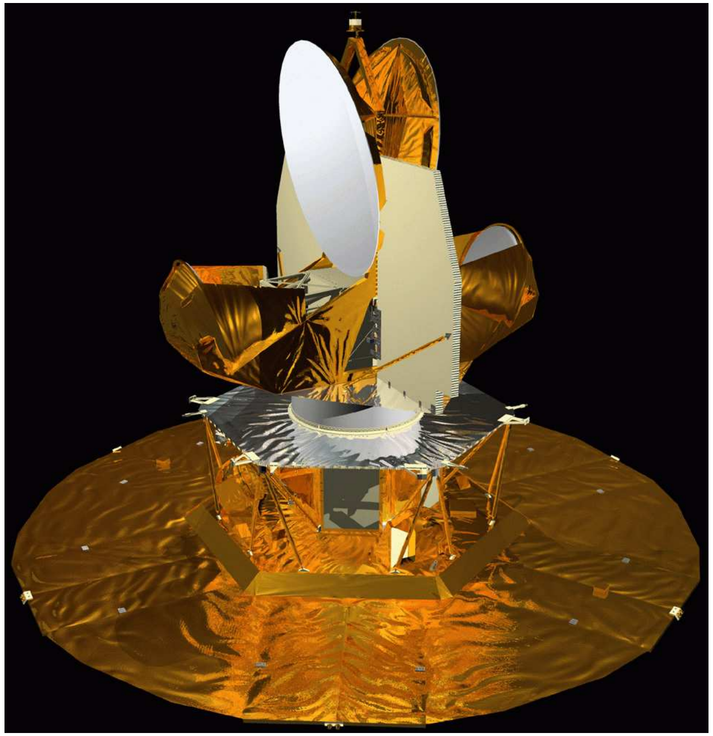


direct proof
of primordial nature of fluctuations

E- and B-modes (gradient and rotor field)
density fluctuations (E) and gravitational waves (E & B)

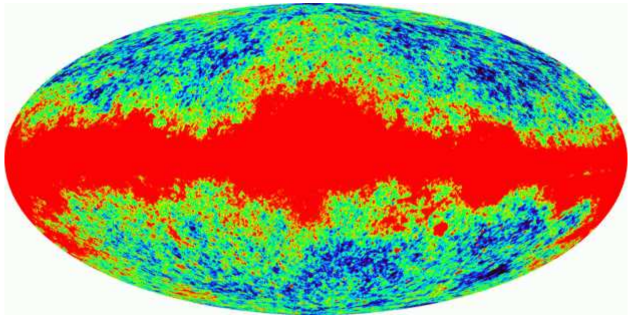
Observations of the microwave sky

BOOMERanG

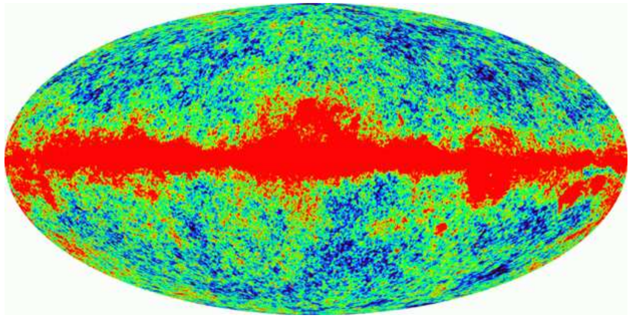


WMAP

WMAP: 5 frequency bands (K,Ka,Q,V,W)



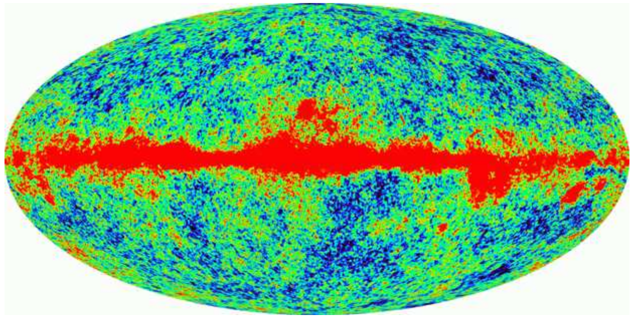
23GHz



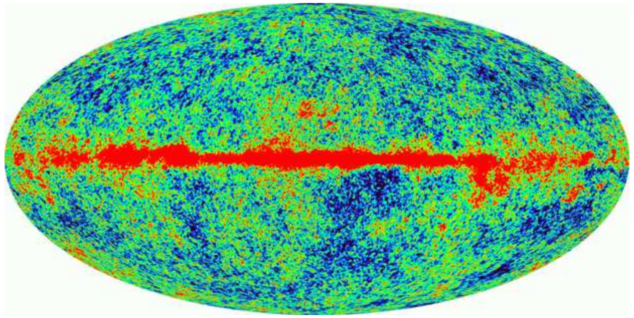
33GHz

$-200\mu\text{K} < \Delta T < +200\mu\text{K}$

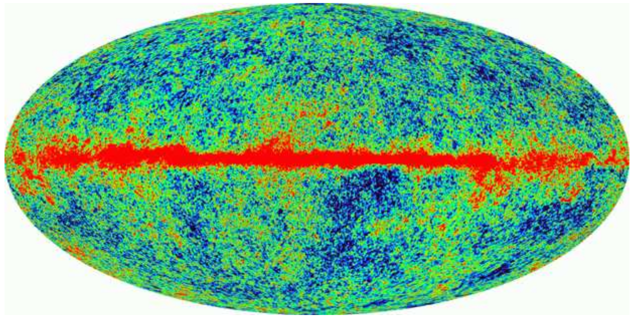
WMAP: Bennett et al 2003



41GHz

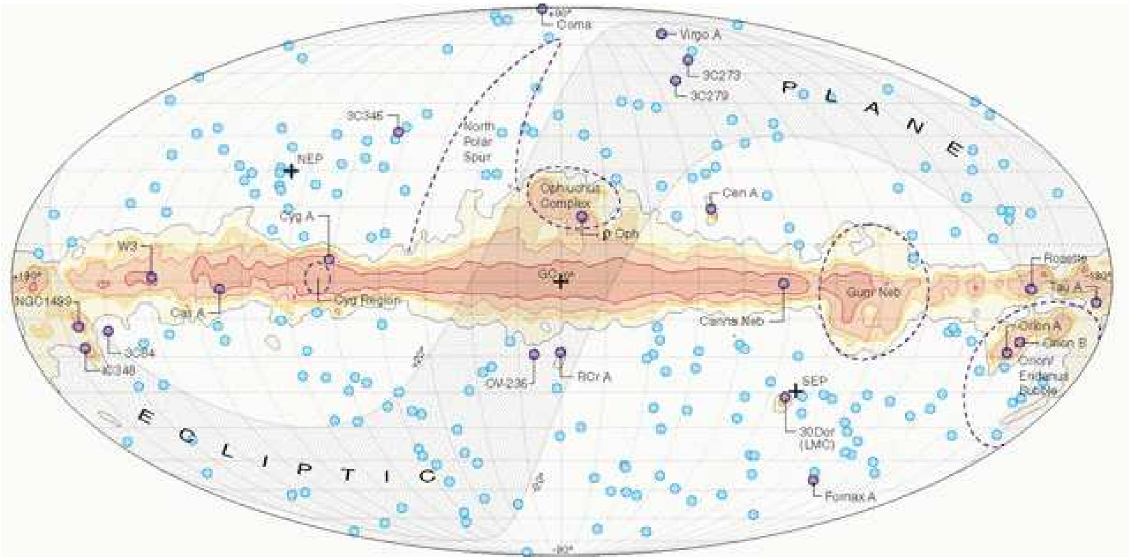


61GHz

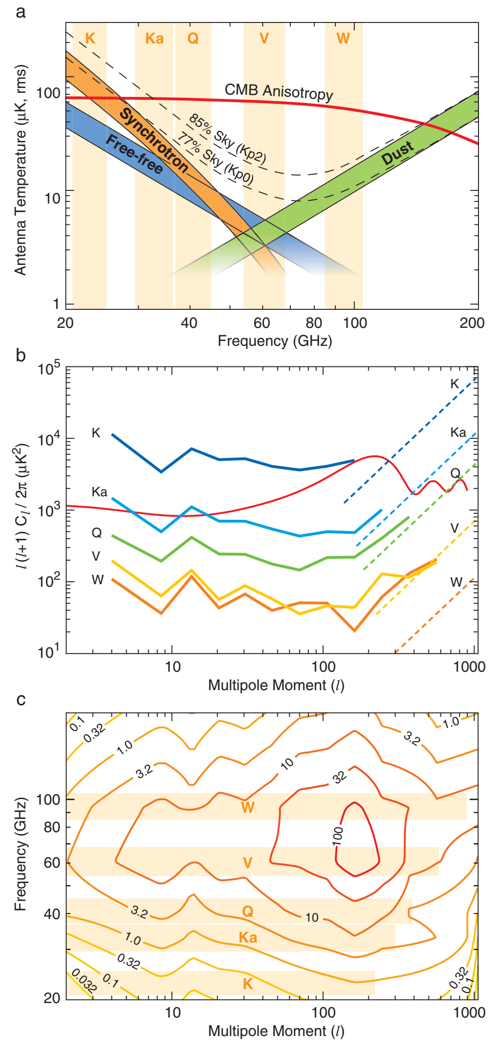


94GHz

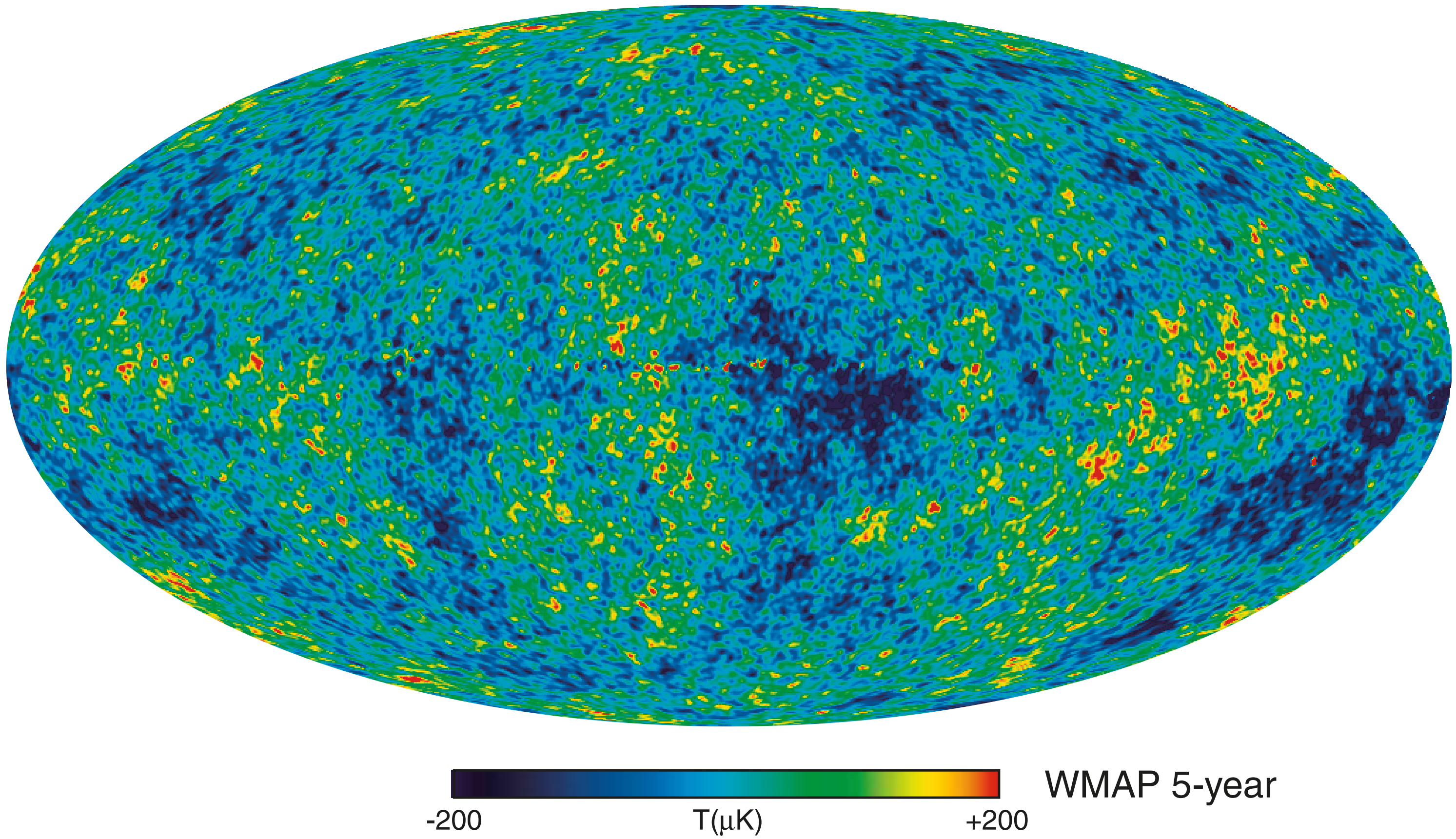
WMAP: foreground



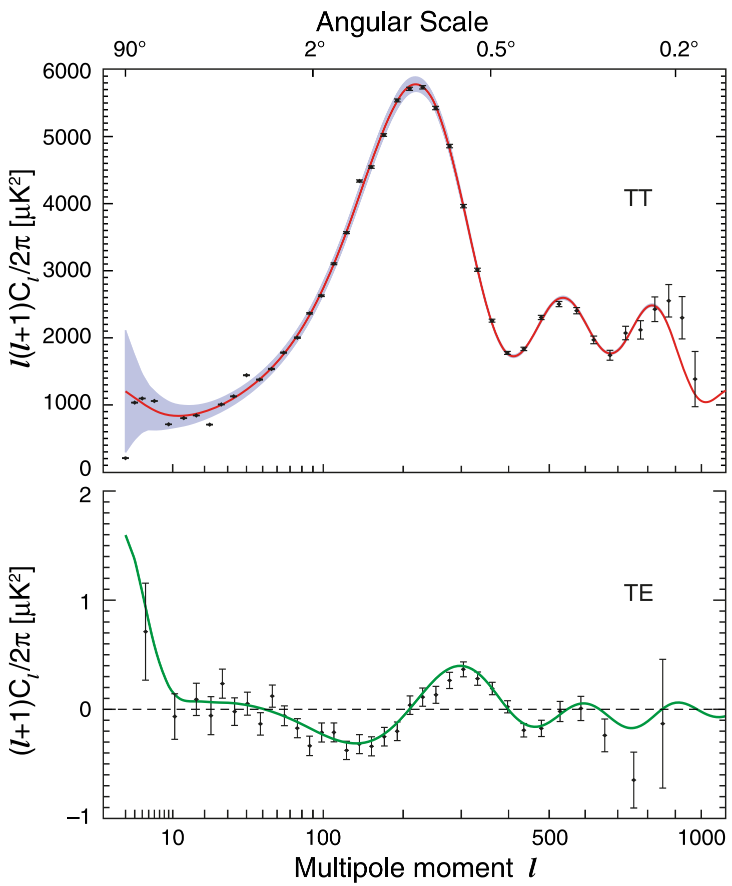
WMAP: Bennett et al 2003



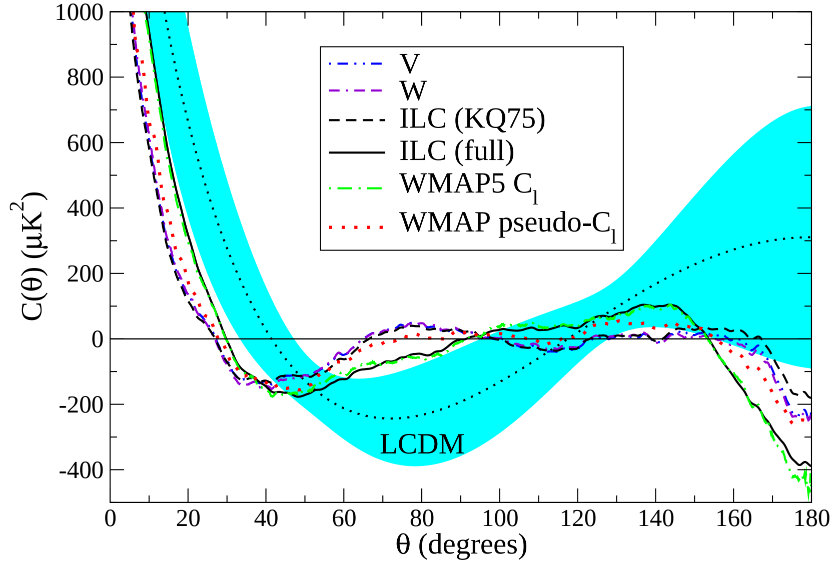
WMAP: Internal Linear Combination Map



Angular power spectrum and two-point correlation functions



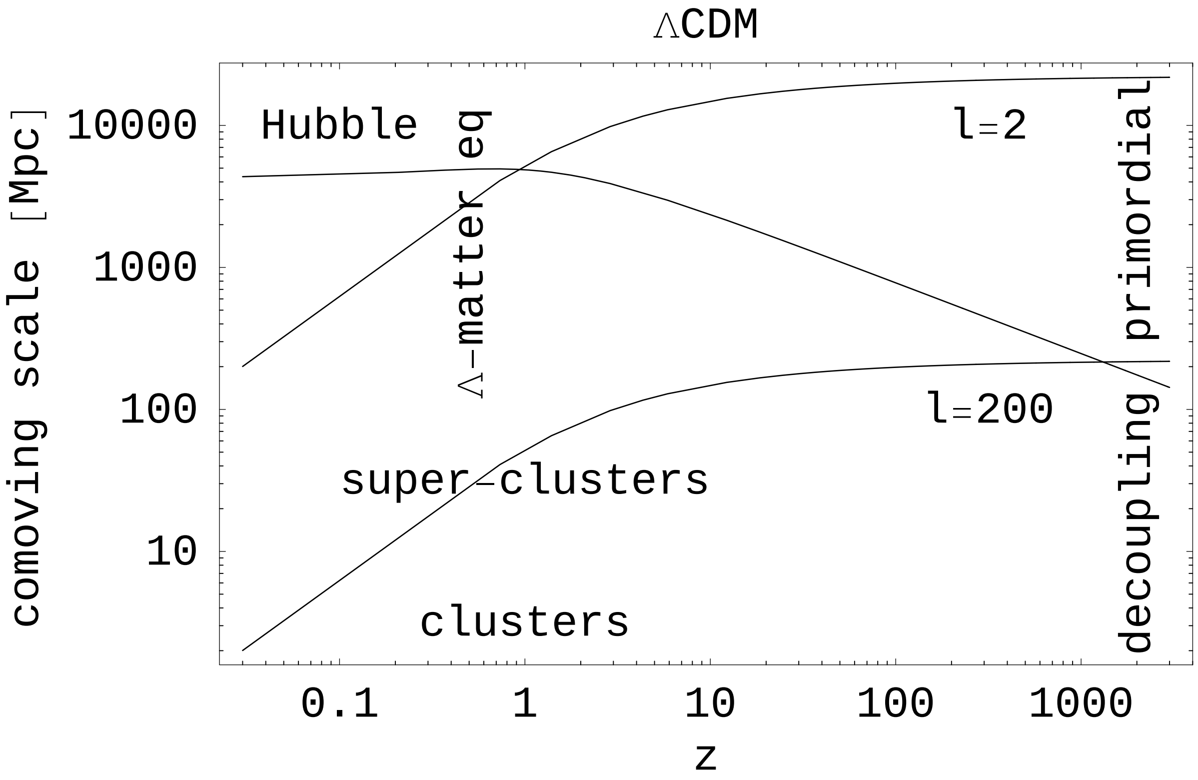
Hinshaw et al. 2008



Copi et al. 2008

Angular scales of cosmic microwave background

CMB probes physics back to photon decoupling $z_{\text{dec}} \approx 1100$



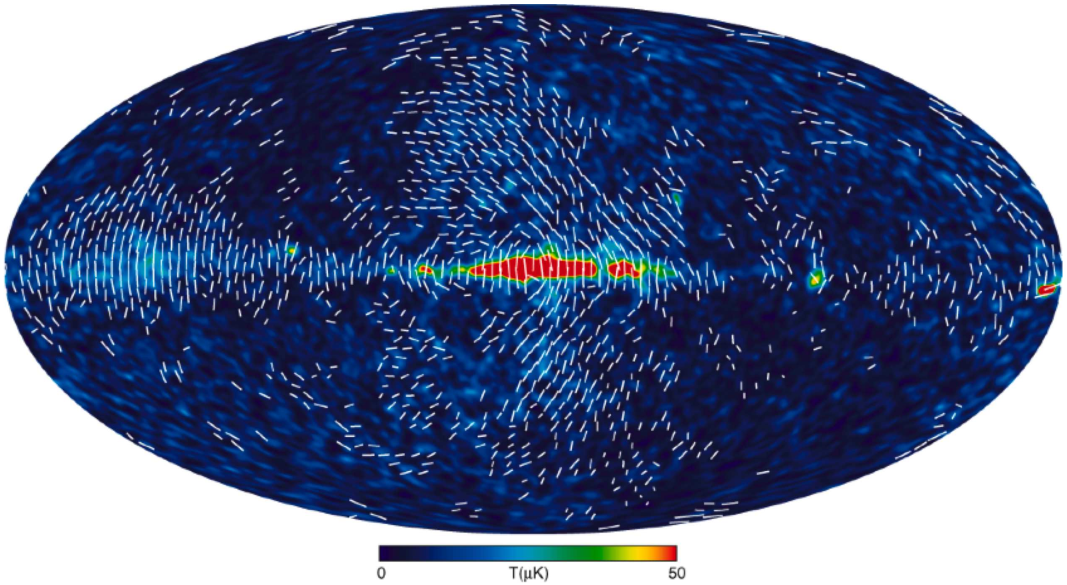
Geometry of the Universe

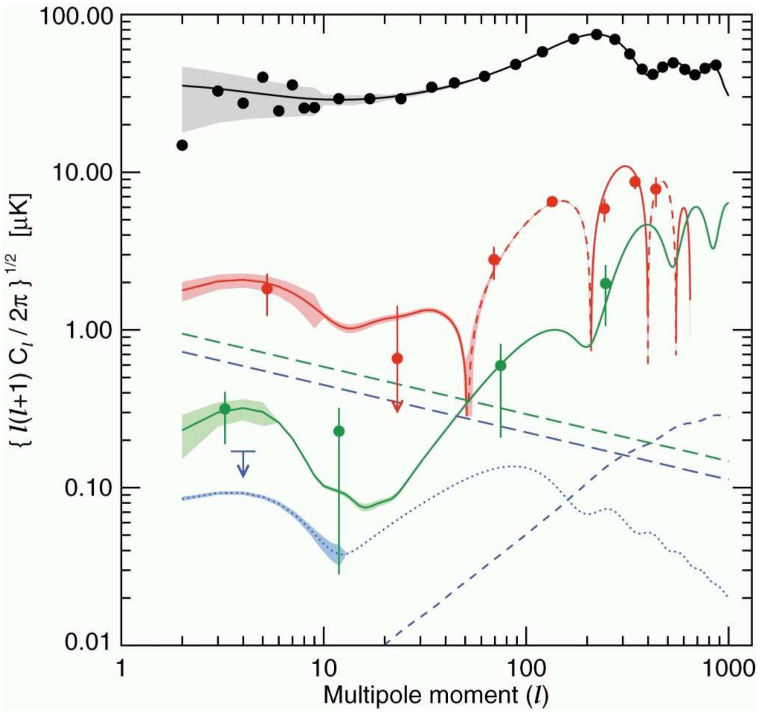
acoustic oscillations of photon-baryon plasma

$\lambda_{\text{ph}}/2 = (c_s/H)_{\text{dec}}$ and t_{dec} fixed (H-atom) $\Rightarrow$
triangle with all sides and one angle known determines the geometry

WMAP & BAO & SN $\Rightarrow$
$-0.0175 < \Omega - 1 < 0.0085$ Hinshaw et al. 2008

First full sky maps of polarisation





WMAP Q band polarisation
dominated by foreground, low S/N

Page et al. 2006

Cosmological parameters: power-law Λ CDM

inflationary parameters:

$\mathcal{P}_\zeta = (2.41 \pm 0.11) \times 10^{-9}$, $n = 0.963^{+0.014}_{-0.015}$

dynamic parameters:

$h = 0.719^{+0.026}_{-0.027}$, $\Omega_{\rm dm} h^2 = 0.1099 \pm 0.0062$, $\Omega_{\rm b} h^2 = 0.02273 \pm 0.0062$

astrophysical parameter:

$\tau = 0.087 \pm 0.017$

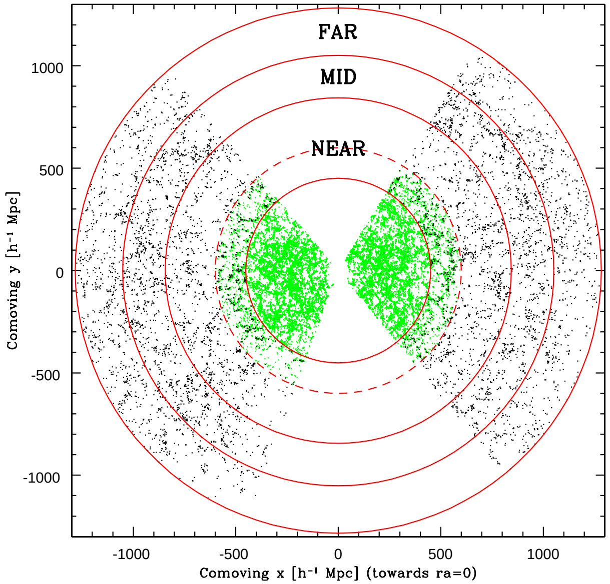
fit to WMAP-only

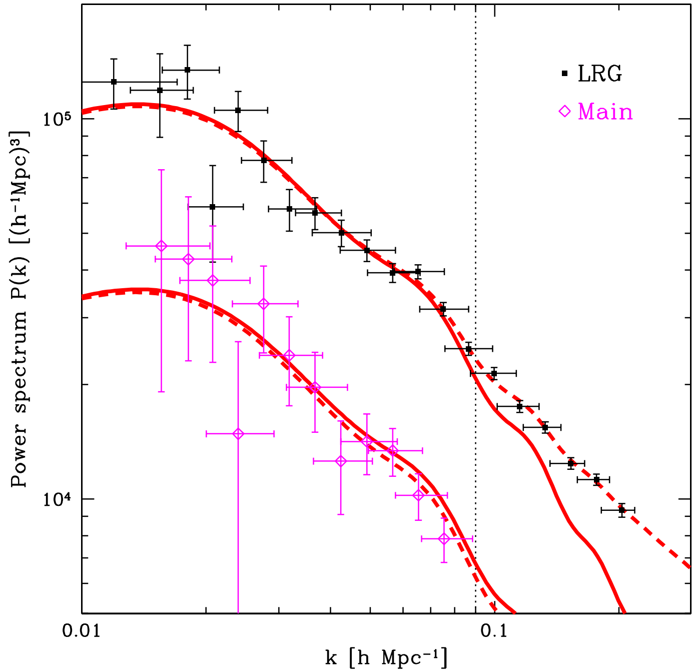
Hinshaw et al. 2008

WMAP 5yr: Minimal model fits data very nicely!

5% atoms, 23% cold dark matter and 72% dark energy

Observation of large scale structure and power spectrum





SDSS LRG (black) & main (green)

Tegmark et al. 2006

extra parameters:

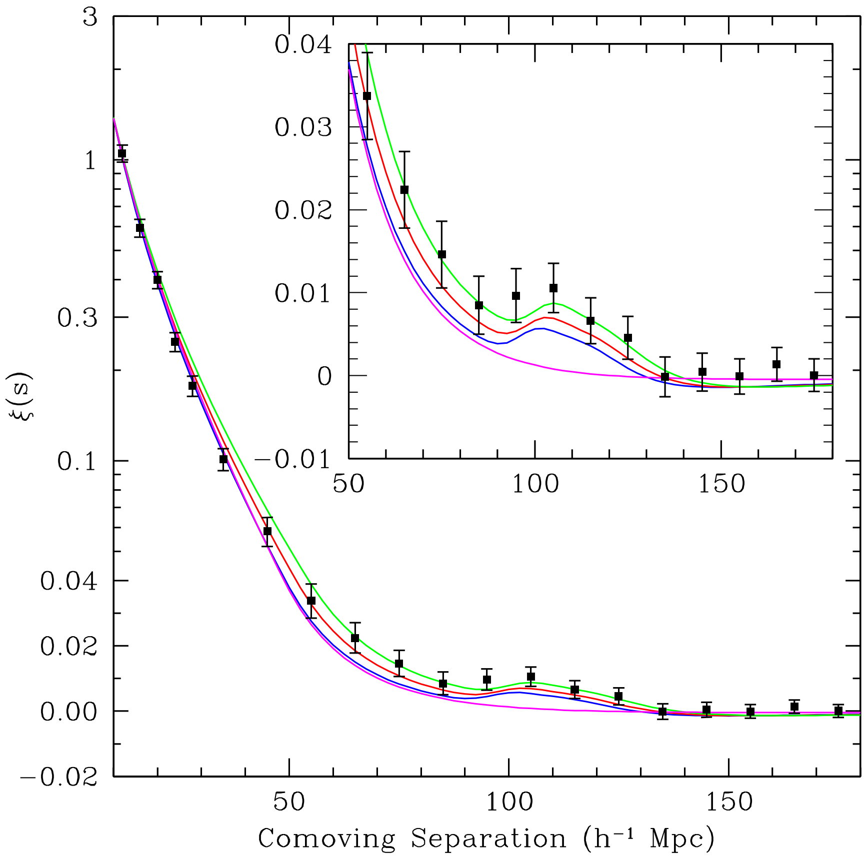
bias for each sample: $P_{\mathrm{s}} = b_s^2 P_{\mathrm{m}}$; nonlinear corrections (dashed): Q_{nl}

Baryon acoustic oscillations

baryon acoustic oscillations
in matter power spectrum $P(k)$
and
peak in correlation function $\xi(r)$

acoustic scale at $z \simeq 0.35$
constrain on
$d_\mathsf{V}(z) = [d_\mathsf{a}^2(z) d_\mathsf{H}(z) z]^{1/3}$

compare to acoustic scale
in CMB at $z \simeq 1100$



SDSS LRG Eisenstein et al. 2005

Summary of 4th lecture

structure forms via gravitational instability
seeds from quantum fluctuations during inflation

cosmic microwave background: most detailed and well defined probe

galaxy redshift surveys: less precise, extra parameter b

galaxy clusters, weak lensing surveys, Lyα forest, etc.

CMB polarisation (B-modes) interesting for fundamental physics
scale of inflation, higher accuracy thus additional cross-checks;
Planck or next generation of CMB experiments