Five lectures on

INTRODUCTION TO COSMOLOGY

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Lecture 1: The large picture
  observations, cosmological principle, Friedmann model, Hubble diagram, thermal history

Lecture 2: From quantum to classical
  cosmological inflation, isotropy & homogeneity, causality, flatness, metric & matter fluctuations

Lecture 3: Hot big bang
  radiation domination, hot phase transitions, relics, nucleosythesis, cosmic microwave radiation

Lecture 4: Cosmic structure
  primary and secondary cmb fluctuations, large scale structure, gravitational instability

Lecture 5: Cosmic substratum
  evidence and candidates for dark matter and dark energy, direct and indirect dm searches
Diffuse cosmic background radiation

Halpern & Scott 1999
Cosmic microwave background

1. isotropic distribution
   Planck spectrum $T = 2.725 \pm 0.001$ K

COBE/DMR: Bennett et al 1996

2. dipole $\Delta T = 3.346 \pm 0.017$ mK

COBE/FIRAS: Fixsen et al 1996

3. Milky Way and fluctuations

Nobel prize 1978: Penzias & Wilson

Nobel prize 2006: Mather & Smooth
Radio galaxies

isotropic distribution

\( f = 1.4 \text{ GHz} \)

top: \( S > 140 \text{ mJy}, \delta > -40^\circ \)

bottom: \( S > 2.5 \text{ mJy}, \delta > +75^\circ \)

1 Jy = \( 10^{-26} \text{ W m}^{-2} \text{Hz}^{-1} \)

NRAO VLA Sky Survey
Condon 1999
isotropic distribution

Briggs 2000
Isotropy

observational fact:
(statistically) isotropic distribution of light

possibilities:
1. isotropic around one point (we are at the centre)
2. isotropic around many points (we are preferred observers)
   \( \Rightarrow \) fractal space
3. isotropic around any point
   (Copernican principle: we are typical observers)
   \( \Rightarrow \) continuous homogenous space

Cosmological Principle (CP):
Universe is (statistically) isotropic and homogeneous
Large scale structure

galaxies, visible light

Sloan Digital Sky Survey

Tegmark et al 2004
Homogeneity I

luminous red galaxies
Sloan Digital Sky Survey

$N(r)/r^3 \rightarrow \text{constant}$
for $r > 70h^{-1}$ Mpc
mean density exists!
fractal is excluded

probability distribution

$$dP(x) = n(x)dV = \bar{n}[1 + \delta(x)]dV$$
Homogeneity II

2dF QSO redshift survey

Croom et al 2005

two-point correlation vanishes at $r > 100h^{-1}\text{Mpc} \Rightarrow \text{homogeneity}$

$dP_{12} = \bar{n}^2[1 + \xi(r)]dV_1dV_2$
Friedmann model I

assume cosmological principle holds for space-time itself

isotropic & homogenous line element \((c = 1)\):

\[
ds^2 = -dt^2 + a(t)^2 \left( \frac{dr^2}{1 - Kr^2} + r^2 d\Omega^2 \right)
\]

\(a\) scale factor, \(K/a^2\) spatial curvature \((K = -1, 0, +1)\)

\(H \equiv \dot{a}/a\) expansion rate

physical (radial) distance: \(r_p = a \sin(\sqrt{Kr})/\sqrt{K} = ar[1 - \frac{K}{6}r^2 + \mathcal{O}(K^2r^4)]\)

physical area (ball): \(A_p = 4\pi a^2 r^2\)

physical volume (ball): \(V_p = \frac{4\pi}{3} a^3 r^3[1 + \frac{3K}{10}r^2 + \mathcal{O}(K^2r^4)]\)

line element is unique up to coordinate transformations (Robertson & Walker)
Velocities and redshift

4-velocity of observer: $u^\mu \equiv dx^\mu/d\tau$, $u^\mu u_\mu = -1$ (time-like)

$\tau$ proper time of observer

**free falling observers** $u^\mu;_{\nu}u^\nu = 0$, are slowing down for growing $a(t)$:

$|u| \equiv \sqrt{g_{ij}u^i u^j} \propto 1/a$

free falling observers are asymptotically **comoving** ($u^i \equiv 0$)

photons are **redshifted**, i.e. $f \propto 1/a$

**redshift** $z \equiv \frac{f_e - f_o}{f_o} = \frac{a_o}{a_e} - 1$
Luminosity distance and angular distance

comoving distance

\[ d_{\text{com}} \equiv r_p = \frac{a_0}{\sqrt{K}} \sin \left( \sqrt{K} \frac{a_0}{a_0} \int_0^z \frac{dz'}{H(z')} \right) \]

luminosity distance \( d_L \equiv \sqrt{\frac{L}{4\pi F}} = (1 + z)d_{\text{com}} \)

Hubble diagram

angular distance \( d_a \equiv \frac{D}{\theta} = d_{\text{com}}/(1 + z) \)

acoustic oscillations in cmb and lss
Expanding universe

red-shift
\[ z \equiv \frac{f_e - f_o}{f_o} = \frac{a_o}{a_e} - 1 \]

Hubble expansion
\[ H_0d_L = z + O(z^2) \]

Red-shift and Hubble expansion are a direct consequence of the cosmological principle (without Einstein’s equations)

\[ H_0 \equiv 100h \text{ km/s/Mpc} \]

\[ h = 0.72 \pm 0.03 \pm 0.07 \]

Freedman et al 2001
Time and distance scales

astronomer’s unit:

1 pc ≡ 1 AU/1 arc sec
1 Mpc = 3.086 × 10^{22} m ≈ 3.262 × 10^6 light-years
typical distance between two galaxies

natural units of cosmology:
Hubble time, Hubble distance

\[ t_H \equiv \frac{1}{H_0} = 9.78h^{-1}\text{Gyr}, \quad d_H \equiv t_H = 3000h^{-1}\text{Mpc} \]

\( H_0 \) sets the time and length scale of local causal processes

curvature radius: \( r_c \equiv a_0/\sqrt{|K|} \)
Acceleration of the expansion

\[ d_L(z) = \frac{1}{H_0} \left[ z + (1 - q_0) \frac{z^2}{2} + \left( -j_0 + 3q_0^2 + q_0 - 1 - \frac{K}{a_0^2 H_0^2} \right) \frac{z^3}{6} + \mathcal{O}(z^4) \right] \]

deceleration \( q \equiv -\frac{\ddot{a}}{a}/H^2 \), jerk \( j \equiv \frac{\dddot{a}}{a}/H^3 \)

SN Ia data suggest \( q_0 < 0 \)

Here \( K = 0 \); SN Ia \cite{Riess:2004}
Phase space distribution of matter and light

Planck spectrum of CMB: equilibrium? isotropy and homogeneity on large scales: equilibrium?

number of quanta in phase space at time $t$

$$dN_t = f_t(x, p)dx dp$$

homogeneity and isotropy: $f_t(x, p) = f_t(p)$

gravitational interaction only: $L(f) = 0 \Rightarrow$

equilibrium in an expanding Friedmann Universe possible for

1. massless particles; Bose-Einstein or Fermi-Dirac $T \propto 1/a$, $\mu \propto 1/a$

2. massive particles, iff $m \gg T$; Maxwell-Boltzmann $T \propto 1/a^2$, $\mu \approx m$
“Hot Big Bang”

Universe expands:
- cosmic Joule-Thomson effect

\[ T(z) = T_0(1 + z) \]

measure CMB temperature in distant molecular clouds via absorption spectra

LoSecco, Mathews & Wang 2001
Radiation domination in the early Universe

photons: \( \epsilon = \frac{2\pi^2}{30} T^4 \) (Stefan-Boltzmann law) \( \propto a^{-4} \)

dust (matter): \( \epsilon = \frac{m_{\text{n}}}{V_p} \propto a^{-3} \)

curvature: \( Ka^{-2} \)

cosmological constant: \( \Lambda \propto a^0 \)

as \( a \ll a_0 \), at \( T > T_{\text{eq}} \): radiation (\( \gamma, \nu s \), etc.) dominates early on

\( T_{\text{eq}} \equiv (1 + z_{\text{eq}})T_0, \quad (1 + z_{\text{eq}}) \equiv \frac{\epsilon_{\text{m}0}}{\epsilon_{r0}} \sim 4000 \) matter-radiation equality
History of the Universe

- LHC dipole
- RHIC-event (STAR)
- Sky from WMAP
- Hubble Deep Field

- geometry & fluctuations
- masses
- hadrons
- nucleosynthesis
- 3K radiation
- galaxies
- today

- $10^{-12}$ s
- $10^{-6}$ s
- $10^0$ s
- $10^6$ s
- $10^{12}$ s
- $10^{18}$ s

- inflation
- radiation
- matter
- ?
Einstein equation

Lovelock’s theorem:
1. covariant second order equation for the metric \( g_{\mu\nu} \)
2. covariant conservation of energy-momentum

\[ G_{\mu\nu} - \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu} \]

\( G \) Newton constant
\( \Lambda \) cosmological constant

**Einstein equations fix geometry, but not topology**
assume trivial topology, no convincing observational evidence for more complex one
Friedmann model II

from $dU = -pdV + \delta Q$ (CP: $\nabla \delta Q = 0^*$) and Einstein's equation

$$\dot{\epsilon} + 3H(\epsilon + p) = 0 \quad \text{and} \quad 3H^2 + \frac{3K}{a^2} - \Lambda = 8\pi G\epsilon$$

$\epsilon$ energy density, $p$ pressure

to solve need equation of state $p = p(\epsilon)$

examples:
- dust (matter) $p = 0$; radiation (light) $p = \epsilon/3$

*bulk dissipation is not excluded by CP, usually it is assumed to be irrelevant
Energy density and spatial curvature

\[ \Omega \equiv \frac{\epsilon}{\epsilon_c}, \quad \text{with} \quad \epsilon_c \equiv \frac{3H^2}{8\pi G} \]

\[ \Omega - 1 = \frac{K}{a^2H^2} \]

to know \( \Omega \), \( H_0 \) must be known, thus measure \( \omega \equiv h^2\Omega \)
Einstein-de Sitter model

\( \Lambda = 0 \), flat dust solution

\[ \epsilon(a) = \epsilon_0 \left( \frac{a_0}{a} \right)^3, \quad a(t) = a_0 \left( \frac{t}{t_0} \right)^{2/3}, \quad t_0 = \frac{2}{3} t_H, \quad q_0 = \frac{1}{2} \]

in conflict with age of Universe \( t_0 \geq 12 \text{Gyr} \) (oldest stars) and
in conflict with Hubble diagram \( q_0 < 0 \)
Dark energy

acceleration possible for

\[-3\frac{\ddot{a}}{a} = 4\pi G (\epsilon + 3p) - \Lambda < 0\]

cosmological constant or other form of “dark energy” required

simplest model: \(\Lambda > 0, p = 0, K = 0\) flat \(\Lambda\)CDM
Cosmological parameters of ΛCDM: $h, \Omega_\Lambda, \Omega_m$

Spergel et al. 2006
CMB (WMAP) and $H_0$ (HST key project)
$\Omega - 1 = -0.014 \pm 0.017 \Rightarrow r_c > 21 \text{Gpc}$

Wood-Vasey et al. 2007
supernovae Ia
$\Omega_\Lambda > 0$
Summary of 1st lecture

statistical isotropy is an observational fact

cosmological principle implies redshift and Hubble expansion

hot big bang: radiation domination followed by matter domination

Einstein equations and the CP lead to Friedmann cosmology

SN 1a Hubble diagram indicates accelerated expansion and dark energy domination today

minimal model of cosmology: \( K = 0, p_m = 0, \Lambda > 0 \) plus radiation
free parameters of the minimal model: \( T_0, h, \omega_m \)