Far-from-equilibrium anisotropic collective flow

Nicolas BORGHINI

Universität Bielefeld

QUARK MATTER
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Far-from-equilibrium anisotropic flow: onset of collectivity

- Do you need many collisions to build up "collective behavior"?

  flow of massless particles diffusing on fixed scattering centers

- Further effects...
  - initial anisotropic flow
  - anisotropic differential cross-section
  - non-Gaussian initial spatial distribution

(a 15-minute summary of) N.B. & C.Gombeaud, Eur. Phys. J. C **71** (2011) 1612
+ work in progress

Far-from-equilibrium anisotropic flow: a warning

A few things you should not expect to find in this talk

- Fits to experimental data (no η/s!)

  I shall present toy models, with 1 or 2 parameters only:

  my purpose is to identify qualitative behaviors

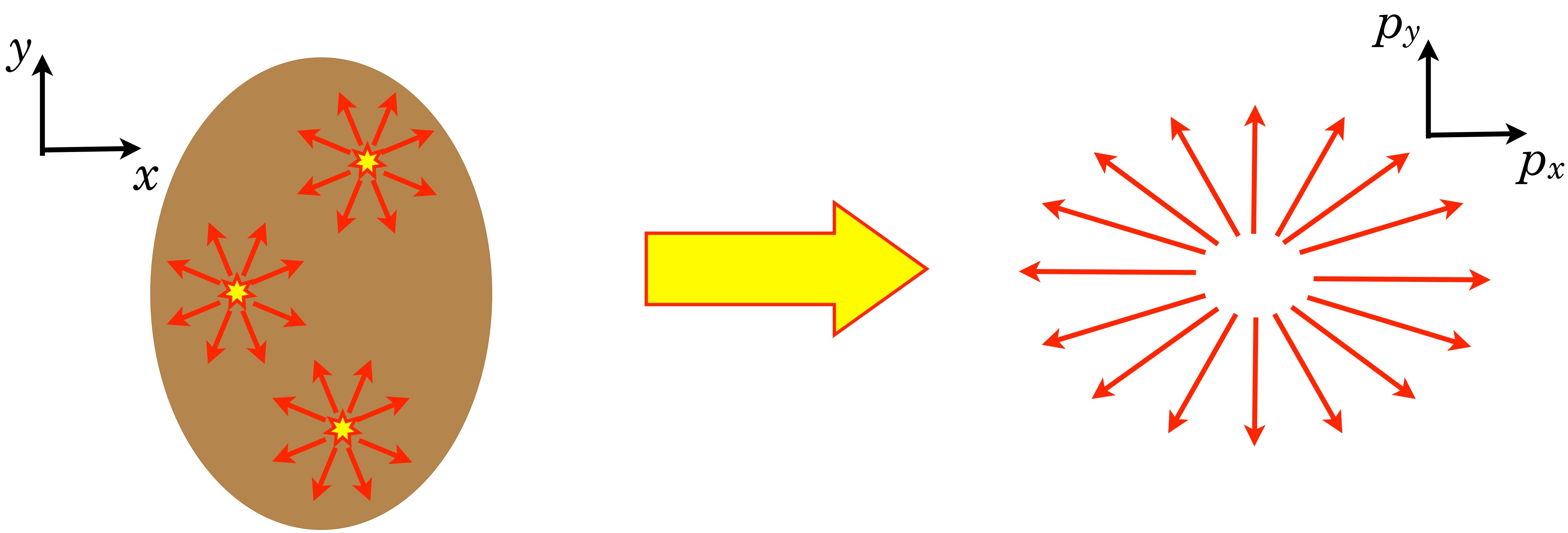
  (+ understand the origin of these behaviors… & have fun?)

- Pocket formulae

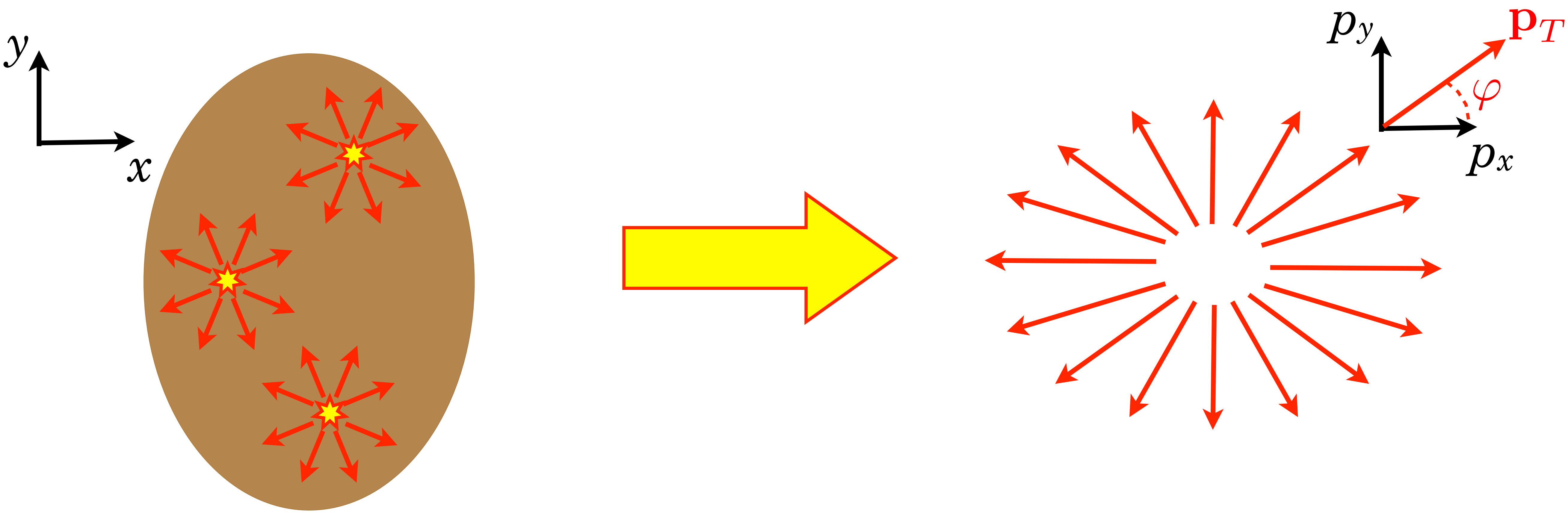
dear convenor, don't worry!

Anisotropic flow

In non-central nucleus-nucleus collisions, the initial spatial asymmetry of the overlap region in the transverse plane is converted by particle rescatterings into an anisotropic transverse-momentum distribution of the outgoing particles: anisotropic (transverse) flow.



Anisotropic flow



$$\epsilon_2 \equiv \frac{\langle y^2 - x^2 \rangle}{\langle x^2 + y^2 \rangle} \neq 0 \quad \Rightarrow \quad v_n(p_T) \equiv \frac{\displaystyle\int \mathrm{d}\varphi \, \frac{\mathrm{d}^2 N}{\mathrm{d}^2\mathbf{p}_T} \cos n\varphi}{\displaystyle\int \mathrm{d}\varphi \, \frac{\mathrm{d}^2 N}{\mathrm{d}^2\mathbf{p}_T}} \neq 0$$

$$\frac{\mathrm{d}^2 N}{\mathrm{d}^2\mathbf{p}_T} = \frac{1}{2\pi} \frac{\mathrm{d}N}{p_T \, \mathrm{d}p_T} \left[1 + \sum_{n=1}^{\infty} 2 v_n(p_T) \cos n\varphi \right]$$

Far-from-equilibrium anisotropic flow: onset of collectivity

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  flow of massless particles diffusing on fixed scattering centers

- Further effects...
  - initial anisotropic flow
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The model

- System: 2-dimensional dilute mixture of components with masses m_i, m_k..., which scatter elastically on each other with an isotropic and constant differential cross-section $\sigma_{\rm d}$.
  - 2-dimensional: I'm only interested in the transverse expansion.
  - $\sigma_{\rm d}$ isotropic, constant, p_T-independent: a single parameter!
  - dilute system: kinetic description à la Boltzmann is meaningful.
  - distribution functions $f_i(t, \mathbf{x}, \mathbf{p}_i)$, $f_k(t, \mathbf{x}, \mathbf{p}_k)$.

The model

- Initial condition ($t = 0$): isotropic distribution $\tilde{f}_0$ in momentum space, asymmetric distribution in position space (identical for i and k).
  - in position space: Gaussian profile with mean square radii $R_x^2 < R_y^2$.

$$f(0,\mathbf{x},\mathbf{p}_T) = \frac{N}{4\pi^2 R_x R_y}\tilde{f}_0(p_T)\exp\left(-\frac{x^2}{2R_x^2} - \frac{y^2}{2R_y^2}\right)$$

Let $R_x^2 \equiv \dfrac{R^2}{1+\epsilon}$, $\quad R_y^2 \equiv \dfrac{R^2}{1-\epsilon}$; then $\epsilon_2(0) = \dfrac{\langle y^2 - x^2\rangle}{\langle x^2 + y^2\rangle} = \dfrac{R_y^2 - R_x^2}{R_x^2 + R_y^2} = \epsilon$!

  - $\tilde{f}_0$ normalized to $\displaystyle\int_0^\infty \mathrm{d}p_T\, p_T\, \tilde{f}_0(p_T) = 1.$

The model

(independent of the choice of particle masses)

Once the distribution function $f(t, \mathbf{x}, \mathbf{p}_T)$ is known, the (transverse-) momentum spectrum

$$\frac{\mathrm{d}^2 N}{\mathrm{d}^2\mathbf{p}_T}(t, \mathbf{p}_T) = \int \mathrm{d}^2\mathbf{x}\, f(t, \mathbf{x}, \mathbf{p}_T)$$

at time t follows at once.

One can thus obtain the time-dependence of the anisotropic flow coefficients $v_n(t, p_T)$.

The usual, experimentally accessible harmonic $v_n(p_T)$ is the large-time limit $v_n(t \to \infty, p_T)$.

The model: evolution equation

(independent of the choice of particle masses)

$$\frac{\partial f_i}{\partial t} + \mathbf{v}_i \cdot \nabla_{\mathbf{x}} f_i = \left[\frac{\partial f_i}{\partial t}\right]_{\rm gain} - \left[\frac{\partial f_i}{\partial t}\right]_{\rm loss}$$

Gain and loss terms:

$$\sim f_i(t,\mathbf{x},\mathbf{p}_i)\, f_k(t,\mathbf{x},\mathbf{p}_k)\, v_{ik}\sigma_{\rm d}$$

with v_{ik} the relative velocity.

In general $v_{ik} = \sqrt{(\mathbf{v}_i - \mathbf{v}_k)^2 - \frac{(\mathbf{v}_i \times \mathbf{v}_k)^2}{c^2}}$, but we won't need that...

The model: evolution equation

(independent of the choice of particle masses)

Integrating the evolution equation

$$\frac{\partial f_i}{\partial t} + \mathbf{v}_i \cdot \nabla_{\mathbf{x}} f_i = \left[\frac{\partial f_i}{\partial t}\right]_{\rm gain} - \left[\frac{\partial f_i}{\partial t}\right]_{\rm loss}$$

over $\mathbf{x}$, the gradient part disappears:

$$\frac{\partial}{\partial t}\frac{{\rm d}^2 N_i}{{\rm d}^2\mathbf{p}_i} = \int {\rm d}^2\mathbf{x} \left(\left[\frac{\partial f_i}{\partial t}\right]_{\rm gain} - \left[\frac{\partial f_i}{\partial t}\right]_{\rm loss} \right)$$

Then $$v_n(p_i) \equiv \frac{\displaystyle\int {\rm d}\varphi_i \frac{{\rm d}^2 N_i}{{\rm d}^2\mathbf{p}_i} \cos n\varphi_i}{\displaystyle\int {\rm d}\varphi_i \frac{{\rm d}^2 N_i}{{\rm d}^2\mathbf{p}_i}}$$ ...easy, no?

The model: first solution

(independent of the choice of particle masses)

If there are no rescattering between i and k particles: $\sigma_{\rm d}$ = 0.

$$\frac{\partial f_i}{\partial t} + \mathbf{v}_i \cdot \nabla_{\mathbf{x}} f_i = 0$$

☞ free-streaming solutions:

$$f_i^{(0)}(t, \mathbf{x}, \mathbf{p}_i) = f_i^{(0)}(0, \mathbf{x} - \mathbf{v}_i t, \mathbf{p}_i)$$

If one starts with an isotropic distribution in momentum space, it remains so as the system evolves: no anisotropies develop...

$v_n(t, p_T)$ = 0 at all times

Let's turn on the rescatterings...

(independent of the choice of particle masses)

... but only few of them!

New solution: $f_i(t,\mathbf{x},\mathbf{p}_i) = f_i^{(0)}(t,\mathbf{x},\mathbf{p}_i) + f_i^{(1)}(t,\mathbf{x},\mathbf{p}_i) + \cdots$

with $f_i^{(1)} \ll f_i^{(0)}$, and so on.*

☞ momentum anisotropies of f_i are those of $f_i^{(1)}$.

*small parameter in the expansion: $\approx \sigma_{\rm d}$ (divided by R, for dimensional reasons)

… but only few rescatterings

(independent of the choice of particle masses)

$f_i^{(1)} \ll f_i^{(0)}$: need to ensure a small number of scatterings per particle.

Collision rate: $\dfrac{\mathrm{d}N_{\mathrm{coll}}}{\mathrm{d}t} = \int \mathrm{d}^2\mathbf{x} \int \mathrm{d}^2\mathbf{p}_i\, \mathrm{d}^2\mathbf{p}_k\, \mathrm{d}\Theta\, f_i f_k v_{ik} \sigma_{\mathrm{d}}$, which should be

integrated over the whole evolution, with $f_i = f_i^{(0)}$, and be kept small.

Simple model: Lorentz gas

- massless diffusing particles: $|\mathbf{v}_i| = c$
- fixed scattering centers: $|\mathbf{v}_k| = 0$

☞ $v_{ik} = c$ ...much easier!

In particular, v_{ik} is independent of the particle azimuths.

Lorentz gas: further simplification

The momentum anisotropies of f_i are those of $f_i^{(1)}$.

- the loss term of the evolution equation does lead to anisotropies: the number of particles with azimuth φ_i lost in a rescattering is directly related to the initial geometry.

- the gain term of the evolution equation does NOT (to leading order) lead to anisotropies in the case of an isotropic cross-section: it involves the distribution functions before the rescatterings, while the azimuth φ_i is that of the outgoing momentum.

$$\frac{\partial v_n}{\partial t}(t, p_i) \propto -\int \mathrm{d}^2\mathbf{x}\,\mathrm{d}\varphi_i \left[\frac{\partial f_i}{\partial t}\right]_{\mathrm{loss}} \cos n\varphi_i$$

Anisotropic flow of a Lorentz gas: phenomenological relevance?

- A gas of massless diffusing particles scattering on infinitely massive centers is the (regular) limiting case for light particles scattering on massive ones.

  Invoking (local) momentum conservation at each scattering, this also describes the flow of massive particles in a wind of light ones.

- Considering a single rescattering may be relevant for particles/states that are "destroyed" after a single collision:
  - high-momentum particles, which lose a sizable amount of their momentum, thus are gone from their initial p_{T} bin;
  - fragile states (quarkonia? ϕ-meson?).

- Obvious(?): photons(?)

Simple model: Lorentz gas

- Rescattering rate:

$$\frac{\mathrm{d}N_{\mathrm{coll}}}{\mathrm{d}t} = \int \mathrm{d}^2\mathbf{x}\,\mathrm{d}^2\mathbf{p}_i\,\mathrm{d}^2\mathbf{p}_k\,\mathrm{d}\Theta\, f_i^{(0)}(t,\mathbf{x},\mathbf{p}_i) f_k^{(0)}(t,\mathbf{x},\mathbf{p}_k) v_{ik}\sigma_{\mathrm{d}}$$

- Anisotropic flow evolution:

$$\frac{\partial v_n}{\partial t}(t,p_i) \propto -\int \mathrm{d}^2\mathbf{x}\,\mathrm{d}\varphi_i\,\mathrm{d}^2\mathbf{p}_k\,\mathrm{d}\Theta\, f_i^{(0)}(t,\mathbf{x},\mathbf{p}_i) f_k^{(0)}(t,\mathbf{x},\mathbf{p}_k)\, v_{ik}\sigma_{\mathrm{d}} \cos n\varphi_i$$

The integrals over $\mathbf{x}$, Θ, φ_k, $|\mathbf{p}_k|$ are easy or even trivial!

Lorentz gas: number of rescatterings

- Rescattering rate:

$$\frac{\mathrm{d}N_{\rm coll}}{\mathrm{d}t} = \frac{N_i N_k \sigma_{\rm d} c\sqrt{1-\epsilon^2}}{2R^2}\, \mathrm{e}^{-c^2t^2/4R^2}\, I_0\!\left(\frac{c^2t^2}{4R^2}\,\epsilon\right)$$

so that the total number of rescatterings is (K: elliptic integral)

$$N_{\rm coll} = \frac{N_i N_k \sigma_{\rm d}}{\sqrt{\pi}R}\,\sqrt{1-\epsilon}\,K\!\left(\sqrt{\frac{2\epsilon}{1+\epsilon}}\right)$$

Lorentz gas: number of rescatterings

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so that the total number of rescatterings is (K: elliptic integral)

$$N_{\mathrm{coll}} = \frac{N_i N_k \sigma_{\mathrm{d}}}{\sqrt{\pi}R}\,\sqrt{1-\epsilon}\,K\!\left(\sqrt{\frac{2\epsilon}{1+\epsilon}}\right)$$

i.e. maximal for central collisions [$K(0)=\frac{\pi}{2}$] at a given cross-section: the choice

$$\sigma_{\mathrm{d}}^{\max} = \frac{2}{N_k\sqrt{\pi}}R$$

ensures at most one rescattering per diffusing particle for all ϵ.

☞ consistency of the approach!

Lorentz gas: anisotropic flow

- Anisotropic flow (even harmonics):

(do not forget the − sign from our considering the loss term!)

$$\frac{\mathrm{d}v_n}{\mathrm{d}t} = (-1)^{\frac{n}{2}+1} \frac{N_k \sigma_{\mathrm{d}} c \sqrt{1-\epsilon^2}}{2R^2} \, \mathrm{e}^{-c^2 t^2/4R^2} I_{\frac{n}{2}}\!\left(\frac{c^2 t^2}{4R^2}\epsilon\right)$$

Lorentz gas: anisotropic flow

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(do not forget the − sign from our considering the loss term!)

$$\frac{\mathrm{d}v_n}{\mathrm{d}t} = (-1)^{\frac{n}{2}+1}\frac{N_k\sigma_{\mathrm{d}}c\sqrt{1-\epsilon^2}}{2R^2}\,\mathrm{e}^{-c^2t^2/4R^2}I_{\frac{n}{2}}\!\left(\frac{c^2t^2}{4R^2}\epsilon\right)$$

that is $\sim (-1)^{\frac{n}{2}+1}\dfrac{N_k\sigma_{\mathrm{d}}c\sqrt{1-\epsilon^2}}{2(\frac{n}{2})!\,R^2}\left(\dfrac{ct\sqrt{\epsilon}}{4R}\right)^n$ for $t \ll \dfrac{2R}{c}$

so that $v_n(t) \propto (-1)^{\frac{n}{2}+1}t^{n+1}$ at early times.

- behavior already seen in transport codes (Gombeaud & Ollitrault);
- differs from the slower rise $\propto t^n$ in fluid dynamics.

Lorentz gas: anisotropic flow

Integrating $\frac{\mathrm{d}v_n}{\mathrm{d}t}$ from $t = 0$ to ∞, one obtains v_n, e.g.

$$v_2(p_i) = \frac{N_k \sigma_\mathrm{d} \sqrt{\pi}}{8R} \sqrt{1-\epsilon^2}\, {}_2F_1\!\left(\frac{3}{4}, \frac{5}{4}; 2; \epsilon^2\right) \epsilon$$

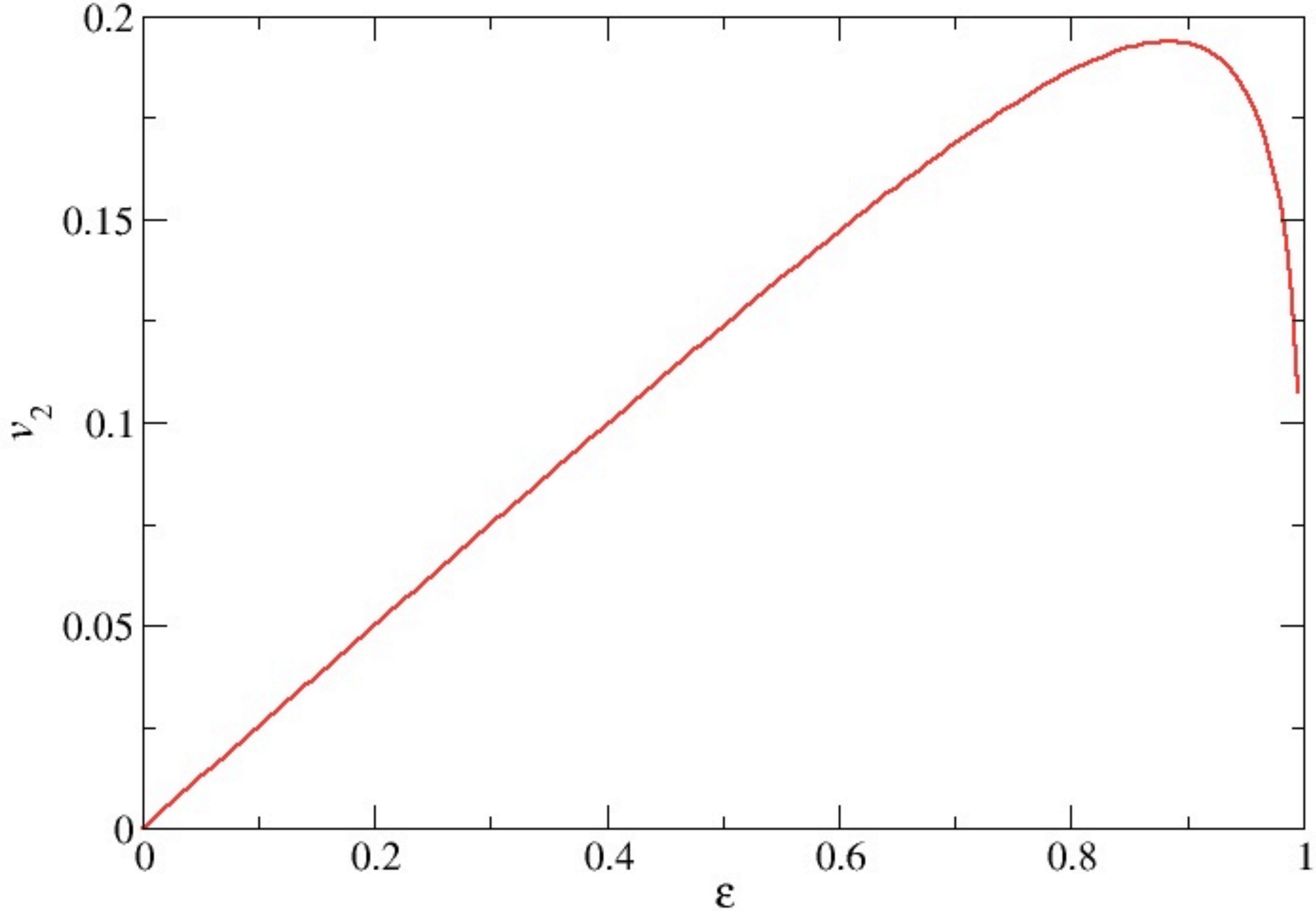
Gauss hypergeometric function

Requiring at most one rescattering per diffusing particles, i.e. fixing σ_d to $\sigma_\mathrm{d}^{\max} = 2R/N_k\sqrt{\pi}$, gives the parameter-free result

$$v_2(p_i) = \frac{1}{4} \sqrt{1-\epsilon^2}\, {}_2F_1\!\left(\frac{3}{4}, \frac{5}{4}; 2; \epsilon^2\right) \epsilon$$

Lorentz gas: Centrality dependence of v_2

$$v_2(p_i) = \frac{1}{4}\sqrt{1-\epsilon^2}\,{}_2F_1\!\left(\frac{3}{4},\frac{5}{4};2;\epsilon^2\right)\epsilon$$

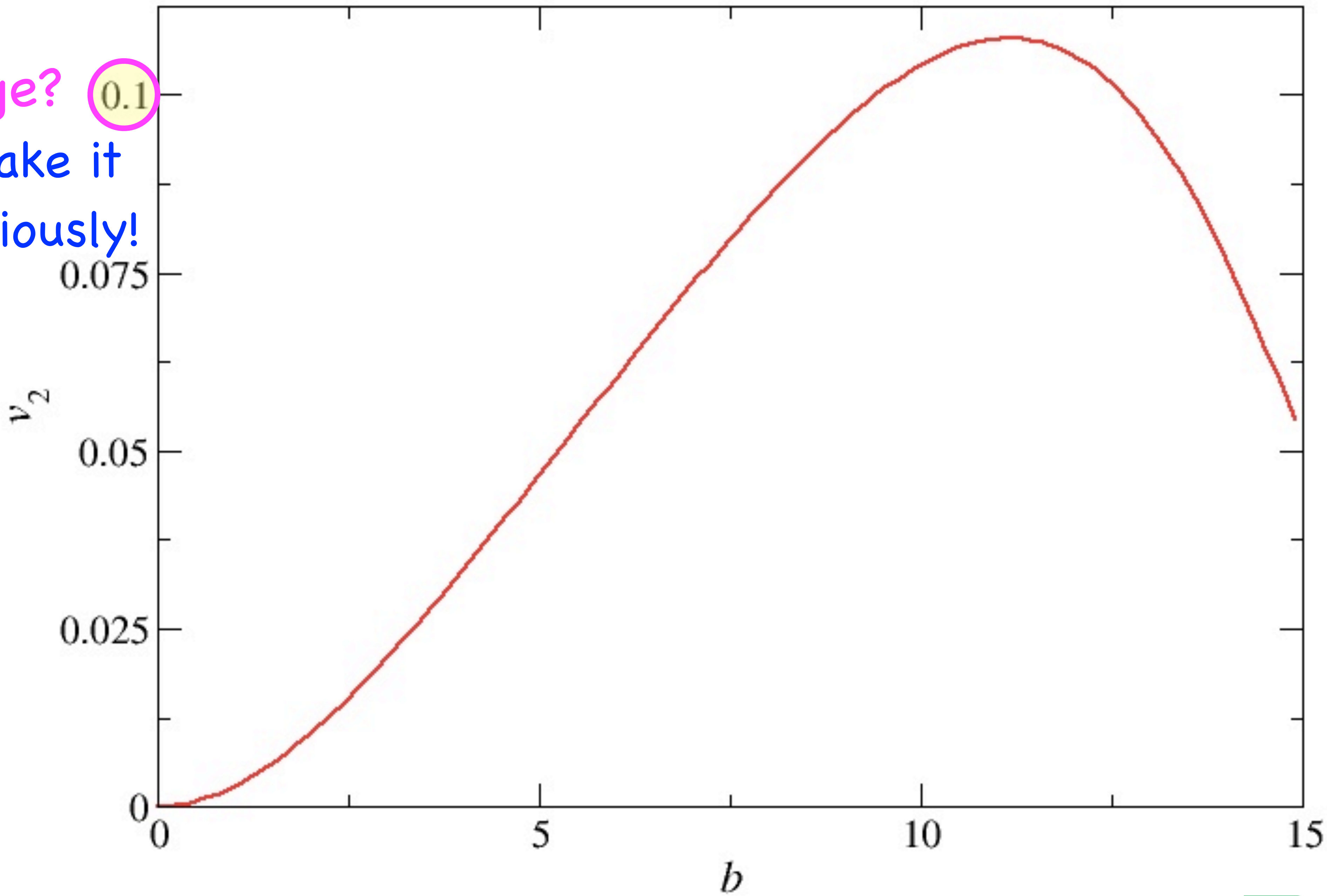


Lorentz gas: Centrality dependence of v_2

Glauber optical model to relate b and ϵ

large?

don't take it too seriously!



Far-from-equilibrium anisotropic flow: onset of collectivity

- Do you need many collisions to build up "collective behavior"?

  flow of massless particles diffusing on fixed scattering centers

- Further effects... more parameters!
  - initial anisotropic flow
  - anisotropic differential cross-section ← here, the gain term plays a role!
  - non-Gaussian initial spatial distribution
  - ...

  } not shown today!

The next model

- Initial condition ($t = 0$): anisotropic distribution $\tilde{f}$ in momentum space, asymmetric distribution in position space (identical for i and k).
  - anisotropic initial distribution in momentum space

$$\tilde{f}(\mathbf{p}_T) = \tilde{f}_0(p_T)\left(1 + 2\sum_{k\geq 1}\left[w_{k,c}(p_T)\cos k\varphi + w_{k,s}(p_T)\sin k\varphi\right]\right)$$

  - $\tilde{f}_0$ normalized to $\displaystyle\int_0^\infty \mathrm{d}p_T\, p_T\, \tilde{f}_0(p_T) = 1\,.$
  - in position space: Gaussian profile with mean square radii $R_x^2 < R_y^2$

$$f(0, \mathbf{x}, \mathbf{p}_T) = \frac{N}{4\pi^2 R_x R_y}\tilde{f}(\mathbf{p}_T)\exp\left(-\frac{x^2}{2R_x^2} - \frac{y^2}{2R_y^2}\right)$$

(side-remark: including $w_{k,s}$ might account for $\Psi_2 \neq \Psi_3 \neq \ldots$)

Lorentz gas with initial flow

The computation proceeds as before:

- Rescattering rate:

$$\frac{\mathrm{d}N_{\rm coll}}{\mathrm{d}t} = \frac{N_c N \sigma_{\rm d} c}{2R^2}\sqrt{1-\epsilon^2}\,\mathrm{e}^{-c^2t^2/4R^2}\left[I_0\!\left(\frac{c^2t^2}{4R^2}\epsilon\right) + 2\sum_{q\geq 1}(-1)^q w_{2q,c} I_q\!\left(\frac{c^2t^2}{4R^2}\epsilon\right)\right]$$

- Anisotropic flow evolution:

$$\frac{\partial v_{2m}}{\partial t}(t) = (-1)^{m+1}\frac{N_c\sigma_{\rm d}c}{2R^2}\sqrt{1-\epsilon^2}\,\mathrm{e}^{-c^2t^2/4R^2} \times\left(I_m\!\left(\frac{c^2t^2}{4R^2}\epsilon\right) + \sum_{q\geq 1}(-1)^q w_{2q,c}\left[I_{m+q}\!\left(\frac{c^2t^2}{4R^2}\epsilon\right) + I_{m-q}\!\left(\frac{c^2t^2}{4R^2}\epsilon\right)\right]\right)$$

$$\frac{\partial v_{2m+1}}{\partial t}(t) = (-1)^{m+1}\frac{N_c\sigma_{\rm d}c}{2R^2}\sqrt{1-\epsilon^2}\,\mathrm{e}^{-c^2t^2/4R^2} \times\sum_{q\geq 1}(-1)^q w_{2q-1,c}\left[I_{m+q}\!\left(\frac{c^2t^2}{4R^2}\epsilon\right) + I_{m-q}\!\left(\frac{c^2t^2}{4R^2}\epsilon\right)\right]$$

Lorentz gas with initial flow

- Anisotropic flow development at early times $t \ll R/c$

  - elliptic flow:

$$\frac{\partial v_2}{\partial t}(t) \sim \frac{N_c \sigma_{\rm d} c}{2R^2}\sqrt{1-\epsilon^2}\left[-w_{2,c} + (1+w_{4,c})\frac{c^2}{8R^2}\epsilon t^2 + \mathcal{O}(t^4)\right]$$

    - evolves even if there is no spatial asymmetry (ϵ = 0)!
    - might decrease (if $w_{2,c}$ = $v_2(t=0)$ > 0) before increasing;

  - triangular flow:

$$\frac{\partial v_3}{\partial t}(t) \sim \frac{N_c \sigma_{\rm d} c}{2R^2}\sqrt{1-\epsilon^2}\left[-w_{1,c} + w_{3,c}\frac{c^2}{8R^2}\epsilon t^2 + \mathcal{O}(t^4)\right]$$

    depends on odd harmonics only.

Far-from-equilibrium anisotropic flow: onset of collectivity

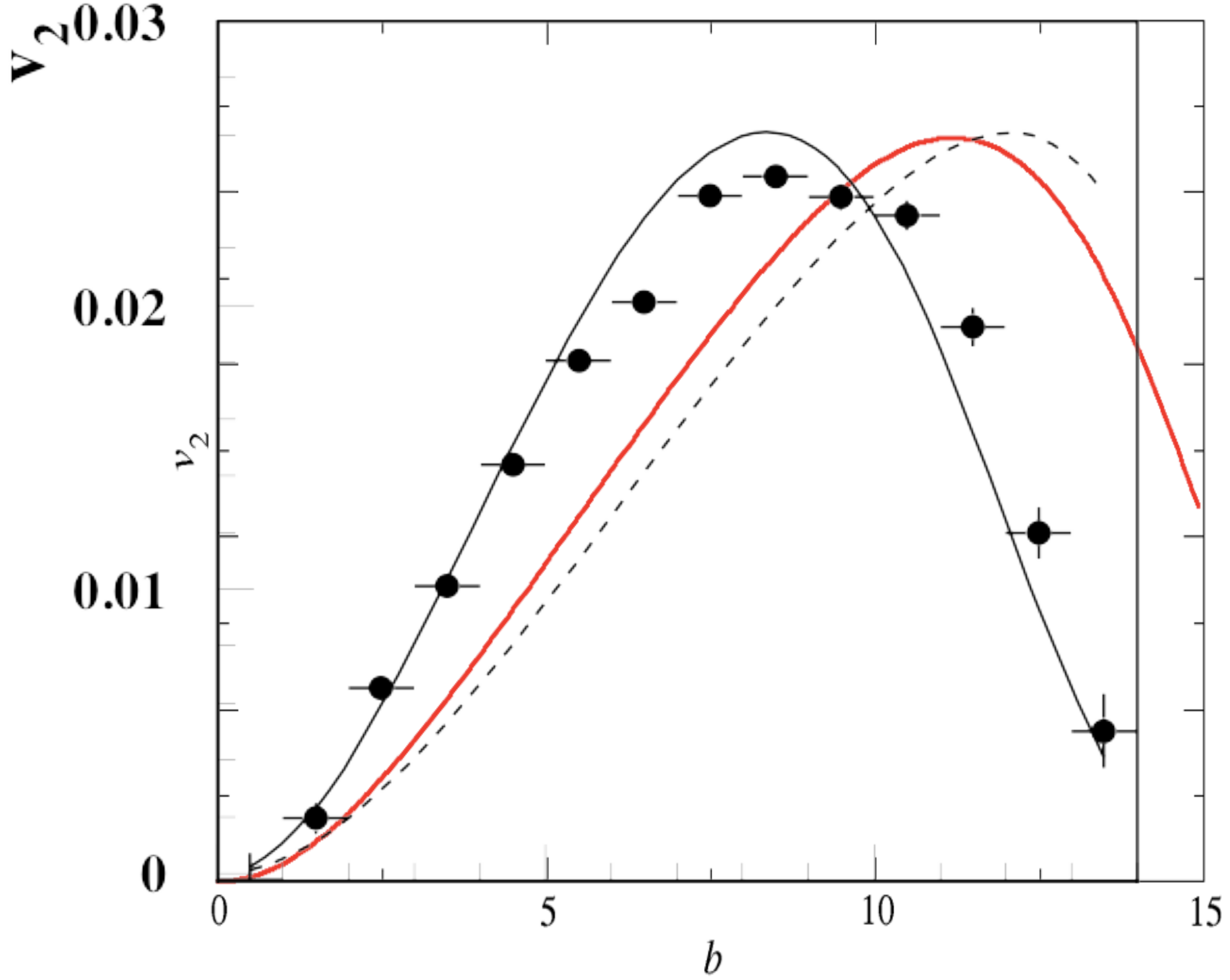
- Do you need many collisions to build up “collective behavior”?

  NO! already significant(?) flow after a single collision

- Further ingredients (initial anisotropic flow, anisotropic differential cross-section...) provide a wealth of possible behaviors:
  - creation of anisotropic flow for $\epsilon = 0$;
  - non-monotonic evolution of anisotropic flow;
  - mixing of different harmonics.

extra slides

Lorentz gas: Centrality dependence of v_2



Black curves (full: "LDL", dashed: hydro) and points (RQMD 2.3) from Voloshin & Poskanzer, Phys. Lett. B **474** (2000) 27

The model: initial condition

Remarks on the Gaussian profile

$$f(0,\mathbf{x},\mathbf{p}_T) = \frac{N}{4\pi^2 R_x R_y}\tilde{f}_0(p_T)\exp\left(-\frac{x^2}{2R_x^2}-\frac{y^2}{2R_y^2}\right)$$

Let $R_x^2 \equiv \dfrac{R^2}{1+\epsilon}$, $R_y^2 \equiv \dfrac{R^2}{1-\epsilon}$; then $\epsilon_2(0) = \dfrac{\langle y^2 - x^2\rangle}{\langle x^2+y^2\rangle} = \dfrac{R_y^2 - R_x^2}{R_x^2+R_y^2} = \epsilon$!

(Note that $\epsilon_2 = -\dfrac{\langle r^2\cos 2\varphi_r\rangle}{\langle r^2\rangle}$, where φ_r denotes the polar angle...)

Now, one finds $\epsilon_4 \equiv -\dfrac{\langle r^4\cos 4\varphi_r\rangle}{\langle r^4\rangle} = -\dfrac{\langle x^4 - 6x^2y^2+y^4\rangle}{\langle x^4+2x^2y^2+y^4\rangle} = -\dfrac{3\epsilon^2}{2+\epsilon^2}$,

that is ϵ_2 and ϵ_4 are of opposite signs.

☞ expect opposite signs for v_2 and v_4 !