Multiparticle methods for measuring anisotropic flow

From large to small systems

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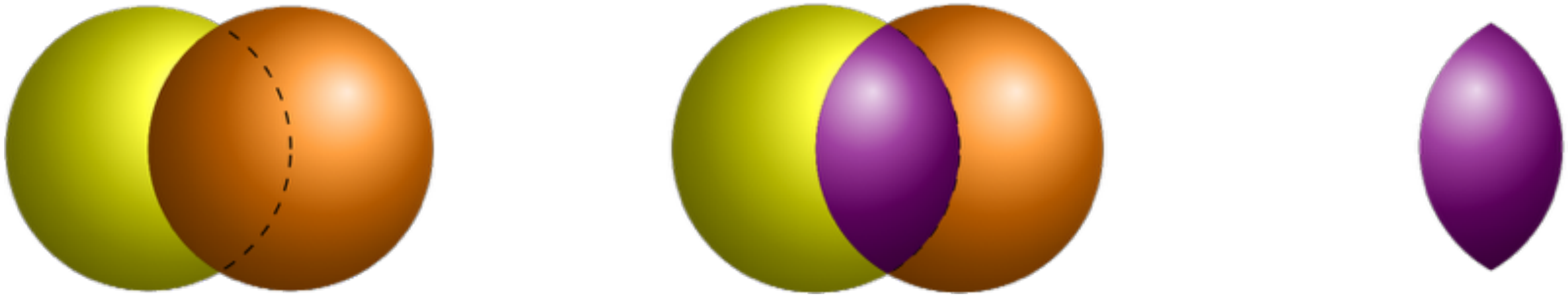
Collectivity in Small Colliding Systems, BNL, March 4-6, 2015

Preamble: Anisotropic (collective) flow

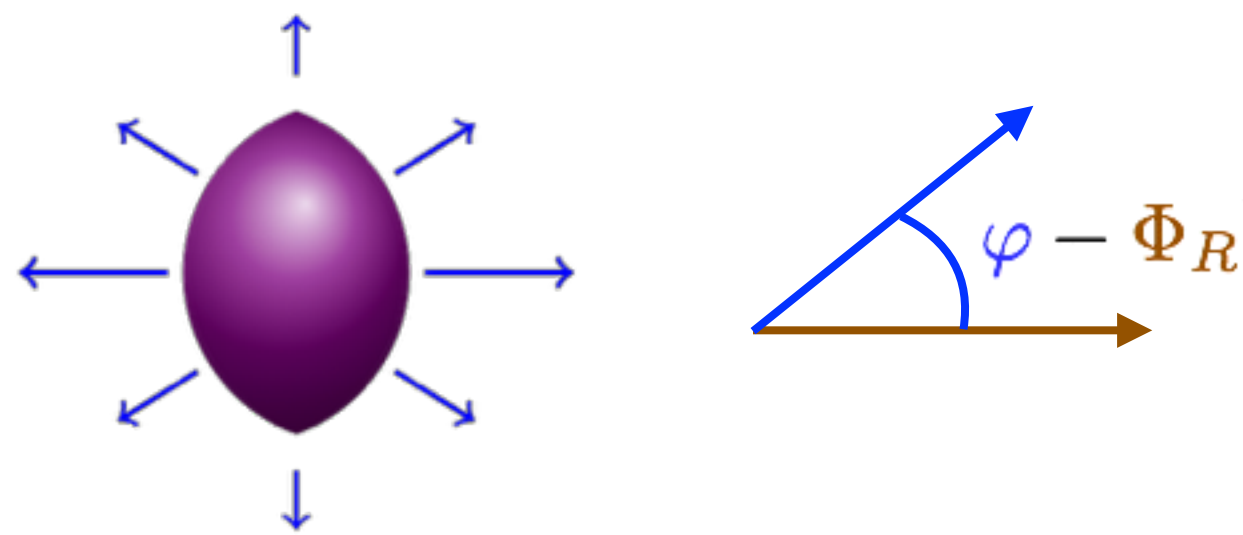
Classical picture (before ca.2010) in collisions of "large" systems:

Initially asymmetric collision zone (in the transverse plane)



$t < 0$ $t = 0$ $t = 0^+$

$\Rightarrow$ anisotropic emission of particles



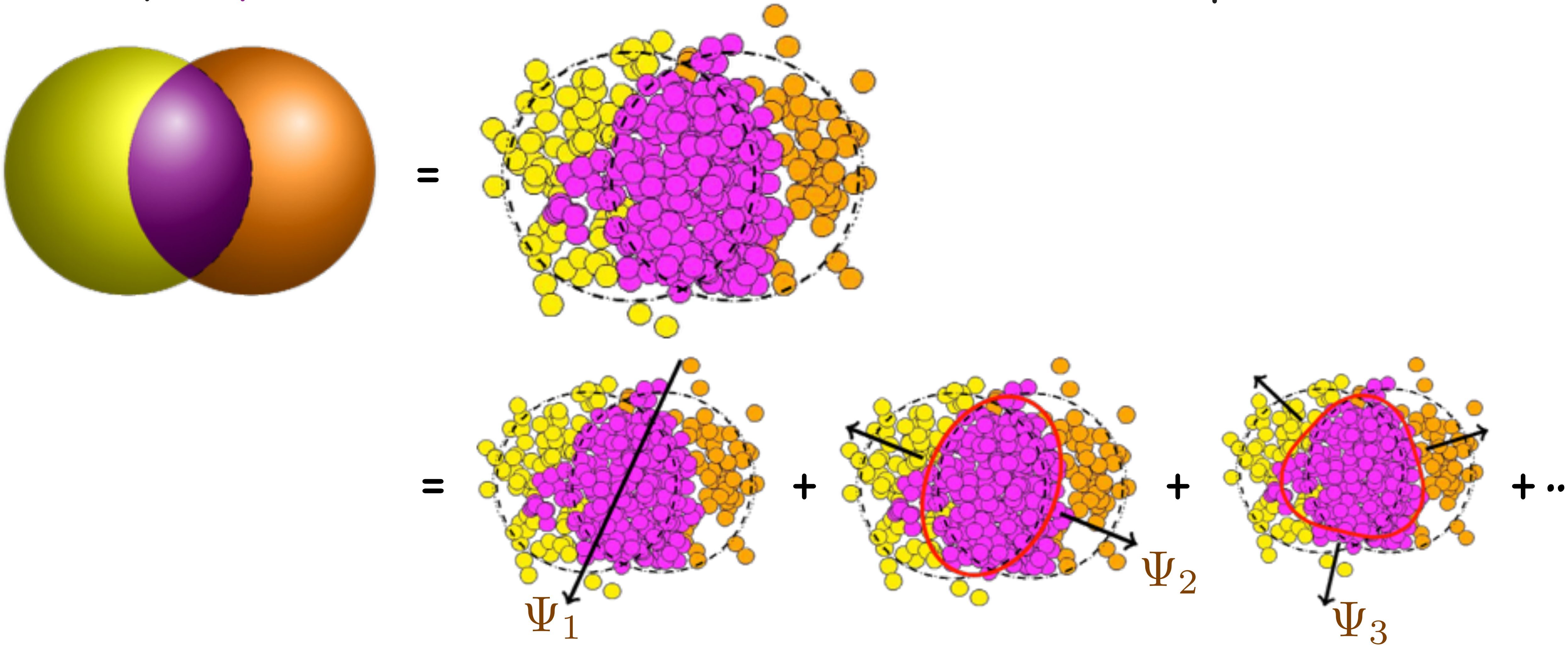
$$E\frac{\mathrm{d}N}{\mathrm{d}^3\mathbf{p}} \propto \frac{\mathrm{d}N}{p_T\,\mathrm{d}p_T\,\mathrm{d}y}\left[1 + 2v_1\cos(\varphi - \Phi_R) + 2v_2\cos 2(\varphi - \Phi_R) + \cdots\right]$$

pictures shamelessly stolen from Matt Luzum

Preamble: Anisotropic (collective) flow

Newer picture (since 2010) in collisions of "large" systems:

Initially asymmetric collision zone (in the transverse plane)



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$$E\frac{\mathrm{d}N}{\mathrm{d}^3\mathbf{p}} \propto \frac{\mathrm{d}N}{p_T\mathrm{d}p_T\mathrm{d}y}\left[1+2\sum_{n=1}^{\infty} v_n \cos n(\varphi-\Psi_n)\right]$$

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Preamble: Anisotropic (collective) flow

$$E\frac{\mathrm{d}N}{\mathrm{d}^3\mathbf{p}} \propto \frac{\mathrm{d}N}{p_T\mathrm{d}p_T\mathrm{d}y}\left[1+2\sum_{n=1}^{\infty} v_n \cos n(\varphi-\Psi_n)\right]$$

v_n: Fourier coefficients of the single-particle distribution;

Ψ_n: n-th harmonic "event plane"

$$v_n \equiv \langle \cos n(\varphi - \Psi_n)\rangle$$

☞ even at (mathematically) fixed impact parameter, v_n & Ψ_n vary from event to event!

☛ initial conditions & system evolution

Goal: measure the v_n coefficients (in a second step: as a function of p_T, y)

Multiparticle methods for measuring anisotropic flow

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- Cumulants, Lee-Yang zeroes: a reminder
  - old motivations & applications
  - newer uses
- Application to smaller(?) systems
  - overview of results
  - caveats from the theory side

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Measuring anisotropic flow with (multiparticle) "cumulants"

Principle:

① Using the measured azimuths of the emitted particles, build appropriate correlators

For instance $\left\langle e^{in(\varphi_j-\varphi_k)}\right\rangle$, $\left\langle e^{in(\varphi_i+\varphi_j-\varphi_k-\varphi_l)}\right\rangle$ ...

brackets? to be discussed later

② Equate with the (theoretical) value of these correlators for events with anisotropic flow only

☛ yields flow estimate(s)

For instance $\left\langle e^{in(\varphi_j-\varphi_k)}\right\rangle = (v_n)^2$ <u>if only flow in the system</u> ☛ $v_n\{2\}$

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If only flow was present, all correlators would be equal.

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If only flow was present, all correlators would be equal.

Due to the presence of additional sources of interparticle correlations ("nonflow") in the system, some correlators are more equal than the others.

☛ Cumulants, in which the impact of the other sources is minimized

e.g. $c_n\{4\} \equiv \left\langle e^{in(\varphi_i + \varphi_j - \varphi_k - \varphi_l)} \right\rangle - \left\langle e^{in(\varphi_i - \varphi_k)} \right\rangle \left\langle e^{in(\varphi_j - \varphi_l)} \right\rangle - \left\langle e^{in(\varphi_i - \varphi_l)} \right\rangle \left\langle e^{in(\varphi_j - \varphi_k)} \right\rangle$

Measuring anisotropic flow with (multiparticle) "cumulants"

Principle:

① Using the measured $\dfrac{\text{azimuths of the emitted particles}}{\text{multiplicity } M}$, build appropriate correlators

☛ Old motivation of cumulants: minimize the bias from nonflow.

N.B., P.M.Dinh, J.-Y.Ollitrault, PRC **63** (2001) 054904, **64** (2001) 054901

Measuring anisotropic flow with (multiparticle) "cumulants"

Principle:

① U ... build

appro

☛ Ol

54901

Nonflow effects?

- Quantum-statistics effects
- Resonance decays
- Momentum conservation
- (Mini)jets
- Strong & Coulomb interaction
- ...

k-particle correlator generally scales as

$$\mathcal{O}\left(\frac{1}{M^{k-1}}\right)$$

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② Equate with the theoretical value of the cumulants for events with anisotropic flow only:

$c_n\{2\} = (v_n)^2$, $c_n\{4\} = -(v_n)^4$, $c_n\{6\} = 4(v_n)^6$, $c_n\{8\} = -33(v_n)^8$ ...

☛ yields flow estimate $v_n\{k\}$ if $c_n\{k\} \gg \mathcal{O}\left(\frac{1}{M^{k-1}}\right)$

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(AGS), SPS, early-RHIC era!

☛ yields flow estimate $v_n\{k\}$ if $v_n \gg \mathcal{O}\left(\frac{1}{M^{1-1/k}}\right)$

... and concern

Measuring anisotropic flow with "Lee-Yang zeroes"

Principle of the method:

① Using the measured azimuths of the emitted particles, build a generating function $G_n(z)$ (which generates multiparticle averages)

② Look for the position of the first zero of $G_n(z)$ (in practice, the first minimum of its modulus) ☞ z_0

③ Under the assumption of events with anisotropic flow only, deduce from z_0 an estimate of v_n:

$$v_n\{\infty\} \equiv \frac{j_{01}}{M z_0} \qquad j_{01} = 2.40483\ldots$$

R.S.Bhalerao, N.B., J.-Y.Ollitrault, NPA **727** (2003) 373

Measuring anisotropic flow with "Lee-Yang zeroes"

"Lee-Yang zeroes" shared the same original motivation as cumulants: minimize the bias from nonflow.

☛ yields flow estimate $v_n\{\infty\}$ if $v_n \gg \mathcal{O}\left(\frac{1}{M}\right)$

From a theorist's point of view, LYZ-method is more aesthetic, since it directly measures "many-body collectivity" in the system under study. (The position of the first zero controls the asymptotic behavior of cumulants)

Measuring anisotropic flow with multiparticle methods

Let us discuss a few assumptions & points which I left aside till now.

The meaning of angular brackets? represent an average!

- On the experimental side, over many (all) detected particles in an event (M), then over many events (N_{ev})...

  - Both M and N_{ev} are finite, there will be statistical fluctuations!
  - nightmarish appendices / sections in

    PRC **63** (2001) 054904, PRC **64** (2001) 054901, & NPA **727** (2003) 373

  - were much talked about in 2002-05 ("limitation of the methods") yet have gone missing since then thanks to large M and N_{ev}.

Measuring anisotropic flow with multiparticle methods

- Average over M detected particles in N_{ev} events
  ☛ define the "resolution parameter" $\chi \equiv v_n \sqrt{M}$
  - for "large" $\chi \gtrsim 1$, life is easy, the statistical fluctuations decrease like $1/\sqrt{M\,N_{\text{ev}}}$ both on the $v_n\{k\}$ and on $v_n\{\infty\}$
    ☞ corresponds to the regime at LHC & high RHIC energies in collisions of large systems (measured with reasonable detectors...)!

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- On the experimental side, over many (all) detected particles in an event, then over many events...
- On the theoretical side — as hidden in the relations between flow and the cumulants or the position of the first zero:
  - first over the particles in an event, with an azimuthal distribution modulated by v_n(!);
  - then over events, assuming an isotropic distribution of Ψ_n and a constant v_n.

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can be corrected for if it only reflects the detector properties

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is it so obvious when there is physics at play?
(☛ engineered events)

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too idealistic!

Multiparticle methods for measuring anisotropic flow

From large to small systems

- Cumulants, Lee–Yang zeroes: a reminder
  - old motivations & applications
  - newer uses
- Application to smaller(?) systems
  - overview of results
  - caveats from the theory side

Anisotropic flow fluctuations

… in an experimental centrality bin are unavoidable; and physical!

⇒ Introduce a probability distribution $p(v_n)$ at fixed Ψ_n.

☛ to first approximation, mean value $\langle v_n\rangle$ and standard deviation δv_n.

- if $\delta v_n \ll \langle v_n\rangle$ then $v_n\{2\} = \langle v_n\rangle + \delta v_n$, $v_n\{4\} = v_n\{6\} = v_n\{8\} = \langle v_n\rangle - \delta v_n$…

  ☞ use cumulants to estimate $\langle v_n\rangle$ and δv_n.

- if $\delta v_n \gtrsim \langle v_n\rangle$ then the above identities no longer hold

  ☞ argue that differences between higher-order cumulant estimates $v_n\{4\}$, $v_n\{6\}$, $v_n\{8\}$… reflect non-Gaussianities of $p(v_n)$.

more in Jiangyong's talk!

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☛ minimize nonflow

☛ statistics not an issue (2002-1?)

☛ reconstruct flow fluctuations

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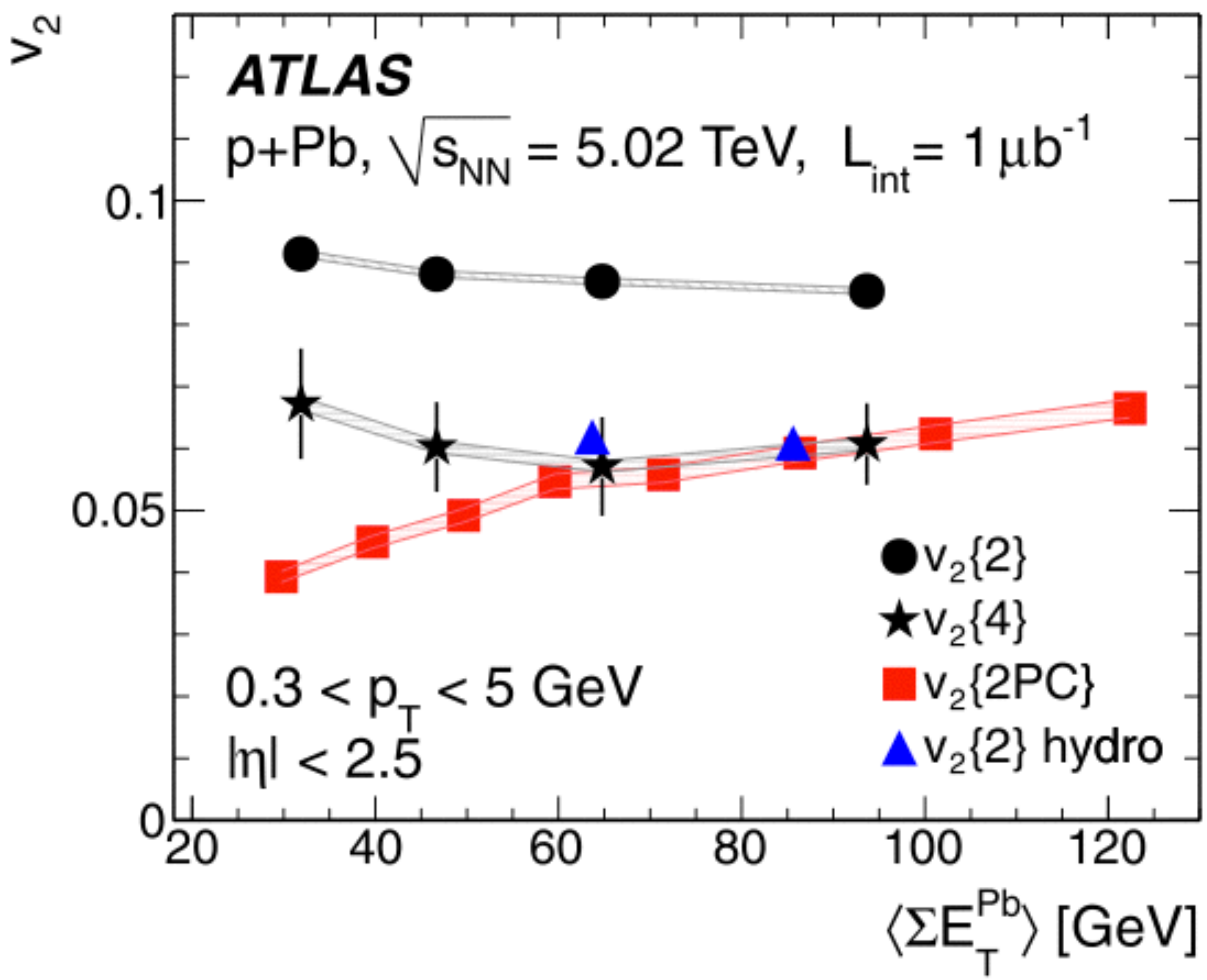
Multiparticle measurements of anisotropic flow in small systems



ATLAS Coll., Phys. Lett. B **725** (2013) 60

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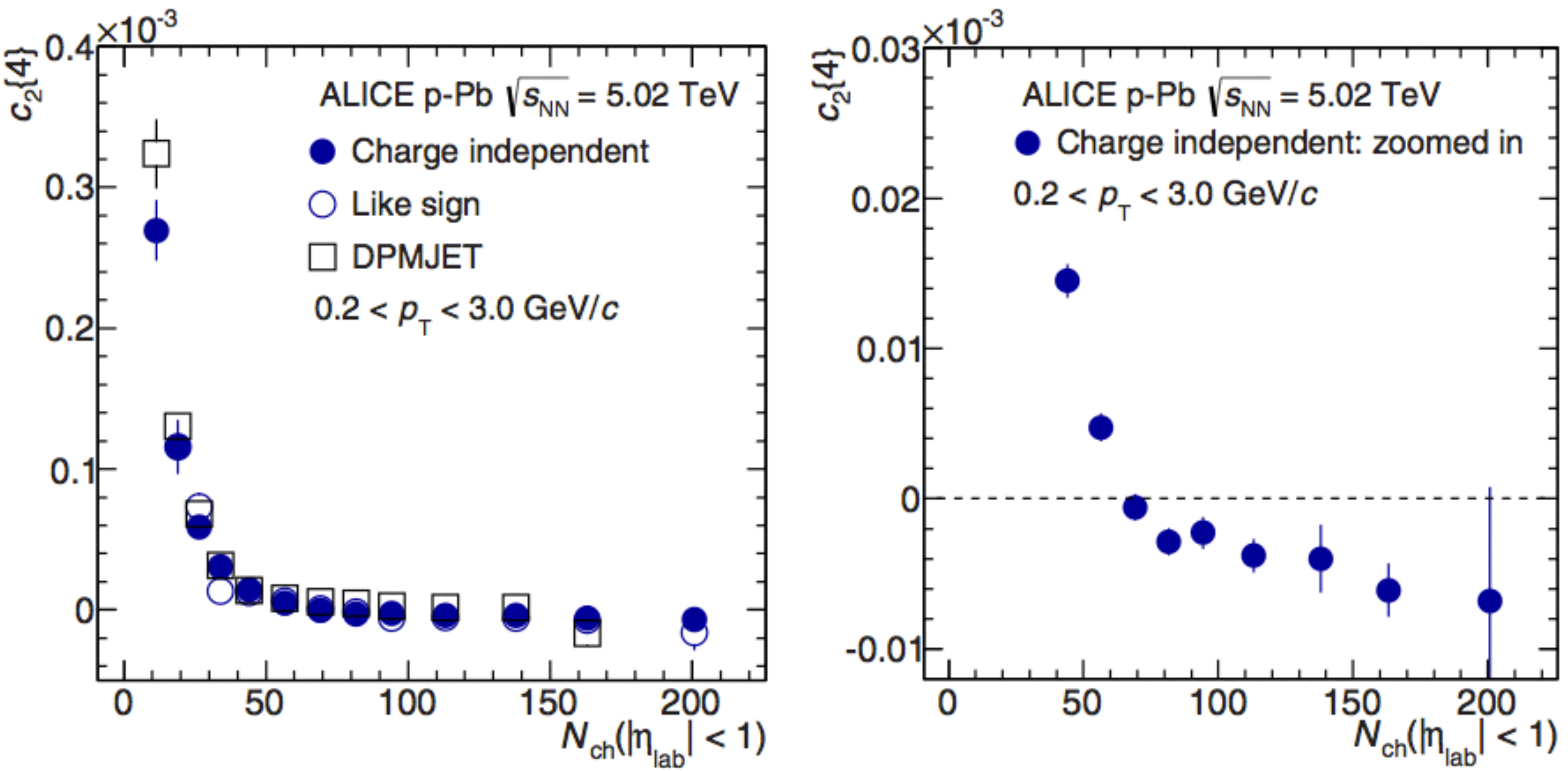

ALICE Coll., Phys. Rev. C **90** (2014) 054901

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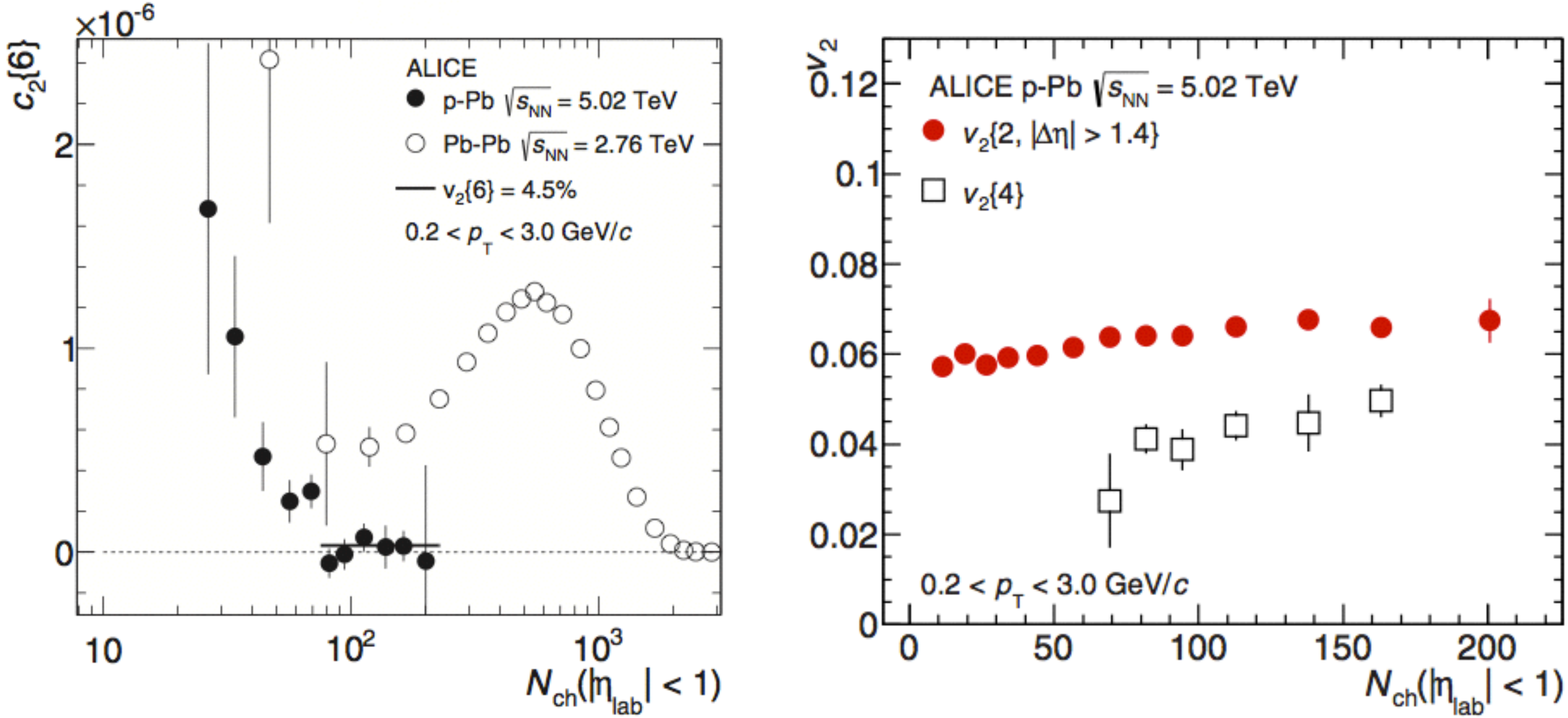

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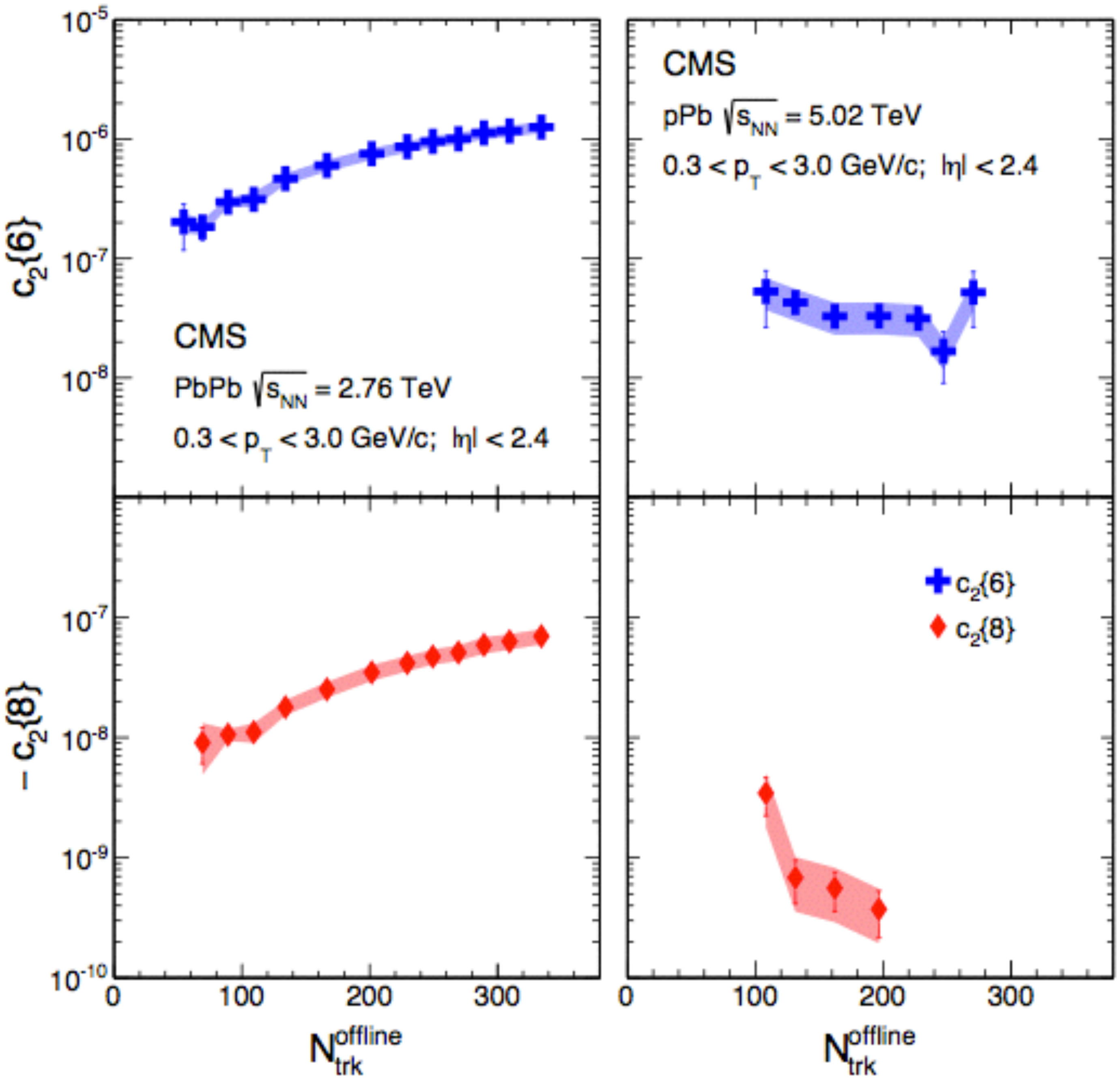

CMS Coll., arXiv:1502:05382

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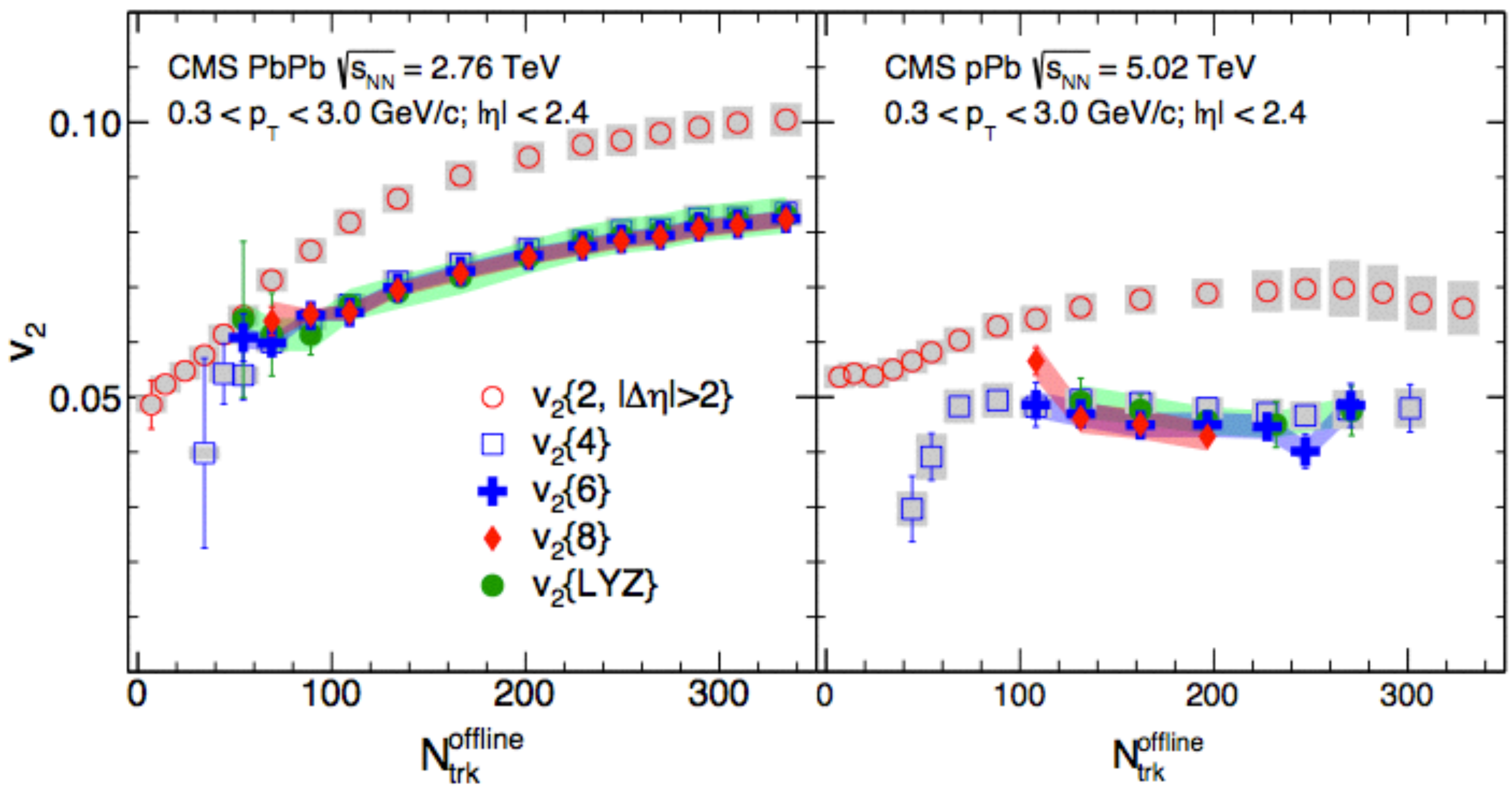

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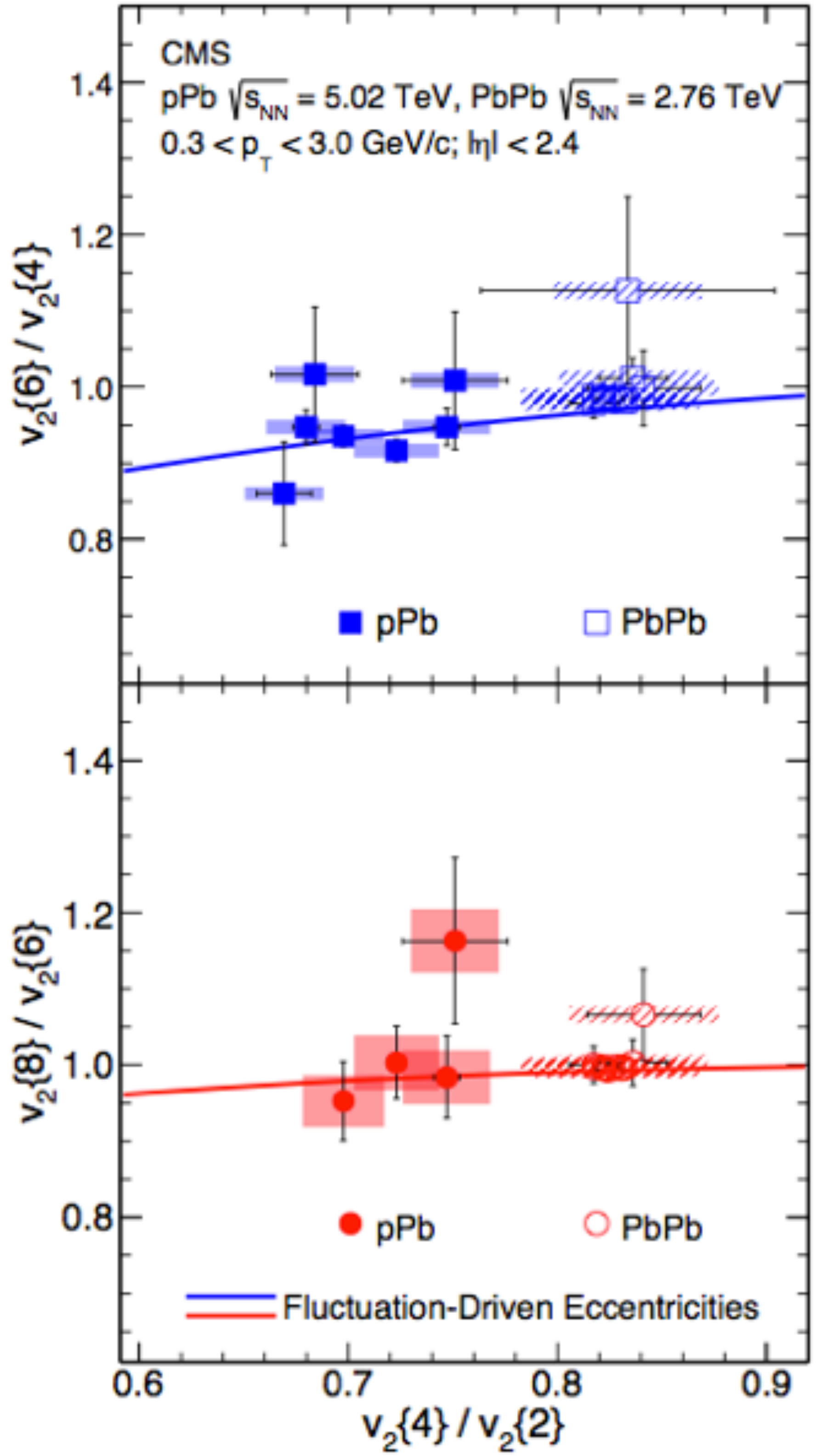

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Measuring anisotropic flow with multiparticle methods

- Average over M detected particles in $N_{\rm ev}$ events
  - ☛ define the "resolution parameter" $\chi \equiv v_n\sqrt{M}$
  - for "large" $\chi \gtrsim 1$, life is easy, the statistical fluctuations decrease like $1/\sqrt{M N_{\rm ev}}$ both on the $v_n\{k\}$ and on $v_n\{\infty\}$

    ☞ corresponds to the regime at LHC & high RHIC energies in collisions of large systems (measured with reasonable detectors...)!

  - when $\chi < 1$, things become more involved...

    ☞ what does this mean for small systems?

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Table 3 from NPA **727** (2003) 373 (page 411...)

Comparison between different methods: the relative statistical error on the integrated flow V_n is shown for the cumulant method [34], and various cumulant orders (denoted by $V_n\{2k\}$ with integer k), and for the present method ($V_n\{\infty\}$)

	$\chi = 0.6$	$\chi = 0.7$	$\chi = 0.8$	$\chi = 1$	$\chi = 1.5$
$\delta V_n\{2\}/V_n$	1.3%	1.0%	0.83%	0.62%	0.37%
$\delta V_n\{4\}/V_n$	4.5%	2.7%	1.8%	1.00%	0.43%
$\delta V_n\{6\}/V_n$	7.7%	3.7%	2.1%	0.99%	0.41%
$\delta V_n\{8\}/V_n$	9.9%	4.1%	2.1%	0.95%	0.41%
$\delta V_n\{\infty\}/V_n$	10.9%	3.9%	2.0%	0.94%	0.41%

As in the previous table, the number of events is $N_{\text{evts}} = 20000$ events, and the resolution parameter χ takes several values.

Multiparticle methods for measuring anisotropic flow in small systems

- Average over M detected particles in N_{ev} events
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  - when χ < 1, things become more involved:

    at small χ:

    - Gaussian errors on the cumulants $c_n\{2k\}$, with width $\dfrac{1}{\sqrt{M^{2k} N_{\text{ev}}}}$

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  ☞ measurement of v_n becomes more difficult;

  ☞ may spoil the attempts to pinpoint $p(v_n)$ (= the "physical" fluctuations of the measured signal).

Multiparticle methods for measuring anisotropic flow in small systems

- Average over M detected particles in N_{ev} events
  ☛ define the "resolution parameter" $\chi \equiv v_n \sqrt{M}$
  - when $\chi < 1$, things become more involved:
    at small χ:
    - Gaussian errors on the cumulants $c_n\{2k\}$, with width $\frac{1}{\sqrt{M^{2k} N_{ev}}}$
    - error on $v_n\{\infty\}$ grows exponentially!

Multiparticle methods for measuring anisotropic flow in small systems

An over-simplified rule of thumb, which emphasizes the role of χ would be the following:

from NPA **727** (2003) 373 (LYZ-method page 385)

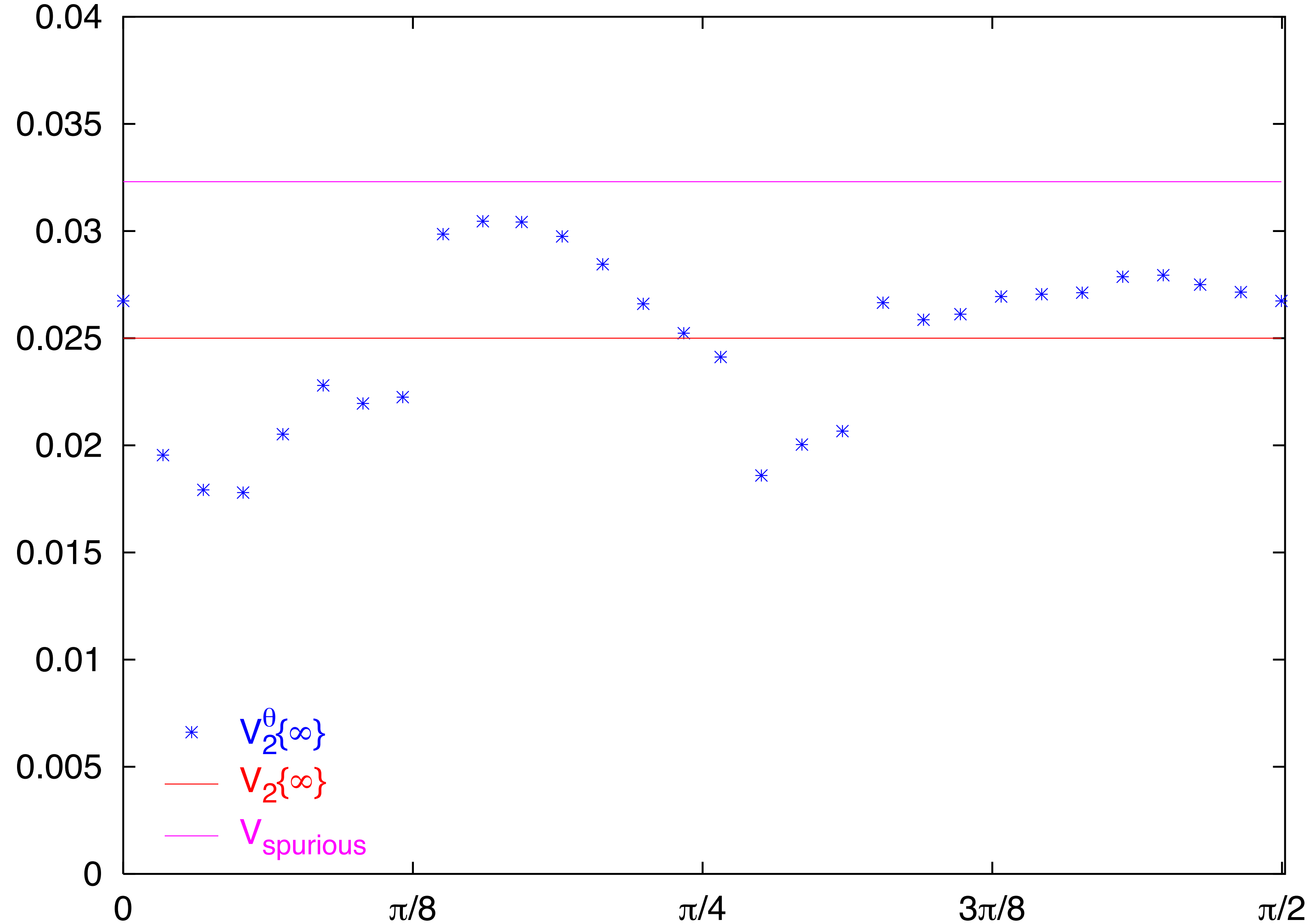
- If $\chi > 1$, the statistical error on the flow is not significantly larger than with the standard method (by a factor of at most 2, see Tables 3, 6 and 7), while systematic errors due to nonflow effects are much smaller. Thus, the present method should be used, and statistics will not be a problem;
- if $0.5 < \chi < 1$, the method can be used, but it really is most important to optimize weights, so as to increase χ, possibly by performing two analyses of the same data sets: in the first analysis, adopting (educated) guesses for the weights; and in the second pass, using as weights w_j the differential flow values obtained in the first analysis;
- if $\chi < 0.5$, statistical errors are too large, and the present method cannot be used; increasing the number of events barely helps here; in this case, one should use the cumulant methods of Refs. [34,35], which still apply if the number of events is large enough [18].

☛ statistical fluctuations (finite $N_{\rm ev}$) of the generating function!

Multiparticle methods for measuring anisotropic flow in small systems

Simulations (N_{ev} = 20000 — yet slow $\sqrt{\ln N_{ev}}$-dependence —, M = 300) without flow, analyzed with Lee–Yang zeroes

from NPA **727** (2003) 373



$$\chi \equiv v_n \sqrt{M}$$

$$\chi = \frac{j_{01}}{\sqrt{2 \ln N_{ev}}}$$

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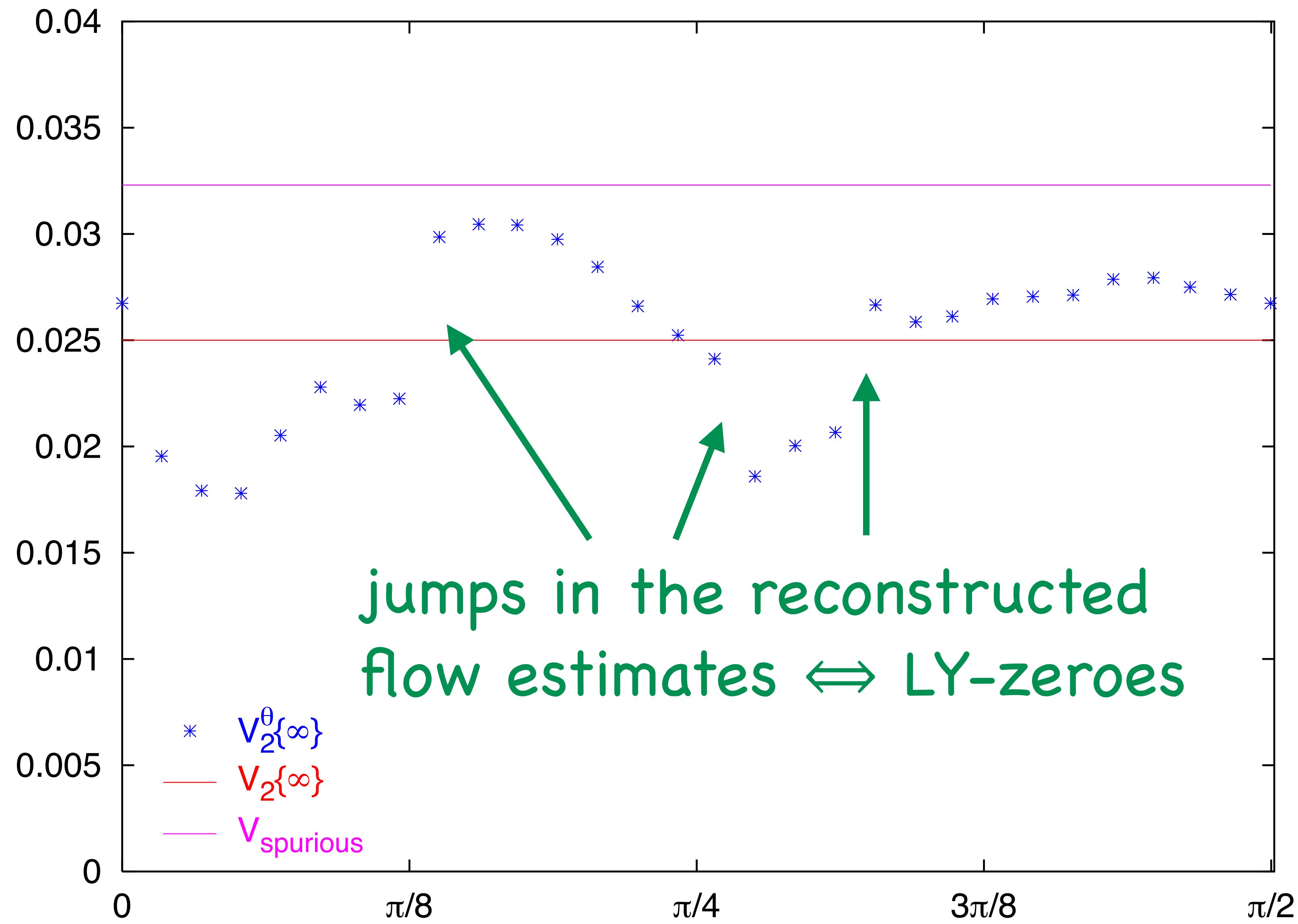

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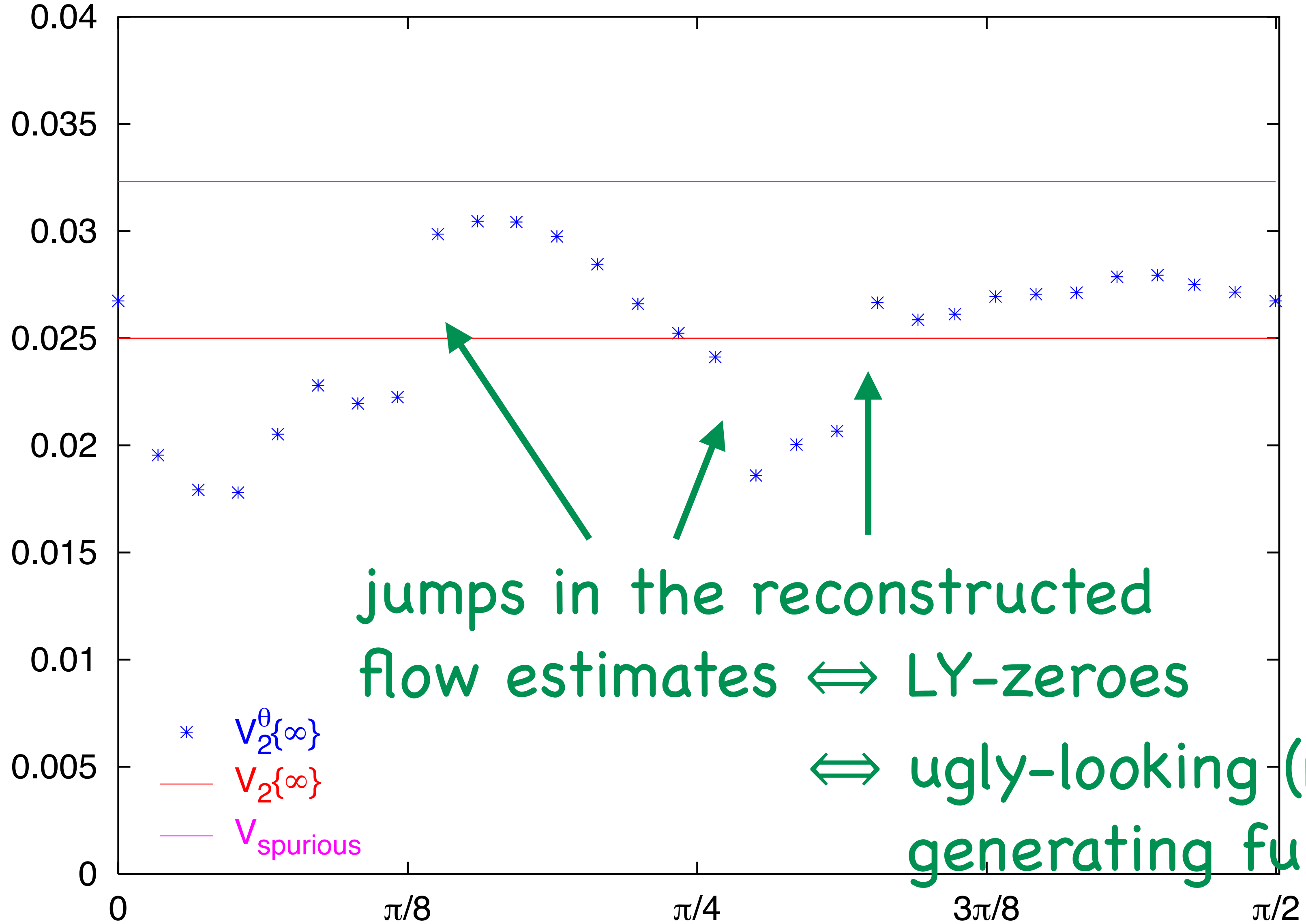

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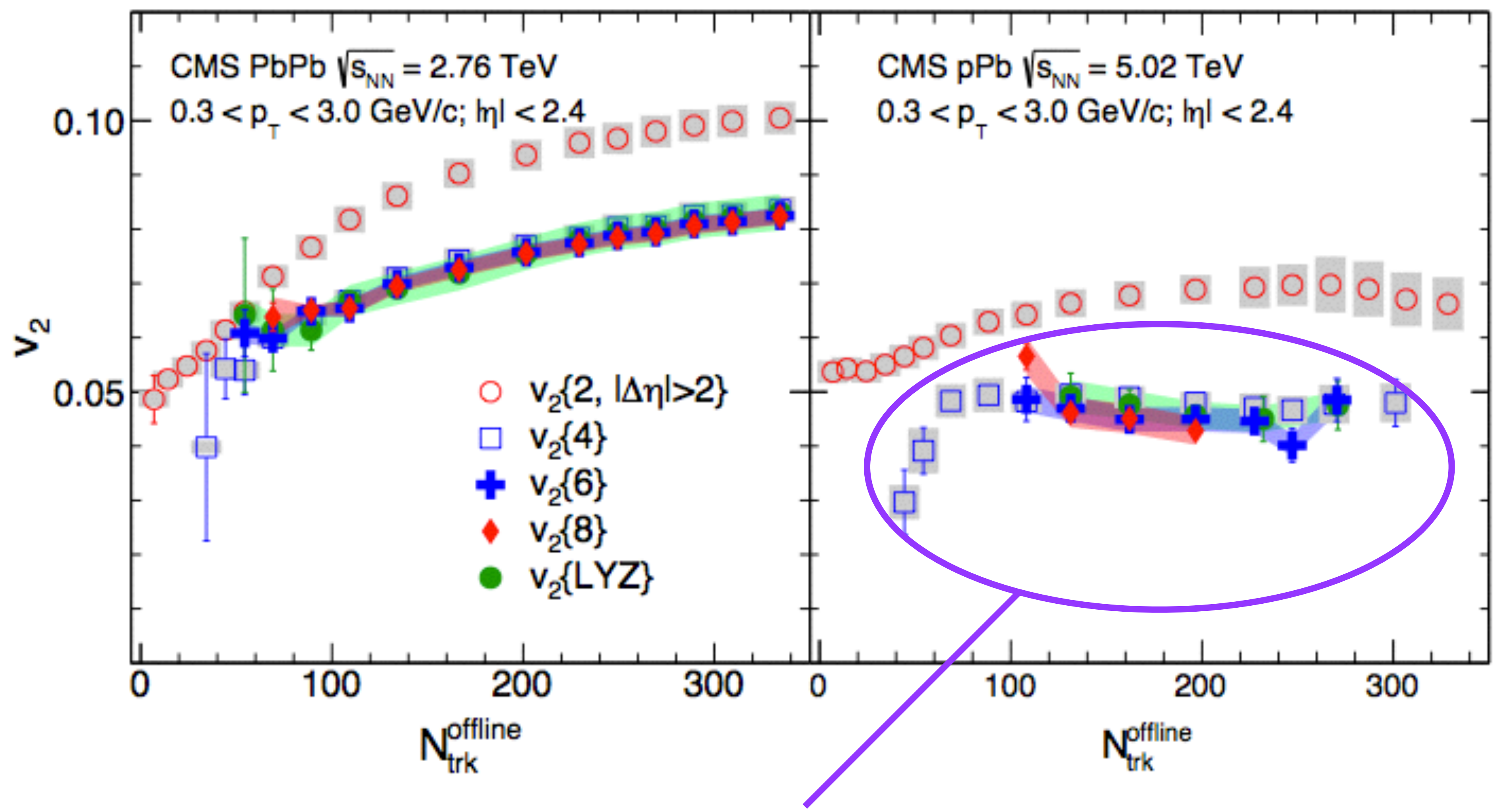
jumps in the reconstructed flow estimates ⟺ LY-zeroes

⟺ ugly-looking (not v_n-driven) generating functions

Multiparticle measurements of anisotropic flow in small systems



@CMS: Are you sure of your error bars?

remember $\chi \equiv v_n \sqrt{M}$

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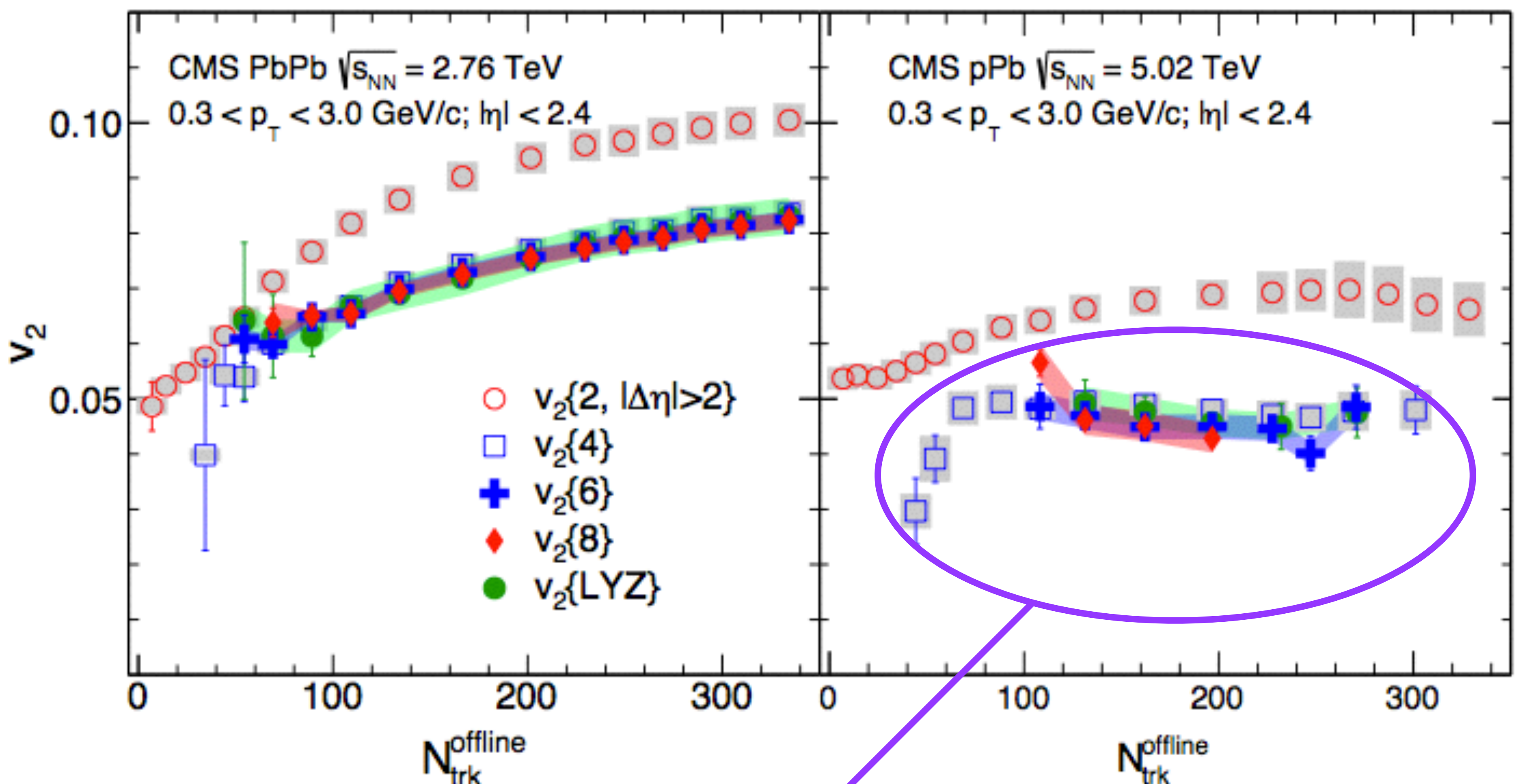

Do not ask cumulants / Lee-Yang-zeroes to provide you with any meaningful "onset of collectivity" (unfortunately!)

remember $\chi \equiv v_n \sqrt{M}$

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- Cumulants, Lee–Yang zeroes: a reminder
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  - newer uses

  ☛ minimize nonflow
  ☛ statistics not an issue (2002–?)
  ☛ reconstruct flow fluctuations

- Application to smaller(?) systems
  - overview of results
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  ☛ minimize nonflow
  ☛ statistics will strike back!

Multiparticle methods for measuring anisotropic flow

From large to small systems

- Event-plane method:
  - born 1984, refinements in 1993, 1997; many successful applications
  - criticized (nonflow) in 1999-2000, when applied to small v_n
- Cumulants (resp. Lee-Yang zeroes):
  - born 2000 (resp. 2003); many successful applications
  - now 15 (resp. 12) year old... Beware of limitations in small systems!