Elements of fluid dynamics for the modeling of heavy-ion collisions

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Elements of fluid dynamics for the modeling of heavy-ion collisions

Lecture III:

- Overview on the dynamics of relativistic perfect fluids
- Application of perfect relativistic fluid dynamics to the description of high-energy nucleus-nucleus collisions
- Beyond perfect relativistic fluid dynamics

theoretical approaches

- many ideas: dissipative fluid dynamics / including hydrodynamical fluctuations / or "non-hydro modes"... ☛ out-of-equilibrium physics
- ... coming from the underlying "microscopic" physics (transport) or invoking strong-coupling scenarios

Fluid dynamics & heavy-ion collisions

hidden "details"

- Fluid dynamics describes the evolution of a continuous medium, starting from an initial condition.

☛ which initial condition?

- Fluid dynamics describes the evolution of a continuous medium at thermodynamic equilibrium.

☛ how do we reach a thermalized "initial state"?

☞ BEFORE the hydro stage

- Fluid dynamics describes the evolution of a continuous medium, yet experiments measure (energy, momentum, tracks...) of particles.

☛ how can / should one relate both?

☞ END of the hydro stage

Fluid vs. particle descriptions

the link

Consider a collection of particles $k = 1, \dots, N$ with respective space-time trajectories ("world-lines") $x_k^\mu(\tau)$.

Their individual 4-velocities are defined as $u_k^\mu(\tau) \equiv \dfrac{\mathrm{d}x_k^\mu(\tau)}{\mathrm{d}\tau}$.

Let a be a conserved quantum number (say baryon number) carried by these particles, with q_a the corresponding "charges" (+1 for baryons, -1 for antibaryons, 0 for mesons, leptons…).

The quantum-number 4-current associated with the collection of particles is defined as

$$N_a^\mu(\mathsf{x}) \equiv \sum_{k=1}^{N} q_{a,k} \int u_k^\mu(\tau)\, \delta^{(4)}\big(x^\nu - x_k^\nu(\tau)\big)\, \mathrm{d}\tau$$

one checks $N_a^0(t,\vec{r}) = \sum_{k=1}^{N} q_{a,k}\delta^{(3)}\big(\vec{r} - \vec{x}_k(t)\big)$, $N_a^i(t,\vec{r}) = \sum_{k=1}^{N} q_{a,k} v_k^i \delta^{(3)}\big(\vec{r} - \vec{x}_k(t)\big)$

Fluid vs. particle descriptions

the link

Consider a collection of particles $k = 1, \ldots, N$ with respective space-time trajectories ("world-lines") $x_k^\mu(\tau)$.

Their individual 4-velocities are defined as $u_k^\mu(\tau) \equiv \frac{\mathrm{d}x_k^\mu(\tau)}{\mathrm{d}\tau}$.

In turn, the energy-momentum tensor associated with the collection of particles is defined as

$$T^{\mu\nu}(\mathsf{x}) \equiv \sum_{k=1}^{N} \int p_k^\mu(\tau)\, u_k^\nu(\tau)\, \delta^{(4)}\big(x^\lambda - x^\lambda(\tau)\big)\, \mathrm{d}\tau$$

Check: $T^{\mu 0}(t,\vec{r}) = \sum_{k=1}^{N} p_k^\mu(t)\, \delta^{(3)}\big(\vec{r} - \vec{x}_k(t)\big)$ is the density of momentum,

$T^{0j}(t,\vec{r}) = \sum_{k=1}^{N} p_k^0(t)\, v_k^j(t)\, \delta^{(3)}\big(\vec{r} - \vec{x}_k(t)\big)$ is the energy flux,

and $T^{ij}(t,\vec{r}) = \sum_{k=1}^{N} p_k^i(t)\, v_k^j(t)\, \delta^{(3)}\big(\vec{r} - \vec{x}_k(t)\big)$ the momentum flux density.

Fluid vs. particle descriptions

the link

If the system of particles is too complicated, its properties are only known on a statistical basis.

Its probabilistic description involves a phase-space distribution f(x,p), such that $f(\mathrm{x},\mathrm{p})\,\mathrm{d}^3\vec{r}\,\mathrm{d}^3\vec{p}$ is the number of particles that at time t are in a volume $\mathrm{d}^3\vec{r}$ around $\vec{r}$ and have a momentum $\vec{p}$ up to $\mathrm{d}^3\vec{p}$.

The quantum-number 4-currents and energy-momentum tensor are then given by

$$N_a^\mu(\mathrm{x}) \equiv \int q_a(\mathrm{x})\, p^\mu f(\mathrm{x},\mathrm{p}) \frac{\mathrm{d}^3\vec{p}}{(2\pi)^3 p^0}$$

$$T^{\mu\nu}(\mathrm{x}) \equiv \int p^\mu p^\nu f(\mathrm{x},\mathrm{p}) \frac{\mathrm{d}^3\vec{p}}{(2\pi)^3 p^0}$$

Note: p^μ/p^0 is the 4-velocity!

Fluid vs. particle descriptions

the link

Given the phase-space distribution f(x,p) for a system, the quantum-number 4-currents and energy-momentum tensor are given by

$$N_a^\mu(\mathsf{x}) \equiv \int q_a(\mathsf{x})\, p^\mu f(\mathsf{x},\mathsf{p}) \frac{\mathrm{d}^3\vec{p}}{(2\pi)^3 p^0} \qquad T^{\mu\nu}(\mathsf{x}) \equiv \int p^\mu p^\nu f(\mathsf{x},\mathsf{p}) \frac{\mathrm{d}^3\vec{p}}{(2\pi)^3 p^0}$$

☛ Why work in terms of hydrodynamic fields?

Couldn't one work at the particle level?

$\Leftrightarrow$

solve equation obeyed by f(x,p)

By the way, what is this equation?

Hints of kinetic theory

heuristic arguments only!

To find the equation obeyed by *f*(x,p), a heuristic possibility is to study particles in a small phase-space volume $\mathrm{d}^3\vec{r}\,\mathrm{d}^3\vec{p}$ and to investigate how their number can evolve with time:

- under the influence of free flight;
- under the influence of external forces (gravity...);
- under the influence of collisions.

More details in: Huang, Statistical mechanics, Chap.3-5

Reif, Fundamental of statistical and thermal physics, Chap.12-14

☛ leads to the "kinetic equation":

$$\partial_t f(\mathsf{x},\mathsf{p}) + \vec{v}\cdot\vec{\nabla}_{\vec{r}} f(\mathsf{x},\mathsf{p}) + \vec{a}\cdot\vec{\nabla}_{\vec{p}} f(\mathsf{x},\mathsf{p}) = \underbrace{\left[\partial_t f(\mathsf{x},\mathsf{p})\right]_{\rm coll.}}_{\text{collision term}}$$

particle acceleration from external forces ($\vec{a}$)

Hints of kinetic theory

heuristic arguments only!

Kinetic equation for the phase-space distribution f(x,p):

$$\partial_t f(\mathsf{x},\mathsf{p}) + \vec{v}\cdot\vec{\nabla}_{\vec{r}} f(\mathsf{x},\mathsf{p}) + \vec{a}\cdot\vec{\nabla}_{\vec{p}} f(\mathsf{x},\mathsf{p}) = \underbrace{\big[\partial_t f(\mathsf{x},\mathsf{p})\big]_{\rm coll.}}_{\text{collision term}}$$

particle acceleration from external forces → $\vec{a}$

Under specific assumptions for the collision term — which encodes the interparticle scatterings —, one obtains the Boltzmann equation, whose steady solution is the Maxwell–Boltzmann distribution that describes the equilibrium statistical physics of ideal classical gases.

(One can easily massage the collision term into giving the equilibrium distributions for ideal quantum gases: Bose–Einstein, Fermi–Dirac).

Hints of kinetic theory

heuristic arguments only!

Kinetic equation for the phase-space distribution f(x,p), dropping the influence of external forces:

$$\partial_t f(\mathsf{x},\mathsf{p}) + \vec{v}\cdot\vec{\nabla}_{\vec{r}} f(\mathsf{x},\mathsf{p}) = \underbrace{\left[\partial_t f(\mathsf{x},\mathsf{p})\right]_{\rm coll.}}_{\text{collision term}}$$

in relativistic form: $u^\mu \partial_\mu f(\mathsf{x},\mathsf{p}) = \left[\partial_t f(\mathsf{x},\mathsf{p})\right]'_{\rm coll.}$

Hints of kinetic theory

heuristic arguments only!

Kinetic equation for the phase-space distribution f(x,p), dropping the influence of external forces:

$$\partial_t f(\mathrm{x},\mathrm{p}) + \underbrace{\vec{v}\cdot\vec{\nabla}_{\vec{r}}}_{\text{defines a macroscopic time scale}} f(\mathrm{x},\mathrm{p}) = \underbrace{\big[\partial_t f(\mathrm{x},\mathrm{p})\big]_{\mathrm{coll.}}}_{\text{involves a microscopic time scale}}$$

The role of the collision term — of interparticle scatterings — is to let the distribution relax, on a microscopic time scale, towards a local equilibrium distribution (i.e. a Maxwell distribution with space and time dependent parameters).

☛ "local equilibration"

Hints of kinetic theory

heuristic arguments only!

Kinetic equation for the phase-space distribution f(x,p), dropping the influence of external forces:

$$\partial_t f(\mathsf{x},\mathsf{p}) + \underbrace{\vec{v}\cdot\vec{\nabla}_{\vec{r}}}_{\text{defines a macroscopic time scale}} f(\mathsf{x},\mathsf{p}) = \underbrace{\big[\partial_t f(\mathsf{x},\mathsf{p})\big]_{\rm coll.}}_{\text{involves a microscopic time scale}}$$

The role of the collision term — of interparticle scatterings — is to let the distribution relax, on a microscopic time scale, towards a local equilibrium distribution (i.e. a Maxwell distribution with space and time dependent parameters) $f^{(0)}(\mathsf{x},\mathsf{p})$.

☛ write $\big[\partial_t f(\mathsf{x},\mathsf{p})\big]_{\rm coll.} = -\dfrac{f(\mathsf{x},\mathsf{p}) - f^{(0)}(\mathsf{x},\mathsf{p})}{\tau_r}$

Hints of kinetic theory

heuristic arguments only!

Kinetic equation for the phase-space distribution $f(\mathrm{x},\mathrm{p})$, dropping the influence of external forces in the "relaxation time approximation":

$$\partial_t f(\mathrm{x},\mathrm{p}) + \underbrace{\vec{v}\cdot\vec{\nabla}_{\vec{r}}}_{\text{defines a macroscopic time scale } \tau_s} f(\mathrm{x},\mathrm{p}) = -\frac{f(\mathrm{x},\mathrm{p}) - f^{(0)}(\mathrm{x},\mathrm{p})}{\tau_r}$$

Write $f(\mathrm{x},\mathrm{p}) = f^{(0)}(\mathrm{x},\mathrm{p}) + f^{(1)}(\mathrm{x},\mathrm{p}) + \cdots$, solve iteratively*: each term is of order τ_r/τ_s with respect to the previous one if you are not too far from equilibrium.

~ Kn! expansion in Knudsen number...

* in the steady case, to make your life easier.

Hints of kinetic theory

heuristic arguments only!

Write $f(\mathrm{x}, \mathrm{p}) = f^{(0)}(\mathrm{x}, \mathrm{p}) + f^{(1)}(\mathrm{x}, \mathrm{p}) + \cdots$ (expansion in Kn), solve your approximate kinetic equation

$$\partial_t f(\mathrm{x}, \mathrm{p}) + \vec{v} \cdot \vec{\nabla}_{\vec{r}} f(\mathrm{x}, \mathrm{p}) = -\frac{f(\mathrm{x}, \mathrm{p}) - f^{(0)}(\mathrm{x}, \mathrm{p})}{\tau_r}$$

iteratively, plug the result for f(x,p) into the relations on slide 5 defining quantum number 4-currents and energy-momentum tensor...

☛ yields expressions for $N_a^\mu(\mathrm{x})$, $T^{\mu\nu}(\mathrm{x})$ as series in Kn.

Look at the leading order terms $\mathcal{O}(\mathrm{Kn}^0)$ carefully... and recognize the expressions for a perfect fluid, together with the equation of state!

The next-to-leading order terms $\mathcal{O}(\mathrm{Kn})$ yield the first-order dissipative corrections, including expressions for the transport coefficients.

Hints of kinetic theory

recovering hydrodynamics!

Write $f(\mathrm{x},\mathrm{p}) = f^{(0)}(\mathrm{x},\mathrm{p}) + f^{(1)}(\mathrm{x},\mathrm{p}) + \cdots$, solve your kinetic equation, plug the result into the relations defining quantum number 4-currents and energy-momentum tensor...

☛ yields expressions for $N_a^\mu(\mathrm{x})$, $T^{\mu\nu}(\mathrm{x})$ as series in Kn.

The leading order $\mathcal{O}(\mathrm{Kn}^0)$ expressions are those for a perfect fluid, and you recover the equation of state of an ideal gas.

The next-to-leading order terms $\mathcal{O}(\mathrm{Kn})$ yield first-order dissipative corrections — including the expressions for the transport coefficients of an ideal gas... which is disappointing because you find $\zeta = 0$; but also η and κ both proportional to the mean free path.

You can now go to NNLO $\mathcal{O}(\mathrm{Kn}^2)$ to get second-order hydrodynamics...

Hints of kinetic theory

recovering hydrodynamics!

Problem: the usual kinetic-theory approach assumes a dilute, weakly interacting (i.e., with kinetic energies much larger than potential energies) system of particles.

☛ hidden in the separation of scales!

Not optimal(!) for heavy-ion physics...

But it should still give the form of the fluid dynamical equations... unless some terms do not show up because the associated transport coefficients vanish in an ideal gas.

☞ need to find other ways to derive a complete set of equations, and to have the values of the transport coefficients.

Fluid vs. particle descriptions

fluid 4-velocity in the dissipative case

We have seen (slides 3–5) that the quantum number 4-currents and energy-momentum tensor of a relativistic fluid are the averages over momentum space, weighted with the phase space distribution, of the respective quantities for particles.

Does this also hold for the fluid 4-velocity?

- in the case of a perfect fluid, yes (easy to check);
- in the case of a dissipative fluid... there is no unique choice for the fluid 4-velocity!
  - you may choose the 4-velocity with which energy is transported ("Landau frame")
  - or the 4-velocity with which a particular conserved number is transported ("Eckart frame")
  - or something in-between...

A subtlety of dissipative relativistic fluid dynamics

- In the "Landau frame", the fluid 4-velocity is taken to coincide with the 4-velocity of energy transport: there is no heat flow, and heat conductivity appears in the quantum number transport 4-velocity(!)

- in the "Eckart frame", it is the opposite: there is no transport of quantum number* — and heat conductivity is associated with heat transport!

Accordingly, the equations of motion look different in both frames... or in any intermediate frame. Although they are fully equivalent!

For heavy-ion collisions, the Landau frame is more convenient, since there is (at mid-rapidity) no conserved number.

* at least, of that with whose transport the Eckart frame has been associated.

A subtlety of dissipative relativistic fluid dynamics

- In the "Landau frame", the fluid 4-velocity is taken to coincide with the 4-velocity of energy transport: there is no heat flow, and heat conductivity appears in the quantum number transport 4-velocity(!)

- in the "Eckart frame", it is the opposite: there is no transport of quantum number* — and heat conductivity is associated with heat transport!

Such a possibility does not arise in the non-relativistic case, where there is only one good definition of the fluid 4-velocity.

* at least, of that with whose transport the Eckart frame has been associated.

Fluid vs. particle descriptions

remarks on the freeze-out scenario

Remember the Cooper–Frye prescription (Lecture II)

$$E_{\vec{p}}\frac{\mathrm{d}^3N}{\mathrm{d}^3\vec{p}} = \frac{g}{(2\pi)^3}\int_\Sigma f\left(\frac{\mathsf{p}\cdot\mathsf{u}(\mathsf{x})}{T}\right)\mathsf{p}\cdot\mathrm{d}^3\sigma(\mathsf{x})$$

It is a nice scenario.

If particles are emitted with an equilibrium phase space distribution (Bose–Einstein, Fermi–Dirac), then on the particle side we have the characteristics ($N_a^\mu(\mathsf{x})$, $T^{\mu\nu}(\mathsf{x})$) of an ideal (quantum) gas!

Accordingly, on the fluid side there should be a perfect fluid, so as to ensure that the conserved currents remain the same as the system crosses the artificial freeze-out hypersurface.

Fluid vs. particle descriptions

remarks on the freeze-out scenario

Remember the Cooper–Frye prescription (Lecture II)

$$E_{\vec{p}}\frac{\mathrm{d}^3N}{\mathrm{d}^3\vec{p}} = \frac{g}{(2\pi)^3}\int f\left(\frac{\mathsf{p}\cdot\mathsf{u}(\mathsf{x})}{T}\right)\mathsf{p}\cdot\mathrm{d}^3\sigma(\mathsf{x})$$

It is a

If par … ibution

(Bose– … ve the

charac

Problem: the purpose of heavy-ion collisions is to create the QGP and study its transport coefficients

[brainwashed crowds cheer: "η over s, η over s!"]

so we do need dissipative fluid dynamics!

Accordingly, on the fluid side there should be a perfect fluid, so as to ensure that the conserved currents remain the same as the system crosses the artificial freeze-out hypersurface.

Fluid vs. particle descriptions

remarks on the freeze-out scenario

Remember the Cooper–Frye prescription

$$E_{\vec{p}}\frac{\mathrm{d}^3N}{\mathrm{d}^3\vec{p}} = \frac{g}{(2\pi)^3}\int_\Sigma f\left(\frac{\mathsf{p}\cdot\mathsf{u}(\mathsf{x})}{T}\right)\mathsf{p}\cdot\mathrm{d}^3\sigma(\mathsf{x})$$

It is a nice scenario, but we should not use the equilibrium phase space distributions, so as to be able to match a dissipative fluid.

OK, please do!

Problem: going from the phase-space occupancy factor f(x,p) to the corresponding conserved currents is easy, "just an integration", the way back is not — and not unique.

☛ f(x,p) is up to a factor a probability distribution, $N_a^\mu(\mathsf{x})$, $T^{\mu\nu}(\mathsf{x})$ are its first two moments.

Fluid vs. particle descriptions

remarks on the freeze-out scenario

Consider the Cooper–Frye prescription

$$E_{\vec{p}}\frac{\mathrm{d}^3N}{\mathrm{d}^3\vec{p}} = \frac{g}{(2\pi)^3}\int_\Sigma f\left(\frac{\mathsf{p}\cdot\mathsf{u}(\mathsf{x})}{T}\right)\mathsf{p}\cdot\mathrm{d}^3\sigma(\mathsf{x})$$

How much does the choice of phase-space occupancy factor f(x,p) affect the computed observables?

A lot!

This is where PID arises in the modeling, before that point there is only a particle-type-blind equation of state.

Particle emission at freeze-out

Consider the Cooper–Frye formula (T is the freeze-out temperature)

$$E_{\vec{p}}\frac{\mathrm{d}^3N}{\mathrm{d}^3\vec{p}} = \frac{g}{(2\pi)^3}\int_\Sigma f\left(\frac{\mathsf{p}\cdot\mathsf{u}(\mathsf{x})}{T}\right)\mathsf{p}\cdot\mathrm{d}^3\sigma(\mathsf{x})$$

The phase space occupation factor f is proportional to $f_{\text{id.}}$, which will be approximated by a Maxwell–Boltzmann distribution.

The integral can be computed with the saddle-point approximation, without needing any detail on the freeze-out surface Σ.

☛ one needs to determine the saddle point, which is* the minimum of $\mathsf{p}\cdot\mathsf{u}(\mathsf{x})/T$.

* at least approximately

Particle emission at freeze-out

Taking the Cooper–Frye formula seriously

$$E_{\vec{p}}\frac{\mathrm{d}^3N}{\mathrm{d}^3\vec{p}} = \frac{g}{(2\pi)^3}\int_\Sigma f\left(\frac{\mathsf{p}\cdot\mathsf{u}(\mathsf{x})}{T}\right)\mathsf{p}\cdot\mathrm{d}^3\sigma(\mathsf{x})$$

one may approximate the integral using the steepest-descent method.

N.B. & J.-Y.Ollitrault 2005, N.B. & Ch.Lang 2013

Two categories of particles:

- "slow particles": velocity $\frac{\mathsf{p}}{m}$ coincides with that of the fluid $\mathsf{u}(\mathsf{x})$ at some point on Σ.
  - ☛ at a given rapidity y, $|\mathbf{p}_t| < m u_{\max}(y)$.

    → $u_{\max}$: maximum fluid velocity in the direction of p^μ

The minimum value of $\mathsf{p}\cdot\mathsf{u}(\mathsf{x})/T$ simply equals m/T, independent of the particle momentum.

Particle emission at freeze-out

Taking the Cooper–Frye formula seriously

$$E_{\vec{p}}\frac{\mathrm{d}^3N}{\mathrm{d}^3\vec{p}} = \frac{g}{(2\pi)^3}\int_\Sigma f\left(\frac{\mathsf{p}\cdot\mathsf{u}(\mathsf{x})}{T}\right)\mathsf{p}\cdot\mathrm{d}^3\sigma(\mathsf{x})$$

one may approximate the integral using the steepest-descent method.

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Two categories of particles:

- "slow particles": velocity $\frac{\mathsf{p}}{m}$ coincides with that of the fluid $\mathsf{u}(\mathsf{x})$ at some point on Σ.
  - ☛ at a given rapidity y, $|\mathbf{p}_t| < m u_{\max}(y)$.
    → $u_{\max}$: maximum fluid velocity in the direction of p^μ
- "fast particles": $|\mathbf{p}_t| > m u_{\max}(y)$.
  The minimum value of $\mathsf{p}\cdot\mathsf{u}(\mathsf{x})/T$ is larger than m/T.

Result for slow particles

"Slow particles": emitted by fluid cells w.r.t. which they are at rest.

Velocity $\frac{\mathsf{p}}{m}$ coincides with that of the fluid $\mathsf{u}(\mathsf{x})$ at the saddle point.

- Freezing out from an ideal fluid:

  The Cooper–Frye integral $E_{\vec{p}} \frac{\mathrm{d}^3 N}{\mathrm{d}^3 \vec{p}} = \frac{g}{(2\pi)^3} \int_\Sigma f_{\mathrm{id.}}\!\left(\frac{\mathsf{p}\cdot\mathsf{u}(\mathsf{x})}{T}\right) \mathsf{p}\cdot\mathrm{d}^3\sigma(\mathsf{x})$

  yields a function of the particle velocity only, with an m-dependent prefactor:

$$E_{\vec{p}} \frac{\mathrm{d}^3 N}{\mathrm{d}^3 \vec{p}} = C(m)\, F\!\left(\frac{\mathbf{p}_t}{m}, y\right)$$

  Fourier-expanding, the flow coefficients v_n for all particles coincide when considered at the same transverse velocity and rapidity.

  $\Rightarrow$ "mass-scaling" of anisotropic flow

N.B. & J.-Y.Ollitrault 2005

Fluid vs. particle descriptions

remarks on the freeze-out scenario

Consider the Cooper–Frye prescription

$$E_{\vec{p}}\frac{\mathrm{d}^3N}{\mathrm{d}^3\vec{p}} = \frac{g}{(2\pi)^3}\int_\Sigma f\left(\frac{\mathsf{p}\cdot\mathsf{u}(\mathsf{x})}{T}\right)\mathsf{p}\cdot\mathrm{d}^3\sigma(\mathsf{x})$$

How much does the choice of phase-space occupancy factor f(x,p) affect the computed observables?

A lot!

For instance, the celebrated "mass-scaling" of anisotropic flow appears at freeze-out in the modeling.

Fluid vs. particle descriptions

In short:

The way to go is to use dissipative fluid dynamics, but we do not know how to match it properly onto a particle description — for instance to model the end of the hydro stage.

The reason is that fluid dynamics is "too simple", it encodes only part of the information present at the particle level.

(Other, similar problem of "gluing" different descriptions properly at the onset of the fluid dynamical stage.)

Are there ways out?

☛ a bunch of more or less baroque ideas

Beyond "traditional" fluid dynamics

let your imagination free!

- Going to higher-order dissipative descriptions.

  ☛ increasing complexity (& number of parameters)

  no obvious gain on the description-matching side

- Importing new features in the fluid dynamics description
  - Fluctuating hydrodynamics
  - "Non-hydro modes" from AdS/CFT

- Writing a (not-too!) different-looking set of hydrodynamics equations, including effects that cannot be obtained in the usual expansion in gradients of the velocity.
  - "anisotropic hydrodynamics"
  - ... ("resummed hydrodynamics"?)

Beyond "traditional" fluid dynamics

let your imagination free!

- Importing new features in the fluid dynamics description
  - Fluctuating hydrodynamics
  - "Non-hydro modes" from AdS/CFT

☛ Common(?) idea: fluid dynamics is an effective, long wavelength description of some more microscopic theory/model.

As every effective model, some of the underlying degrees of freedom have been "integrated out" — here, the fast / short wavelength d.o.f.

Is there a way to re-integrate some of these discarded d.o.f. into the macroscopic description?

Beyond "traditional" fluid dynamics

extending fluid dynamics

- Importing new features in the fluid dynamics description
  - Fluctuating hydrodynamics

Kapusta, Müller, Stepanov; Young; Hirano, Kurita, Murase, Nagai…*

☛ common principle (but different implementation):

add to the conserved currents of dissipative fluid dynamics an extra, stochastic term: fluctuating part / noise

analogy: Langevin model of Brownian motion

e.g. $$T^{\mu\nu} = \mathcal{P} g^{\mu\nu} + (\epsilon + \mathcal{P}) u^\mu u^\nu + \pi^{\mu\nu} + S^{\mu\nu}$$

with $\langle S^{\mu\nu} \rangle = 0$, but a non-vanishing 2-point correlation function.

* sincere apologies to those I have not mentioned.

Beyond "traditional" fluid dynamics

extending fluid dynamics

- Importing new features in the fluid dynamics description
  - "Non-hydro modes" from AdS/CFT

Heller, Janik, Spaliński...* building on Kovtun, Son, Starinets...*

☛ import into 4-D hydrodynamics quantities found in 5-D gravity:

on the gravity side, plane-wave-like perturbations of the metric tensor can either propagate without damping — sound-waves — or admit solutions with complex frequencies: "non-hydro modes".
Mapping the latter (till now, only that with the smallest imaginary part) onto the 4-D setup, the hydrodynamical equations have to be extended with extra terms.

* sincere apologies to those I have not mentioned.

Beyond "traditional" fluid dynamics

modifying fluid dynamics

- Changing the hydrodynamics equations phenomenologically
  - "Anisotropic hydro"

Martinez-Guerrero, Strickland; Florkowski, Ryblewski; Heinz...*

☛ phenomenologically modify the energy-momentum tensor by distorting its spatial part.

For instance:

$$T^{\mu\nu}\big|_{\rm LR} = \begin{pmatrix} \epsilon & 0 & 0 & 0 \\ 0 & \mathcal{P}_\perp & 0 & 0 \\ 0 & 0 & \mathcal{P}_\perp & 0 \\ 0 & 0 & 0 & \mathcal{P}_\parallel \end{pmatrix}$$

originally introduced to facilitate the transition from the initial state (with strong anisotropy along the beam axis) to the hydro stage.

* sincere apologies to those I have not mentioned.

Beyond "traditional" fluid dynamics

modifying fluid dynamics

- Changing the hydrodynamics equations phenomenologically
  - "Anisotropic hydro"

Martinez-Guerrero, Strickland; Florkowski, Ryblewski; Heinz...*

☛ phenomenologically modify the energy-momentum tensor by distorting its spatial part.

Recent proposal (N.B., S.Feld, Ch.Lang 2014) to use that framework also for the freeze-out description (with elongation along the radial direction).

* sincere apologies to those I have not mentioned.

Fluid dynamics & heavy-ion collisions

- Overall, a success story — especially with dissipative fluid dynamics

  ☛ good rendering of a significant set of bulk observables in Pb-Pb @ LHC and Au-Au @ RHIC

- We are reaching a stage where the first successes are too tempting so that we want quantitative results (N.B.'s dream: with error bars)
  - The hydrodynamical framework itself is fine
  - yet it has to be glued to other descriptions (initial & final stages)

  ☛ hydro pushed to its natural limits (hydro in p+Pb??)

- Looking for "beyond hydro" ideas, pictures, models, theories... the heavy-ion community is possibly investigating the boundaries of hydro as its has never been done before

  ☛ fun! (for a theorist)