Five lectures on

INTRODUCTION TO COSMOLOGY

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Lecture 1: The large picture

observations, cosmological principle, Friedmann model, Hubble diagram, thermal history

Lecture 2: From quantum to classical

cosmological inflation, isotropy & homogeneity, causality, flatness, metric & matter fluctuations

Lecture 3: Hot big bang

radiation domination, hot phase transitions, relics, nucleosythesis, cosmic microwave radiation

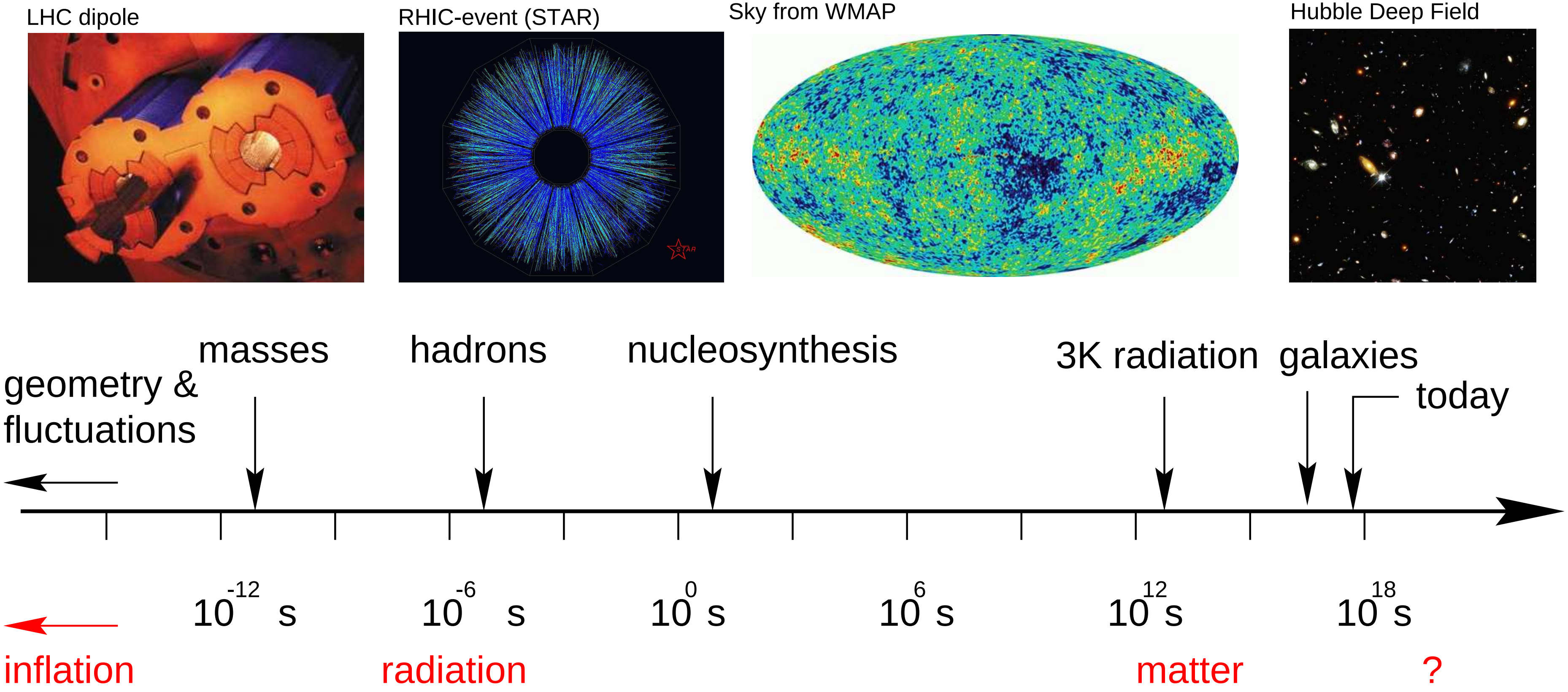
Lecture 4: Cosmic structure

primary and secondary cmb fluctuations, large scale structure, gravitational instability

Lecture 5: Cosmic substratum

evidence and candidates for dark matter and dark energy, direct and indirect dm searches

History of the Universe



Shortcomings of ΛCDM model

observed, but not explained:

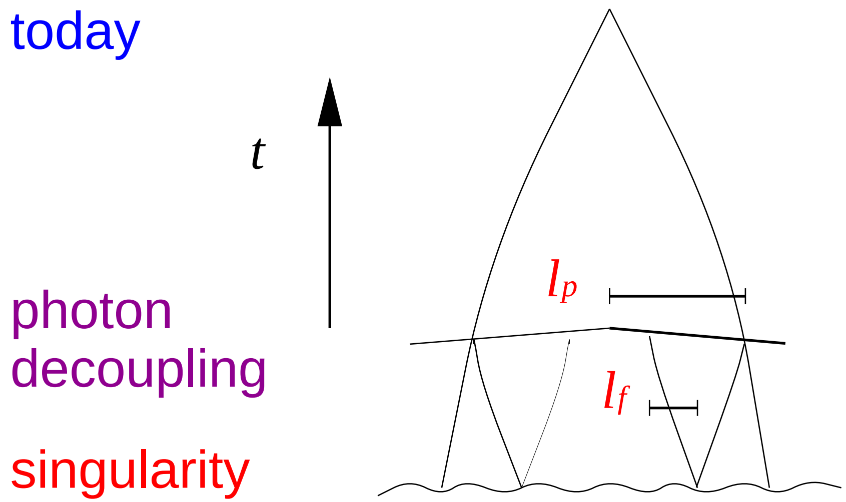
- isotropy and homogeneity
- spatial flatness
- $\Omega_\Lambda \sim \Omega_m$ today

Horizon problem

$\ell_p(t)$ past causal horizon
$\ell_f(t)$ future causal horizon

$(\ell_p/\ell_f)(z_{\rm dec}) \simeq \sqrt{z_{\rm dec}} \gg 1 \;\; {\scriptstyle (z_{\rm dec} \simeq 1100)}$

10^3 causally disconnected patches
have the same temperature. Why?



Flatness problem

Why is $\Omega_0 = \mathcal{O}(1)$?

$$|1-\Omega(z)| = |1-\Omega_0| \begin{cases} (1+z)^{-1} & \text{matter dominated} \\ (1+z)^{-2} & \text{radiation dominated} \end{cases}$$

$$\Rightarrow |1-\Omega(z_{\text{dec}})| = \mathcal{O}(10^{-3}) \ , \quad |1-\Omega(z_{\text{GUT}})| = \mathcal{O}(10^{-60}) \ (z_{\text{GUT}} \sim 10^{30})$$

Singularity problem

singularity ($a \to 0; \epsilon \to M_{\mathrm{P}}^4$) exists, if $\epsilon + 3p > 0$

(strong energy condition; satisfied in matter and radiation dominated universe)

proof: $\ddot{a} < 0$ from

$$-3\frac{\ddot{a}}{a} = 4\pi G(\epsilon + 3p) \qquad \text{(equation of geodesic deviation)}$$

if $\epsilon + 3p > 0$. Thus, $a \to 0$ for $t \ll t_0$. •

N.B. today's cosmological constant cannot change this conclusion

Is quantum-gravity necessary to solve the problems above?

Cosmological inflation

epoch of accelerated expansion in the very early Universe

Starobinsky 1979; Guth 1980

$$\ddot{a} > 0 \qquad \Leftrightarrow \qquad \epsilon + 3p < 0$$

since $-3\frac{\ddot{a}}{a} = 4\pi G\,(\epsilon + 3p)$

number of e-foldings: $N \equiv \ln \frac{a}{a_\mathrm{i}} = \int_{t_\mathrm{i}}^{t} H \mathrm{d}t$

Vacuum energy

ϵ of vacuum is constant, thus

$$\mathrm{d}U = \epsilon \mathrm{d}V = -p\mathrm{d}V \Rightarrow p = -\epsilon$$

equivalent to cosmological constant $\Lambda \equiv 8\pi G\epsilon_{\mathrm{v}}$

from $\ddot{a} - \frac{\Lambda}{3}a = 0$ and $\dot{a}_{\mathrm{i}} > 0$ follows

$$a(t) = a_{\mathrm{i}} \exp\left[\sqrt{\frac{\Lambda}{3}}(t - t_{\mathrm{i}})\right]$$

exponential growth

$H_{\mathrm{inf}} \approx \sqrt{\Lambda/3}$

$N = \sqrt{\Lambda/3}(t - t_{\mathrm{i}}) \sim (m_{\mathrm{inf}}/m_{\mathrm{Pl}})^2(t/t_{\mathrm{Pl}}) \gg 1$ typically

Causality and flatness

horizon problem is solved:

$\ell_p/\ell_f \sim z_{\mathrm{GUT}} \exp(-N) \ll 1$

if $N \equiv H_{\mathrm{inf}}\Delta t > 70$

flatness problem disappears:

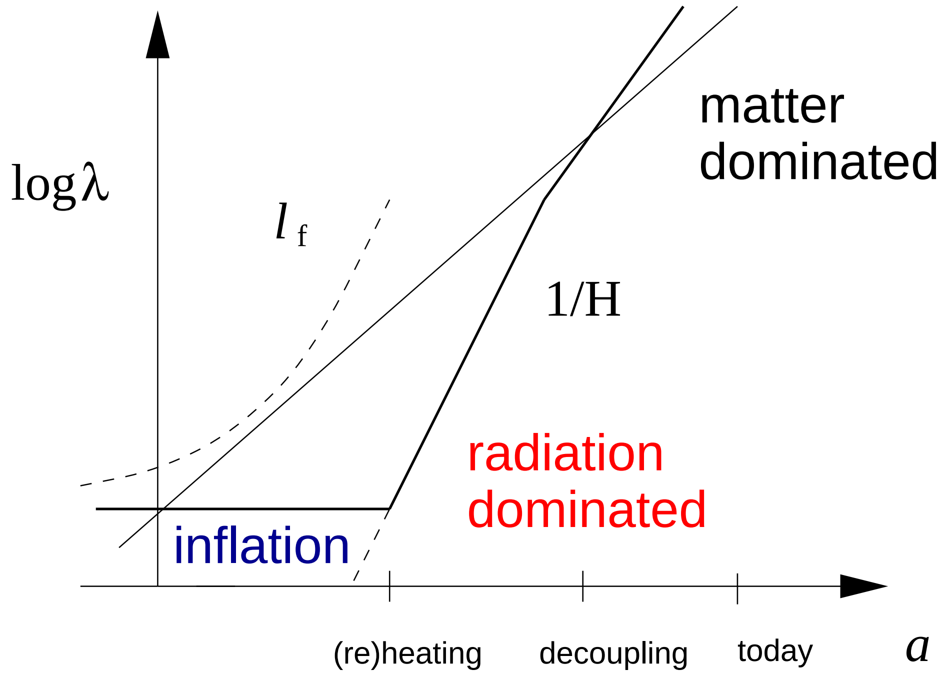
during inflation

$|1 - \Omega(t)| \propto \exp(-2H_{\mathrm{inf}}t)$

after inflation

$\Omega = 1 + \mathcal{O}(\exp[-2N])$

if inflation lasts for
at least 70 e-foldings



prediction 1: spatially flat Universe; $\Omega_0 = 1$

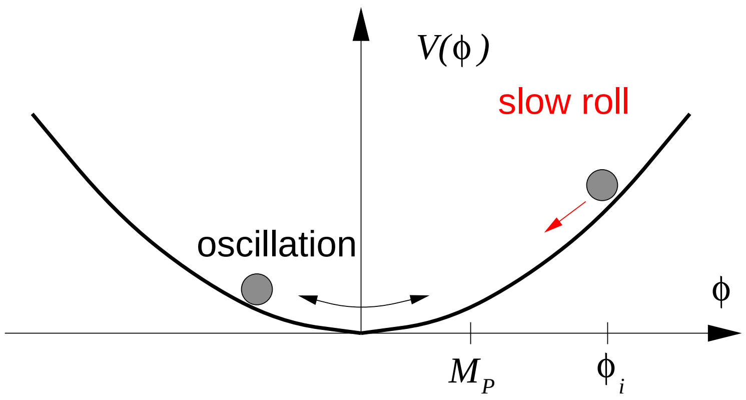
Inflation: Scenarios — History

Starobinskii 1979	R^2-inflation (quantum gravity corrections)
Guth 1980	old inflation (first order GUT transition)
	never stops, because bubbles do not merge
Linde 1982	new inflation (flat potential, slow roll)
Albrecht & Steinhardt 1982	needs special initial conditions
Linde 1983	chaotic inflation (slow roll)
	arbitrary $V(\varphi)$, random initial conditions $\varphi_i, \dot{\varphi}_i$
La & Steinhardt 1989	(hyper-)extended inflation (two scalar fields)
Linde 1993	hybrid inflation (two scalar fields)
. . .	

Chaotic inflation: slow roll   Linde 1983

simple example $V = \lambda\varphi^4/4,\ \lambda \ll 1$

a single scale: $M_{\rm P} \sim 10^{19}$GeV



equations of motion:

$$H^2 = \frac{8\pi}{3M_{\rm P}^2}\left(\frac{1}{2}\dot\varphi^2 + V\right)$$
$$\ddot\varphi + 3H\dot\varphi + V_{,\varphi} = 0$$

generic initial conditions

at $t \sim t_{\rm P}$: $\dot\varphi_i^2 \sim M_{\rm P}^4$ and $V(\varphi_i) \sim M_{\rm P}^4 \Rightarrow$ $\varphi_i \sim \lambda^{-1/4} M_{\rm P} \gg M_{\rm P}$

slow roll: motion of φ is slowed down quickly by the Hubble drag ($H\dot\varphi \gg V_{,\varphi}$)

$\Rightarrow \frac{1}{2}\dot\varphi^2 \ll V$ and $\ddot\varphi \ll -3H\dot\varphi$ $\Rightarrow a(t) \propto \exp(H[\varphi(t)]t)$

with $H(\varphi) \simeq [8\pi V(\varphi)/3M_{\rm P}^2]^{1/2}$ and $\varphi(t) \simeq \varphi_i \exp[-(\lambda/6\pi)^{1/2} t M_{\rm P}]$

Chaotic inflation: end and heating up

Dolgov & Linde 1982; Abbott, Fahri & Wise 1982

inflation terminates at $\varphi \sim M_{\mathrm{P}}$: φ oscillates around its minimum

coherent oscillations decay into other particles

e.g. Yukawa coupling $\frac{1}{2}g^2 v\varphi\chi^2$ to a bosonic particle χ

$$\ddot{\chi}_k + 3H\dot{\chi}_k + [k^2_{\mathrm{ph}} + m^2_\chi + g^2 v\varphi(t)]\chi_k = 0$$

might be very efficient due to parametric resonance $\chi_k \sim \exp(\mu t)$

Traschen & Brandenberger 1990; Kofman, Linde & Starobinskii 1994

these decays produce entropy and (re)heat the Universe to T_{rh}

T_{rh} should be high enough to allow baryogenesis

(probably GUT scale; in any case $T_{\mathrm{rh}} > T_{\mathrm{nuc}}$)

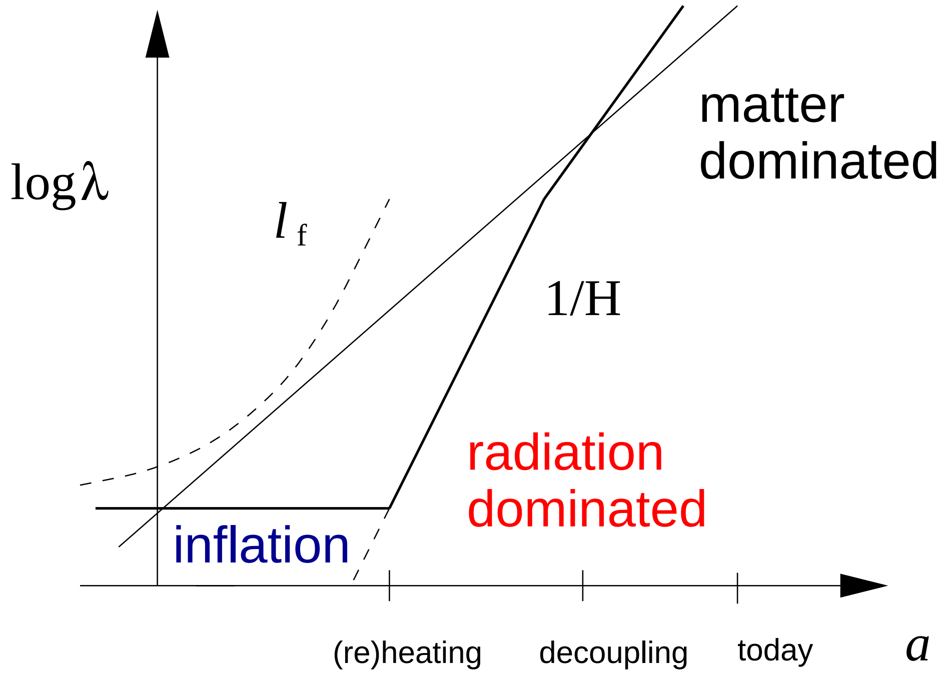
Kinematic considerations

(quantum) fluctuations of energy density and metric

Fourier modes $k = 2\pi/\lambda$

$\lambda_{\text{ph}} \equiv a\lambda$

$\lambda_{\text{ph}} \ll 1/H$ locally Minkowski
$\lambda_{\text{ph}} \gg 1/H$ no causal physics



Structure formation: quantum fluctuations

accelerated expansion provides energy to produce
classical fluctuations from vacuum fluctuations

$$\widehat{\varphi}(\eta,\vec{x}) = \frac{1}{a}\int \frac{\mathrm{d}^3k}{(2\pi)^{3/2}}\frac{1}{\sqrt{2k}}[\widehat{c}_k f_k(\eta)\exp(\imath\vec{k}\vec{x}) + \mathrm{h.c.}]$$

with $\widehat{c}_k|0\rangle = 0$ and $[\widehat{c}_k,\widehat{c}^\dagger_{k'}] = \delta(\vec{k}-\vec{k}')$ [$\eta \equiv \int \mathrm{d}t/a(t)$ conformal time]

$$f_k'' + (k^2 - \frac{a''}{a})f_k = 0$$

subhorizon scales $k_{\mathrm{ph}} \equiv k/a \gg H$: harmonic oscillator
superhorizon scales $k_{\mathrm{ph}} \ll H$: $f_k \simeq a$ rapid amplification of fluctuations

rms amplitude at the moment $k_{\mathrm{ph}} = H$: $\delta\varphi(k=H) \simeq \frac{H(\varphi)}{2\pi}$

power spectrum is almost scale-invariant (Harrison-Zel'dovich)

Structure formation: density perturbations

Chibisov & Mukhanov 1981; Hawking 1982; Guth & Pi 1982

fluctuations $\delta\varphi$ induce fluctuations in the metric

($\phi(\eta, \vec{x}), \psi(\eta, \vec{x})$ . . . metric potentials of longitudinal sector)

$$\mathrm{d}s^2 = a^2(\eta)[-(1+2\phi)\mathrm{d}\eta^2 + (1-2\psi)\mathrm{d}\vec{x}^2] \qquad \text{(longitudinal gauge)}$$

and in the energy density

$$\delta\epsilon(\eta, \vec{x}) = \frac{1}{a^2}(\varphi'\delta\varphi' - \varphi'^2\phi) + V_{,\varphi}\delta\varphi$$

characterise them by a hypersurface-invariant quantity Bardeen 1989

$$\zeta \equiv \frac{\delta\epsilon}{3(\epsilon + p)} - \psi$$

conserved on superhorizon scales, if perturbations are isentropic (see lecture 4)

Primordial power spectra

harmonic oscillator leads to gaussian fluctuations, characterised by two-point functions

def: power spectrum $P_Q(k)$ of some observable Q

$$\langle Q(\vec{0}), Q(\vec{r})\rangle = \int \mathrm{d}(\ln k)\mathrm{j}_0(kr)k^3 P_Q(k) \quad \text{and} \quad \mathcal{P}_Q \equiv k^3 P_Q(k)$$

$Q_{\mathrm{rms}} = \sqrt{\mathcal{P}_Q}$ is the root mean square amplitude in the interval $(k, k + \mathrm{d}k)$

historic ansatz: scale-free power spectrum $\mathcal{P}_\zeta = A_\zeta(k/k_*)^{n-1}$
$n = 1$: scale-invariant Harrison-Zel'dovich, $n - 1$: spectral tilt

Density and metric fluctuations

Chibisov & Mukhanov 1981; Starobinsky 1980

prediction 2: existence of density fluctuations that are

a: gaussian distributed

b: coherent in phase (only growing mode)

c: close to scale-invariant (slow-roll models)

d: isentropic (simplest models)

prediction 3: existence of gravity waves with properties a, b and c

prediction 4: no rotational perturbations at $k < aH$

Slow-roll inflation

attractor in many inflationary scenarios

dynamical (slow-roll) parameters: $\varepsilon_{n+1} \equiv \mathrm{d} \ln \varepsilon_n / \mathrm{d} N$ and $\varepsilon_0 \equiv H_\mathrm{i}/H$

$\varepsilon_1 = \dot{d}_\mathrm{H}$ Schwarz, Terrero-Escalante & Garcia 2001

$$\varepsilon_1 \simeq \frac{M_\mathrm{P}^2}{16\pi}(V'/V)^2, \quad \varepsilon_2 \simeq \frac{M_\mathrm{P}^2}{4\pi}\left[(V'/V)^2 - V''/V\right], \quad \ldots$$

slow-roll inflation: $|\epsilon_n| \ll 1 \ \forall n > 0$

density perturbations $\quad \mathcal{P}_\zeta = \frac{H^2}{\pi \varepsilon_1 M_\mathrm{P}^2}\left(a_0 + a_1 \ln \frac{k}{k_*} + \frac{a_2}{2} \ln^2 \frac{k}{k_*} + \cdots\right)$

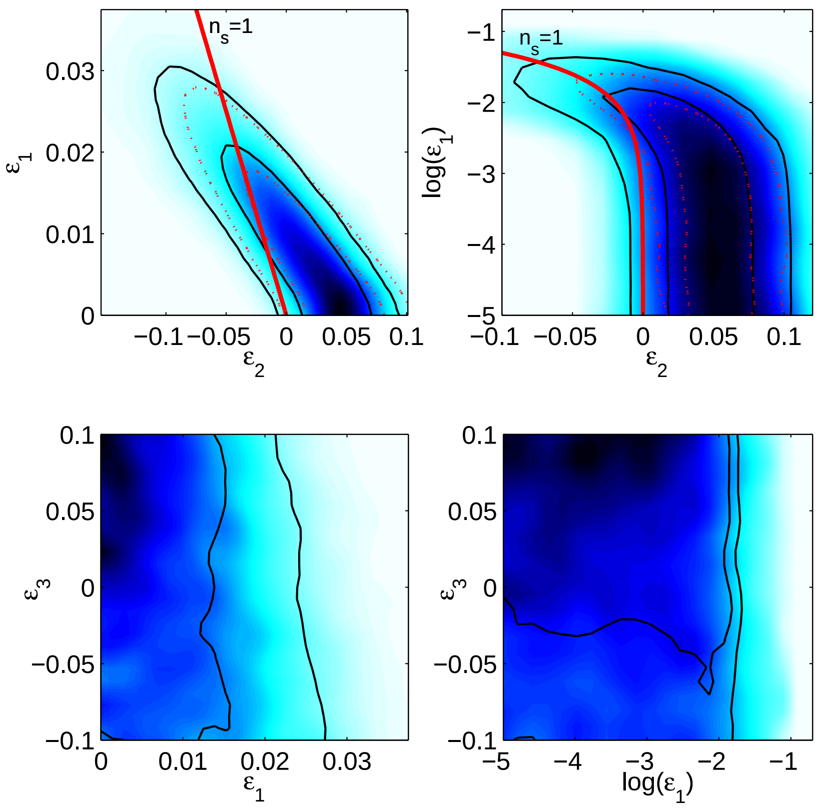
gravitational waves $\quad \mathcal{P}_h = \frac{16 H^2}{\pi M_\mathrm{P}^2}\left(b_0 + b_1 \ln \frac{k}{k_*} + \frac{b_2}{2} \ln^2 \frac{k}{k_*} + \cdots\right)$

with $a_i = a_i(\varepsilon_n), b_i = b_i(\varepsilon_n)$ and k_* pivot scale at which ε_n are evaluated

Stewart & Lyth 1993; Martin & Schwarz 2000;

Stewart & Gong 2001; Leach, Liddle, Martin & Schwarz 2002

Scale of inflation and slow-roll parameters



Martin & Ringeval 2006

CMB data from WMAP

from upper limit on tensor perturbations and the amplitude of scalar perturbations:

$H < 1.6 \times 10^{14}$ GeV $= 1.3 \times 10^{-5} M_{\mathrm{P}}$

$\varepsilon_1 < 0.022$

from deviation from scale-invariance:

$-0.07 < \varepsilon_2 < 0.07$

The largest scales — a multiverse?

Does inflation predict isotropy and homogeneity?

classical dynamics:
inflation produces an isotropic Universe for all homogeneous models except Bianchi IX and Kantowski-Sachs models Turner & Widrow 1986

shown for some inhomogeneous models Calzetta & Sakellariadou 1992
counter examples exist, what are generic initial conditions?

quantum dynamics:
large fluctuations modelled by stochastic inflation
$\Rightarrow$ eternal inflation, multiverse, . . .

Summary of 2nd lecture

cosmological inflation explains
isotropy & homogeneity, causality, spatial flatness and
seeds for structure formation

inflationary parameters (slow-roll):
$H_{\mathsf{inf}}, \varepsilon_1, \varepsilon_2, \ldots$ or $A, n-1, r \equiv \mathcal{P}_h/\mathcal{P}_\zeta, \ldots$

at first order slow-roll approximation: $n - 1 \simeq -2\varepsilon_1 - \varepsilon_2, r \simeq 16\varepsilon_1$

what is the fundamental physics of inflation?
what is it's scale?