Phenomenology of high-energy nucleus-nucleus collisions

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High-energy nucleus-nucleus collisions: Time scale of the experiments

- March 1984: Lausanne workshop, where the idea of a Large Hadron Collider (LHC) at CERN was first officially discussed.
- about 1991: the possibility of letting heavy nuclei ("ions") collide inside the LHC is taken seriously; formation of the ALICE (A Large Ion Collider Experiment) project; later joined by CMS & ATLAS.
- November 4, 2010: ^{208}Pb nuclei are for the first time injected and accelerated in the LHC.
- November 7, 2010: First collisions of Pb nuclei in the LHC.
- November 8, 2010: The Pb beams are declared stable, the physics programme begins (until December 6); first papers on Nov. 17.
- November 14, 2011: Scheduled start of the second Pb-Pb run.

Phenomenology of high-energy nucleus-nucleus collisions

- What is the purpose of colliding heavy nuclei at high energy?
- An observable of nucleus-nucleus collisions: anisotropic flow
  - What kind of physical quantities can you hope to access through this observable?
  - How can you measure this observable?

☛ An introduction to the tools and trade of the phenomenologist.

Interlude

… for those of you who don't care about high-energy physics

You've all heard of the most mistaken physicist / scientist ever:

8:30 COUPÉ IN SILBER. 12:10 HERBST IN GOLD.

DIE ZEIT

WOCHENZEITUNG FÜR POLITIK WIRTSCHAFT WISSEN UND KULTUR

Europcar

29. September 2011 DIE ZEIT Nº 40

5 Wahrheiten über Europa

- Europa wird nie bürgernah sein SEITE 3
- Es liegt nicht nur an den Griechen SEITE 23/24
- Europa vergisst seine kulturellen Wurzeln SEITE 3
- Wie Brüssel in der Krise wirklich funktioniert SEITE 3
- Es geht nicht mehr um Krieg und Frieden SEITE 4

Das andere Paris

Liebeserklärung an eine Stadt, die sich gerade neu erfindet ZEITmagazin

Das gelobte Land

Innovativ, weltoffen, effizient: Für Menschen aus ganz Europa ist Deutschland der neue Sehnsuchtsort

Dossier Seite 15–19

Fragt das Volk!

Es gibt ein Demokratie- und Legitimationsdefizit in Europa. In Deutschland würde ein Referendum helfen VON HEINRICH WEFING

Wenn Einstein irrt

Nun gerät auch noch die Relativitätstheorie ins Wanken. Für die Wissenschaft ist das kein großes Problem VON ANDREAS SENTKER

Abi und los!

Was man für das Leben nach der Schule wissen muss

56 Seiten Chancen-Spezial

Interlude

… for those of you who don't care about high-energy physics

You've all heard of the most mistaken physicist / scientist ever, and in particular of his most speculative theory called Special Relativity.

Zur Elektrodynamik bewegter Körper, Ann. Phys. (Leipzig) **17** (1905) 891

Zur Elektrodynamik bewegter Körper. 903

§ 4. Physikalische Bedeutung der erhaltenen Gleichungen, bewegte starre Körper und bewegte Uhren betreffend.

Wir betrachten eine starre Kugel[1]) vom Radius R, welche relativ zum bewegten System k ruht, und deren Mittelpunkt im Koordinatenursprung von k liegt. Die Gleichung der Oberfläche dieser relativ zum System K mit der Geschwindigkeit v bewegten Kugel ist:

⋮

auch jedes starren Körpers von beliebiger Gestalt) durch die Bewegung nicht modifiziert erscheinen, erscheint die X-Dimension im Verhältnis $1 : \sqrt{1-(v/V)^2}$ verkürzt, also um so stärker, je größer v ist. Für $v = V$ schrumpfen alle bewegten Objekte — vom „ruhenden" System aus betrachtet — in flächenhafte Gebilde zusammen. Für Überlichtgeschwindigkeiten werden

He even made a few predictions…

- Time-dilation seems to be well tested experimentally (lifetime of the muon…)

QUIZ!

- Do you know any experimental verification of the so-called "Lorentz-contraction" of lengths along the direction of motion?

Phenomenology of high-energy nucleus-nucleus collisions

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- An observable of nucleus-nucleus collisions: anisotropic flow

  - What kind of physical quantities can you hope to access through this observable?

  - How can you measure this observable?

☛ An introduction to the tools and trade of the phenomenologist.

Why high-energy heavy-ion collisions?

- What is the purpose of colliding heavy nuclei at high energy?
  - Because we can!
  - To create a medium with an extraordinarily high energy density, i.e. possibly a new state of matter with novel properties.

Phenomenology, lesson 1: know the scales of your problem

A few scales & units to keep in mind...

- Radius of nucleus with atomic mass number A: $R_A \approx 1.1\, A^{1/3}$ fm

  1 fm (femtometer / Fermi) = 10^{-15} m

  ☞ for ^{208}Pb, $R_{\rm Pb} \approx 6{,}8$ fm

- The corresponding "natural" time scale is $\dfrac{R_{\rm Pb}}{c} = 6{,}8\ {\rm fm}/c$ (!) ≈ 23 ys

  1 fm/$c \approx 3{,}3$ ys (yoctosecond) = $3{,}3 \cdot 10^{-24}$ s

- Mass of the ^{208}Pb nucleus $m_{\rm Pb} \approx 208 m_{\rm N}$

  with $m_{\rm N}$ = 0,939 GeV/c^2 = $1.67 \cdot 10^{-27}$ kg

☛ typical length, time, mass scales: fm, fm/c, GeV/c^2.

A few scales & units to keep in mind...

- What does "high-energy collisions" mean?
  - ☞ in 2010-2011 the kinetic energy of a ^{208}Pb nucleus at LHC is

    $E_{\text{kin}} = 287$ TeV = 208×1.38 TeV = $1481\, m_{\text{Pb}}\, c^2$

    ultrarelativistic regime! $v_{\text{Pb}} = (1 - 0{,}23 \cdot 10^{-6})\, c$
  - ☞ in a single Pb-Pb collision at LHC, the available energy is $2E_{\text{kin}}$.

- If 20% of this energy is deposited in a volume of about 1000 fm^3, then the energy density in this volume is $e \approx 100$ GeV/fm^3.

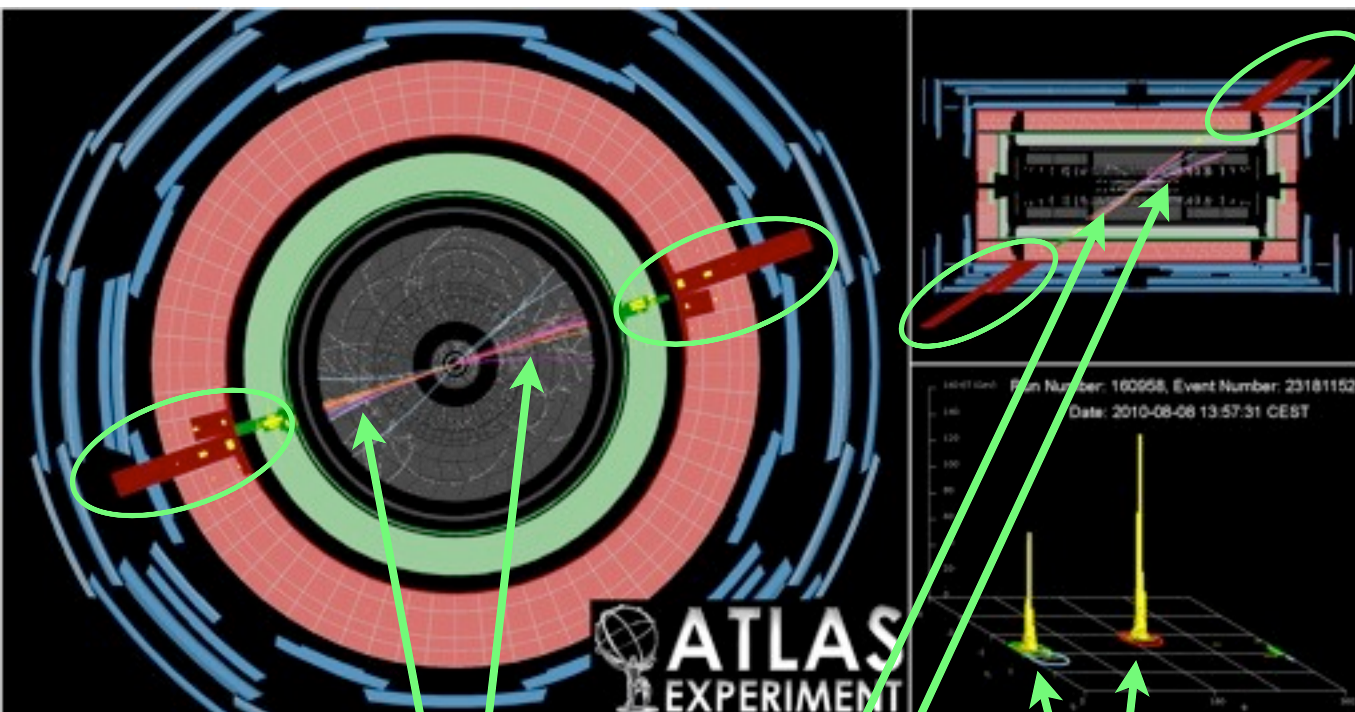
- Such an energy density $e \approx 100$ GeV/fm^3 amounts to a temperature $k_B T \approx 500$ MeV, that is $T \approx 6 \cdot 10^{12}$ K ($\gg 15 \cdot 10^6$ K at the Sun center)

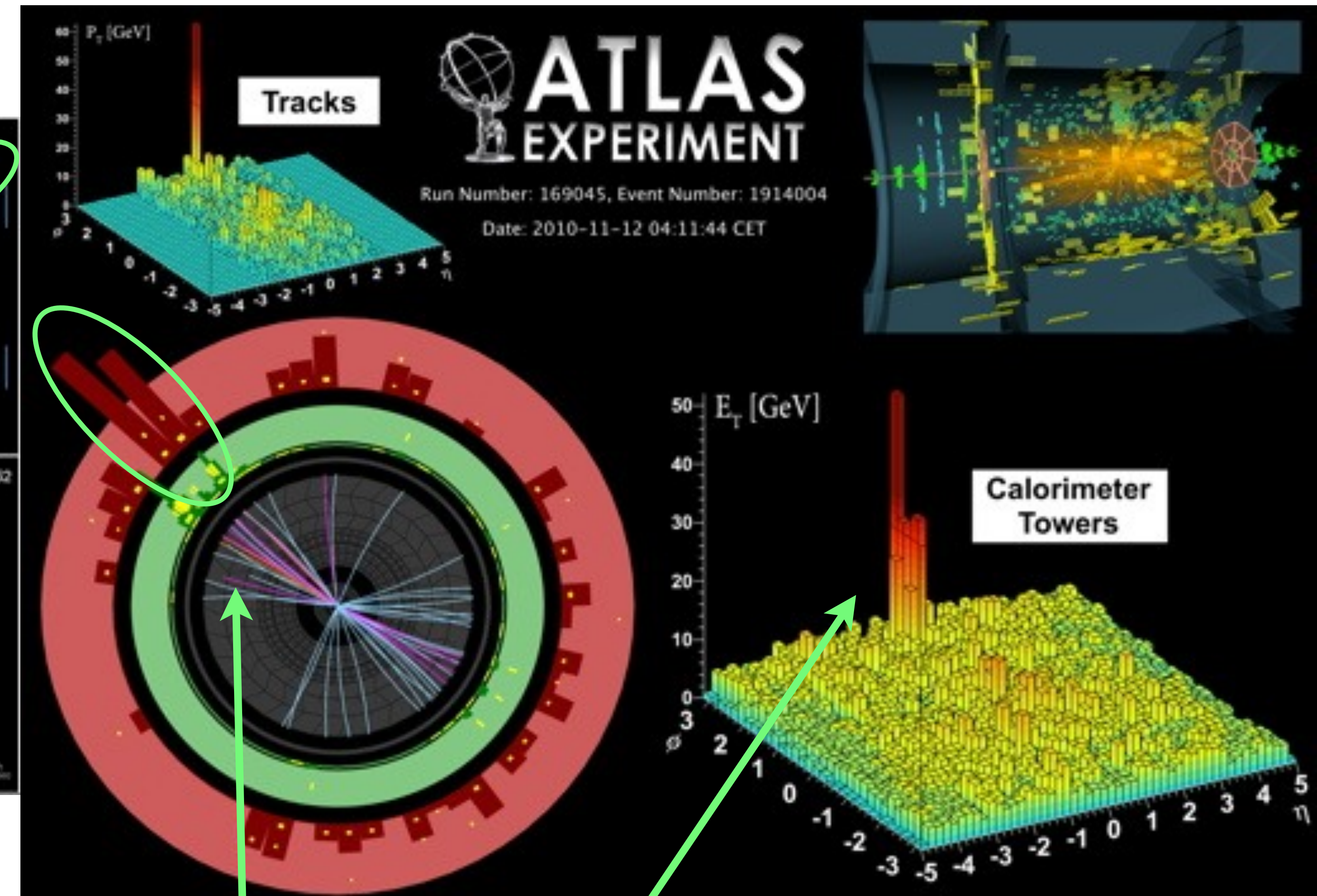
☛ hot (& dense) medium: "new state of matter"

Is there evidence for this medium?

proton-proton collision at the LHC

Pb-Pb collision at the LHC





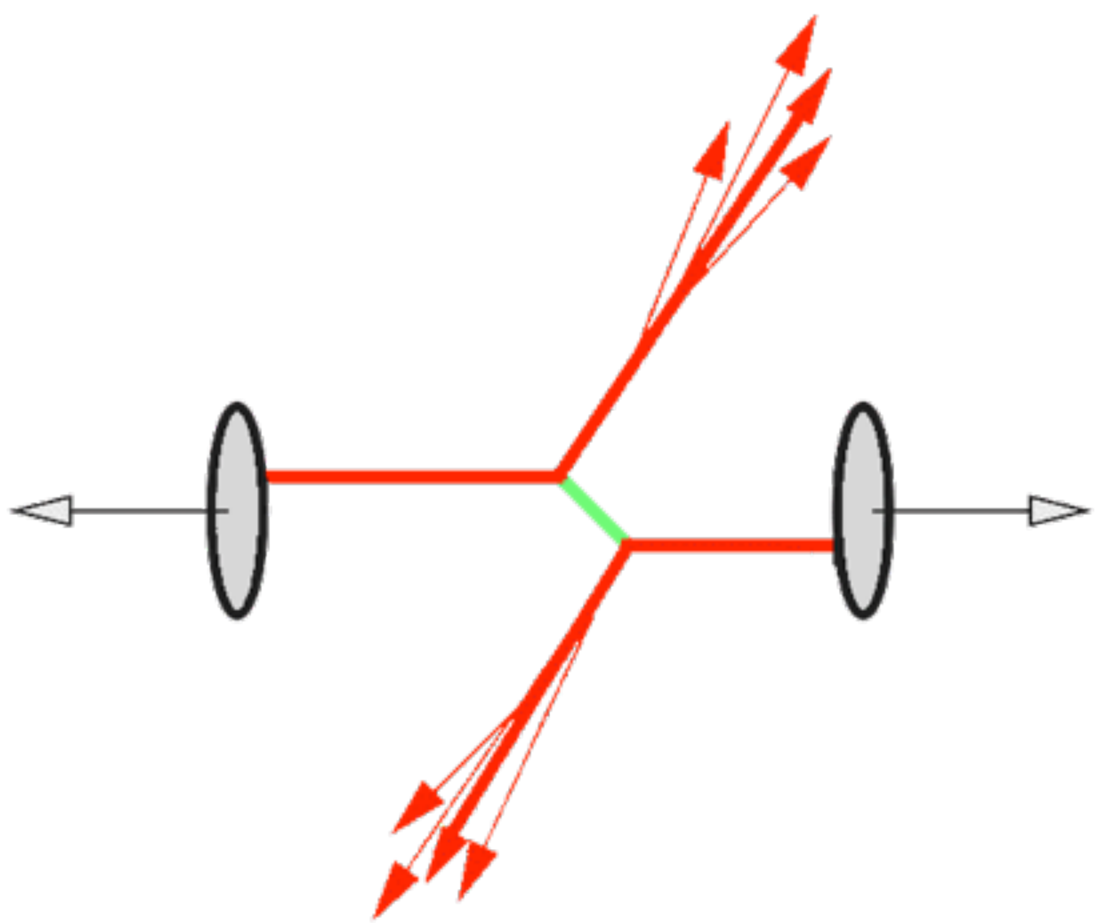
2 back-to-back "jets" of highly energetic particles, that deposit energy in calorimeters

a single "jet", which has lost its back-to-back counterpart...

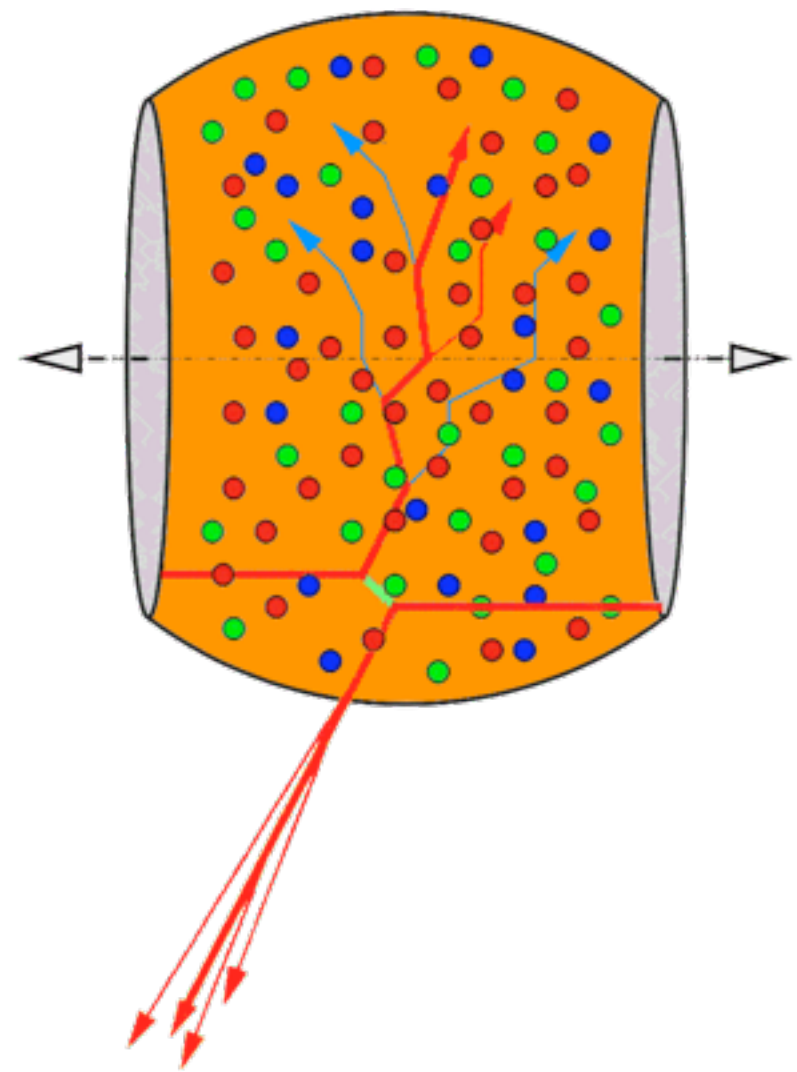
ATLAS Collaboration, Phys. Rev. Lett. **105** (2010) 252303

Is there evidence for this medium?

proton-proton collision
at the LHC



Pb-Pb collision at the LHC



While propagating through the hot and dense medium, the "jet"-to-be has dissipated part of its energy, and does not emerge as a jet!

Is there evidence for this medium?

YES

There is a very opaque medium,
which can stop jets over short distances

What are its properties?

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Evolution of the medium in a Pb–Pb collision at LHC

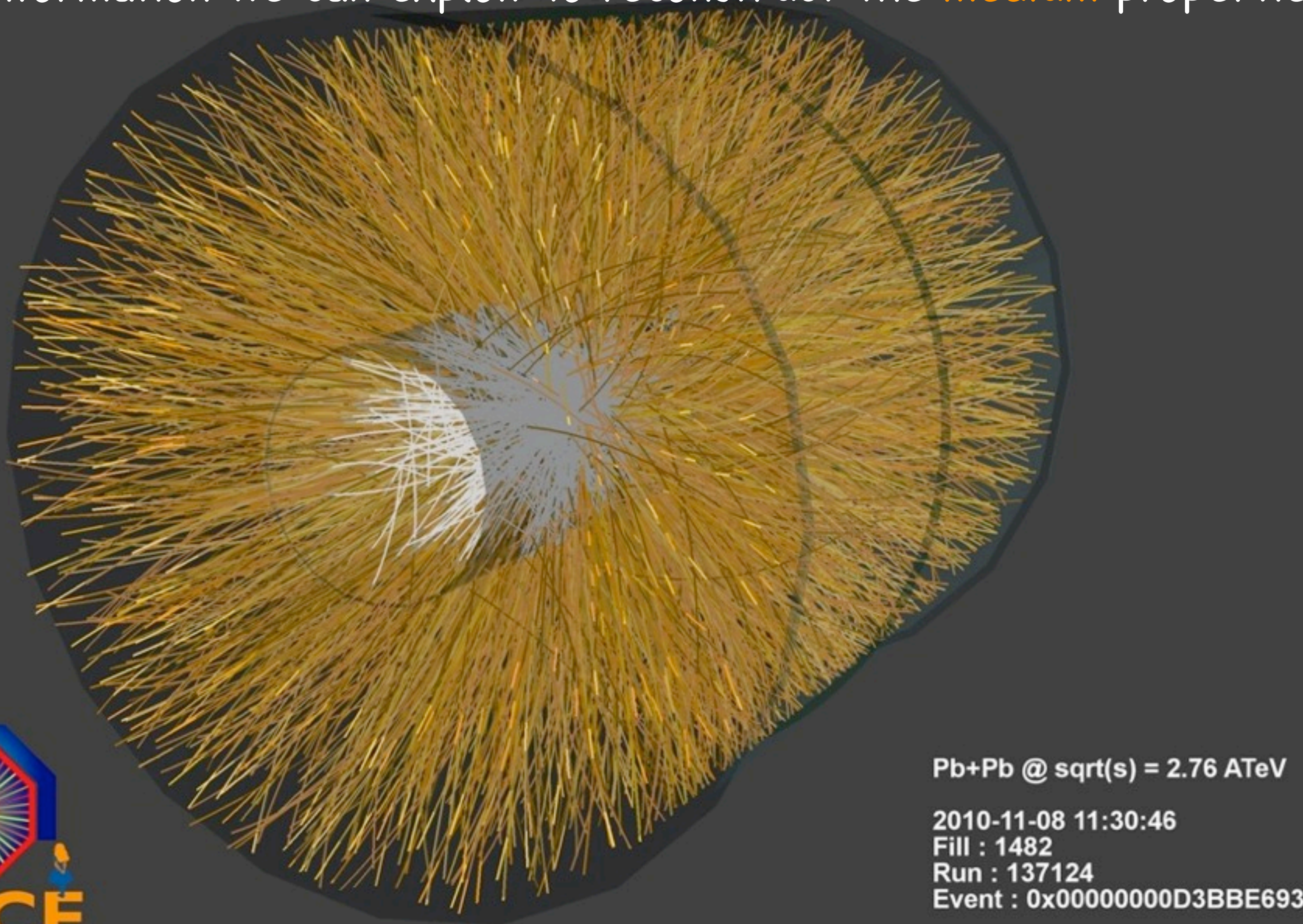
(a sketch!)

- At $t = 0$, the Pb nuclei collide: "event" (10^5–10^7 events in a month run)
  - some of their internal constituents are stopped and set free from the nuclei wavefunctions ☞ QCD fields
  - at $t = 0^+$, the remnants of the nuclei fly away.
- First few fm/c: the liberated degrees of freedom form a "fireball"
  - which rapidly expands and cools down: collective behavior;
  - whose content (relevant degrees of freedom) evolves.
- At $t \approx$ 10–20 fm/c, the fireball stops behaving collectively, particles fly freely to the detectors.

☞ about $2\text{–}10 \cdot 10^3$ particles per event

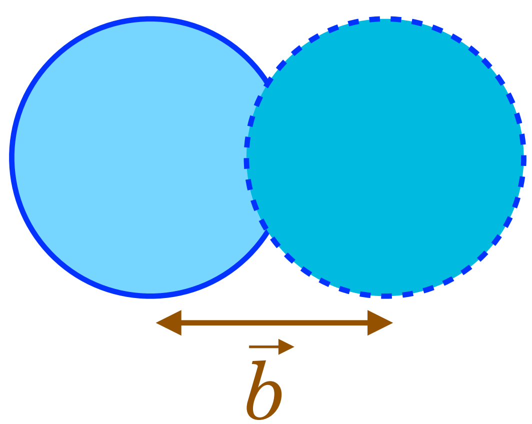
About 2-10·10^3 particles per event…

= the information we can exploit to reconstruct the medium properties

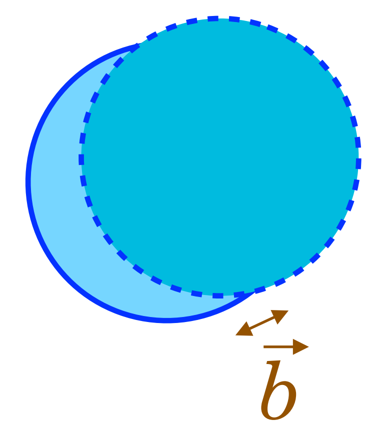


Phenomenology, lesson 2: pay attention to symmetries

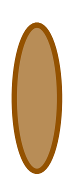
- At $t = 0$, the Pb nuclei collide:
  a typical event looks like this:

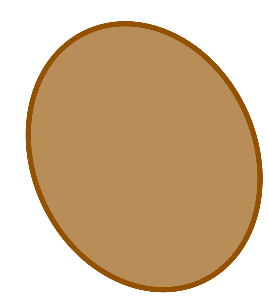


… or this:



i.e. with a non-zero impact parameter: the overlap region of the nuclei is almond-shaped (in the "transverse plane" perpendicular to the beam axis):



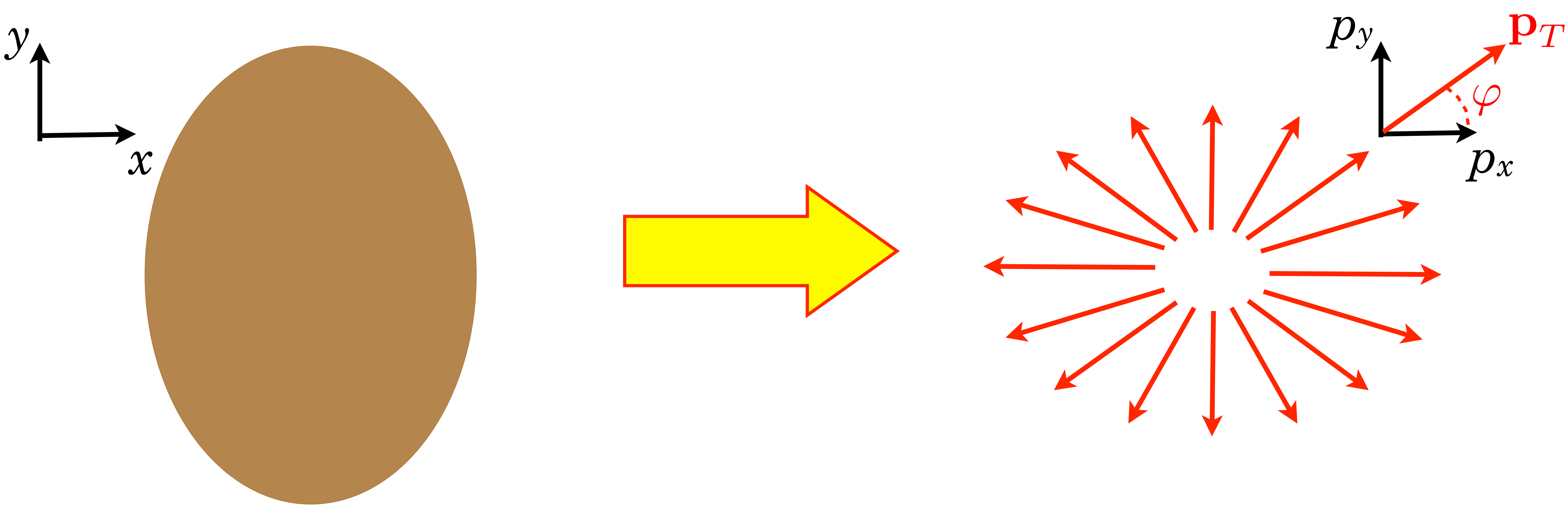


Anisotropic initial state ☛ expect an anisotropic final state!

Anisotropic flow

Initial state asymmetry: in position space (e.g. spatial eccentricity)

≠ final state anisotropy: in momentum space



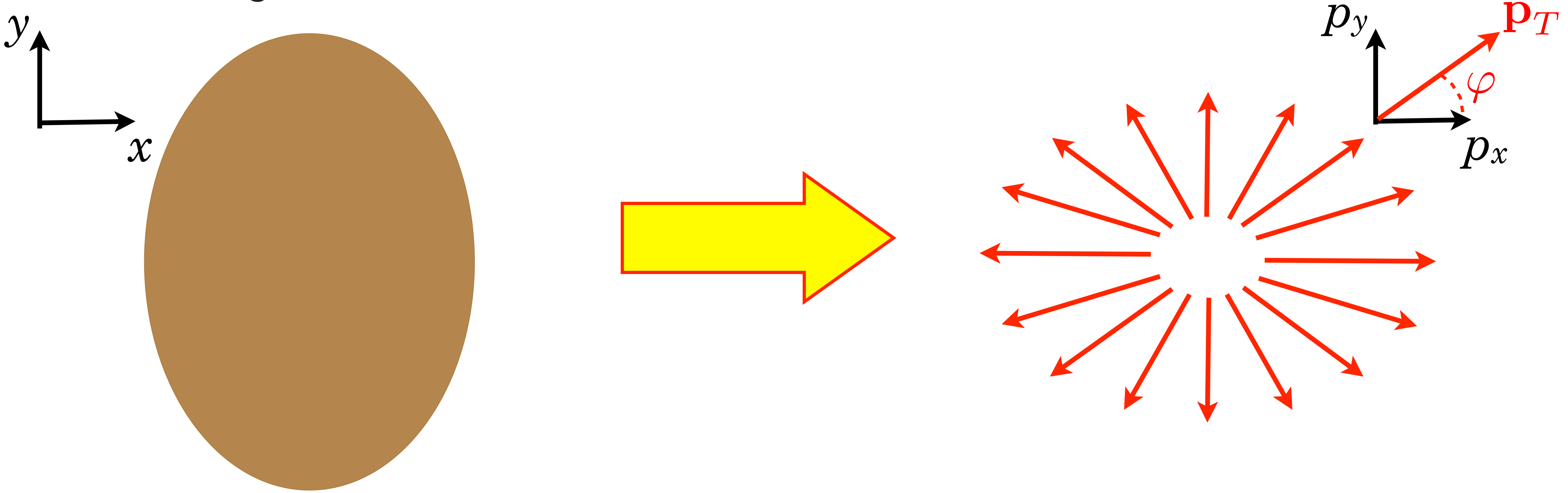
This "anisotropic flow" is quantified by the Fourier coefficients of the transverse-momentum distribution

$$\frac{\mathrm{d}^2 N}{\mathrm{d}^2\mathbf{p}_T} = \frac{1}{2\pi}\frac{\mathrm{d}N}{p_T\,\mathrm{d}p_T}\left[1 + \sum_{n=1}^{\infty} 2v_n(p_T)\cos n\varphi\right]$$

Anisotropic flow

$$\frac{\mathrm{d}^2 N}{\mathrm{d}^2\mathbf{p}_T} = \frac{1}{2\pi}\frac{\mathrm{d}N}{p_T\,\mathrm{d}p_T}\left[1+\sum_{n=1}^{\infty} 2v_n(p_T)\cos n\varphi\right]$$

This is a highly non-trivial effect!



Anisotropic flow

$$\frac{\mathrm{d}^2 N}{\mathrm{d}^2\mathbf{p}_T} = \frac{1}{2\pi}\frac{\mathrm{d}N}{p_T\,\mathrm{d}p_T}\left[1 + \sum_{n=1}^{\infty} 2 v_n(p_T)\cos n\varphi\right]$$

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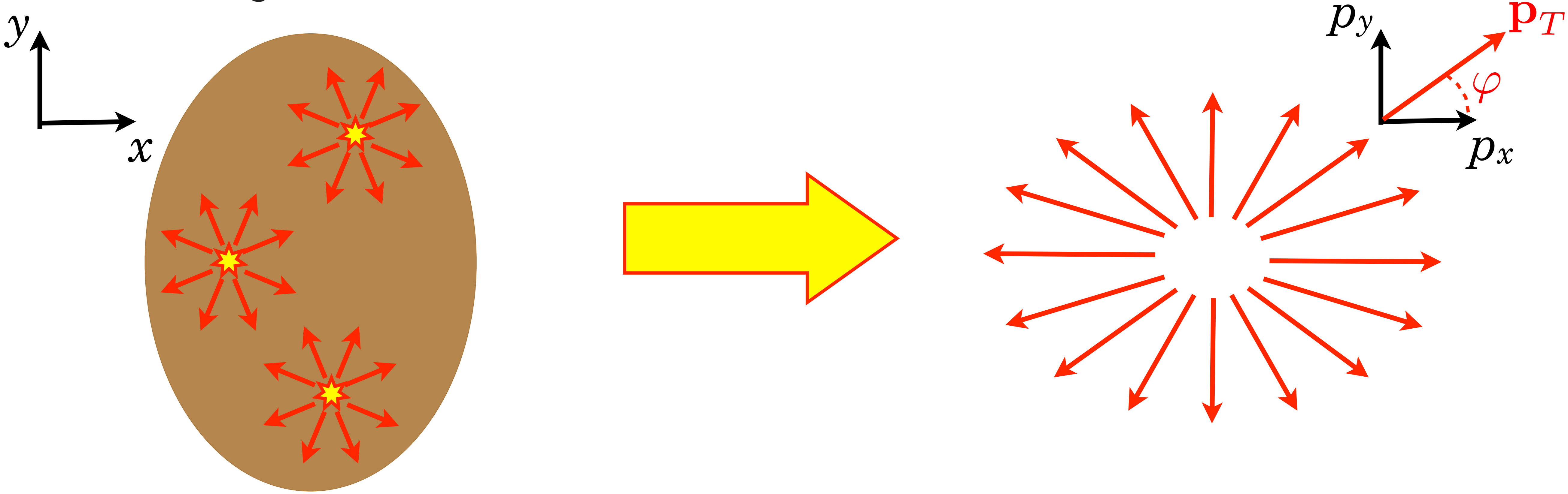


The initial collisions between the nuclei constituents do not know the impact parameter of the Pb-Pb collision and emit particles isotropically.

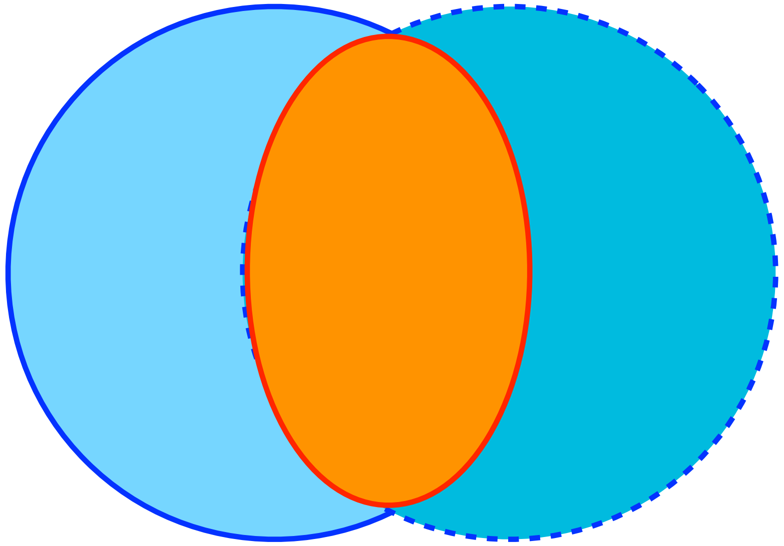
⇒ anisotropic flow is a collective behavior, caused by rescatterings.

N.B., C.Gombeaud, Eur. Phys. J. **71** (2011) 1612

Phenomenology, lesson 3: investigate alternative descriptions

View the fireball as a continuous medium ("fluid") instead of particles.

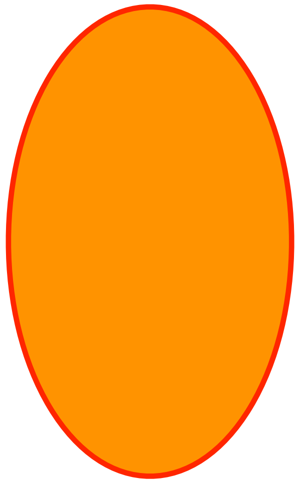
- At $t = 0$, the nuclei collide



Phenomenology, lesson 3: investigate alternative descriptions

View the fireball as a continuous medium ("fluid") instead of particles.

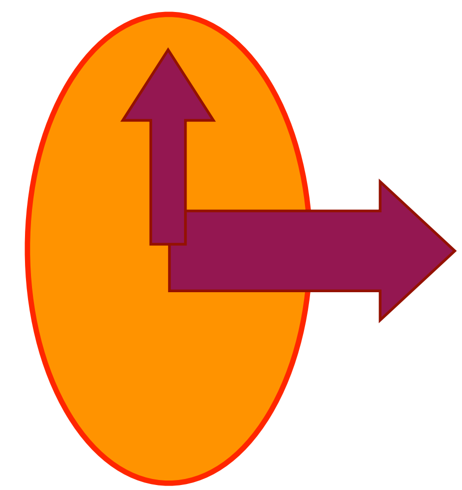
- At $t = 0$, the nuclei collide
- At the LHC, the nuclei remnants quickly fly away



Phenomenology, lesson 3: investigate alternative descriptions

View the fireball as a continuous medium ("fluid") instead of particles.

- At $t = 0$, the nuclei collide
- At the LHC, the nuclei remnants quickly fly away
- In the medium, there is a non-zero pressure; outside, there is vacuum: the pressure gradient is larger along the impact parameter direction (φ = 0 or 180°) than perpendicular to it.



⇒ the fluid accelerates more in the impact parameter direction

(cf. the Euler equation $\rho\left[\frac{\partial\vec{v}}{\partial t} + (\vec{v}\cdot\vec{\nabla})\vec{v}\right] = -\vec{\nabla}p$)

☛ anisotropic momentum distribution, $v_2 = \langle\cos 2\varphi\rangle > 0$, with $\langle\cdot\rangle$ an average over many particles and events.

Anisotropic flow in a fluid picture

View the fireball as a continuous medium, described by the laws of (relativistic) fluid dynamics.

The initial spatial asymmetry of the fluid evolves into an anisotropic velocity pattern.

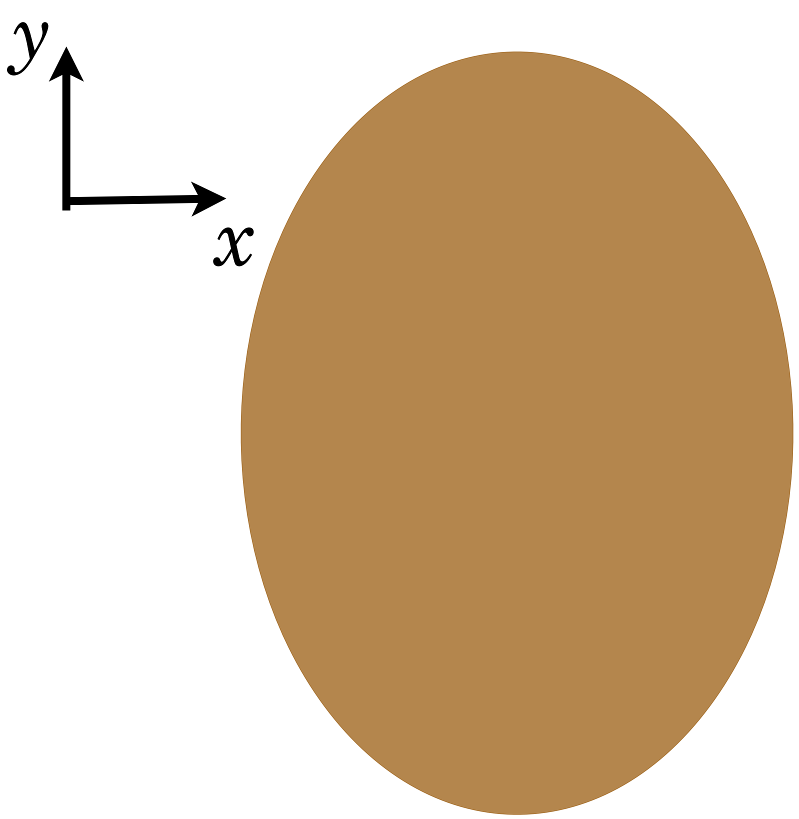
The size of the anisotropy (here, the v_n coefficients) depends on the medium characteristics entering the dynamical equations:

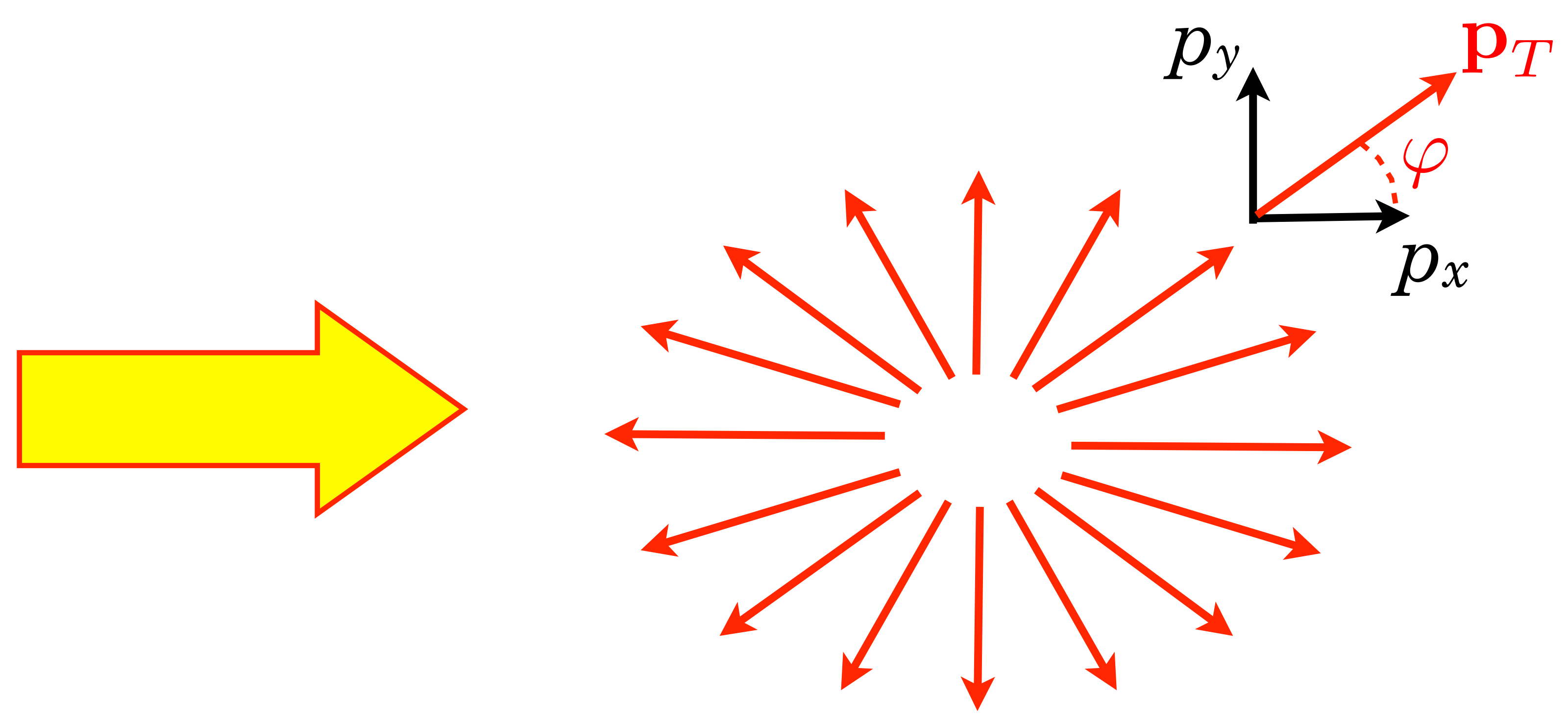
- equation of state;
- transport coefficients (viscosity...) of the fluid.

☛ Measuring the v_n should give access to these quantities!

A small experimental detail

$$\frac{d^2N}{d^2\mathbf{p}_T} = \frac{1}{2\pi}\frac{dN}{p_T\,dp_T}\left[1 + \sum_{n=1}^{\infty} 2v_n(p_T)\cos n\varphi\right]$$





The Fourier coefficients $v_n = \langle\cos n\varphi\rangle$ are defined using azimuths with respect to the impact parameter direction.

This direction is not known, and fluctuates from event to event!

A small experimental detail

Picture by Mona Schweizer

The closest detector to the collision point is 39 mm = $3{,}9 \cdot 10^{13}$ fm away.

Try to measure the orientation of the Earth axis from 6000 light-years away!

Measuring anisotropic flow: a nice problem for a theorist

$$\frac{\mathrm{d}^2N}{\mathrm{d}^2\mathbf{p}_T} = \frac{1}{2\pi}\frac{\mathrm{d}N}{p_T\,\mathrm{d}p_T}\left[1+\sum_{n=1}^{\infty}2v_n(p_T)\cos n(\varphi-\Phi_R)\right]$$

with $v_n = \langle\cos n(\varphi-\Phi_R)\rangle$, φ and Φ_R angles in a fixed frame (lab), φ measured, Φ_R not measured and varying from event to event.

Physical assumption (A):
For a fixed orientation Φ_R and a fixed value b of the impact parameter, each particle in the system is correlated to only a small number of other particles.
Moreover, this number varies only weakly with nuclear size and b.

Reasonable assumption (emission of "clusters" = resonances, jets...).

... except in the vicinity of a second-order phase-transition!

Measuring anisotropic flow

Under assumption (A), define a "generating function":

$$G_n(z) \equiv \left\langle \prod_{j=1}^{M} (1 + z \cos n\varphi_j) \right\rangle$$

where the product runs over all (detected) particles in an event.

Angular brackets denote an average over (infinitely) many events with the same impact parameter value.

A convenient approach:

- Average first over events with the same impact parameter orientation; the corresponding averages will be denoted by $\langle \cdots | \Phi_R \rangle$.
- Then average over Φ_R, assuming its distribution is isotropic.

Measuring anisotropic flow

Consider first events with a fixed impact parameter orientation.

According to hypothesis (A), each event can be split into N independent subsystems. One may write

$$\prod_{j=1}^{M}(1+z\cos n\varphi_j) = \prod_{k=1}^{N}\prod_{j_k}(1+z\cos n\varphi_{j_k})$$

The fixed-Φ_R average is then straightforward:

$$\left\langle \prod_{j=1}^{M}(1+z\cos n\varphi_j)\,\middle|\,\Phi_R \right\rangle = \prod_{k=1}^{N}\left\langle \prod_{j_k}(1+z\cos n\varphi_{j_k})\,\middle|\,\Phi_R \right\rangle$$

If there is no anisotropic flow, Φ_R plays no role, fixed-Φ_R averages are actually independent of Φ_R, and $G_n(z)$ factorizes into the product of generating functions for (independent) subsystems.

The positions of its zeroes do not depend on the multiplicity M, i.e. the first zero of $G_n(z)$ is at $|z_0| = \mathcal{O}(1)$.

Measuring anisotropic flow

Consider now collisions with anisotropic flow.

Assuming $|z| \ll 1$, one may write

$$\ln\left\langle \prod_{j=1}^{M}(1+z\cos n\varphi_j)\,\middle|\,\Phi_R\right\rangle \simeq \left\langle \sum_{j=1}^{M}\cos n\varphi_j\,\middle|\,\Phi_R\right\rangle z = Mv_n\cos(n\Phi_R)z$$

so that
$$\left\langle \prod_{j=1}^{M}(1+z\cos n\varphi_j)\,\middle|\,\Phi_R\right\rangle = \mathrm{e}^{Mv_n\cos(n\Phi_R)z}$$

This is easily averaged over Φ_R, yielding

$$G_n(z) = \int_0^{2\pi}\frac{\mathrm{d}\Phi_R}{2\pi}\left\langle \prod_{j=1}^{M}(1+z\cos n\varphi_j)\,\middle|\,\Phi_R\right\rangle = I_0(Mv_n|z|)$$

☛ The positions of the zeroes of $G_n(z)$ now obviously depend on M !
When M is known, finding the first zero (for instance) gives v_n.

R.S.Bhalerao, N.B., J.-Y.Ollitrault, Nucl. Phys. A **727** (2003) 373

Anisotropic flow is a collective effect

Generating function $G_n(z) \equiv \left\langle \prod_{j=1}^{M} (1 + z\cos n\varphi_j) \right\rangle$

- In the absence of anisotropic flow, the position of the first zero is at some $|z_0| = \mathcal{O}(1)$, independent of the system size M.
- If there is anisotropic flow, the first zero lies at $z_0 = \dfrac{\mathrm{i}j_{01}}{Mv_n}$, where j_{01} denotes the first zero of the Bessel function J_0.

Does this remind you of something?

Phenomenology, lesson 4: scientific culture does not harm

A theory of phase transitions

C.N.Yang & T.D.Lee, Phys. Rev. **87** (1952) 404

- Grand partition function: $Z(T,\mu,\mathcal{V}) = \sum_{N=0}^{+\infty} Z_N(T,\mathcal{V})\, \mathrm{e}^{\mu N/k_BT}$
- Take a reference value μ_c, define $z \equiv (\mu-\mu_c)/k_BT$
- Define $\mathcal{G}(z) \equiv \dfrac{Z(T,\mu,\mathcal{V})}{Z(T,\mu_c,\mathcal{V})} = \sum_{N=0}^{+\infty} P_N \mathrm{e}^{zN}$

  P_N is the probability to find N particles in the system at $\mu = \mu_c$
- Let the system size $\mathcal{V}$ increase:
  - if there is no phase transition, the zeroes of $\mathcal{G}$ are unchanged;
  - if there is a phase transition at $\mu = \mu_c$, the zeroes come closer to the origin.

A phase transition is a collective phenomenon... like anisotropic flow!

Measuring anisotropic flow

Generating function $G_n(z) \equiv \left\langle \prod_{j=1}^{M} (1 + z\cos n\varphi_j) \right\rangle$

- In the absence of anisotropic flow, the position of the first zero is at some $|z_0| = \mathcal{O}(1)$, independent of the system size M.
- If there is anisotropic flow, the first zero lies at $z_0 = \frac{\mathrm{i}\, j_{01}}{M v_n}$, where j_{01} denotes the first zero of the Bessel function J_0.

☛ Method for measuring v_n with Lee–Yang zeroes:

- Compute the generating function using the measured azimuths.
- Find its first zero (or rather the first minimum of its absolute value).
- Since you know M and $j_{01} = 2.40483\ldots$ you know v_n.

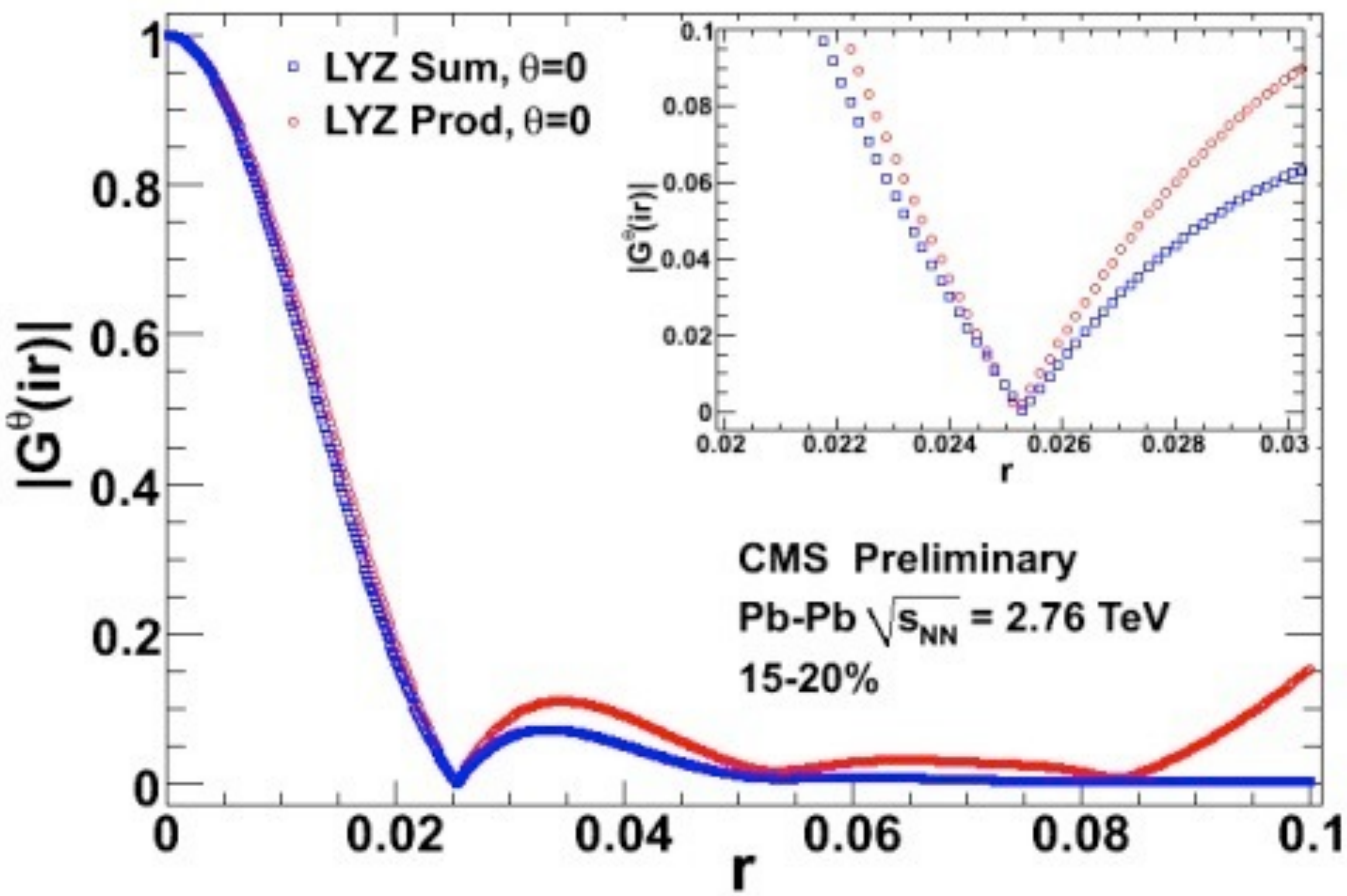
That's all folks!

R.S.Bhalerao, N.B., J.-Y.Ollitrault, Nucl. Phys. A **727** (2003) 373

Anisotropic flow and Lee-Yang zeroes

Measuring a quantity by building a generating function and finding its first zero / minimum... Is this only a theorist's phantasm?

NO!



J.Velkovska (CMS Coll.), talk at Quark Matter 2011

see also ALICE Collaboration, Phys. Rev. Lett. **105** (2010) 252302
(= the 1st paper on results from Pb-Pb collisions at the LHC)

Phenomenology of high-energy nucleus-nucleus collisions

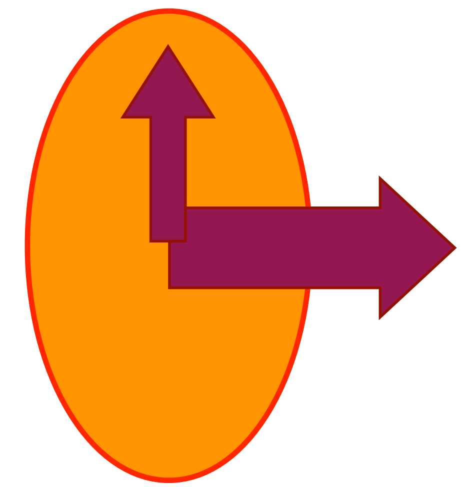
- A rich domain with ongoing (LHC @ CERN, RHIC @ Brookhaven) and planned (FAIR @ GSI, NICA @ Dubna) experiments testing various regimes

- and still plenty of theoretical and phenomenological work to do, using knowledge from various areas: particle physics, nuclear physics, statististical / many-body physics...

Thank you for your attention!

Wait... what about the quiz?

Phenomenology, lesson 5: do not forget your assumptions!



- At $t = 0$, the nuclei collide
- At the LHC, the nuclei remnants quickly fly away
- In the medium, there is a non-zero pressure; outside, there is vacuum:
  the pressure gradient is larger along the impact parameter direction (φ = 0 or 180°) than perpendicular to it.

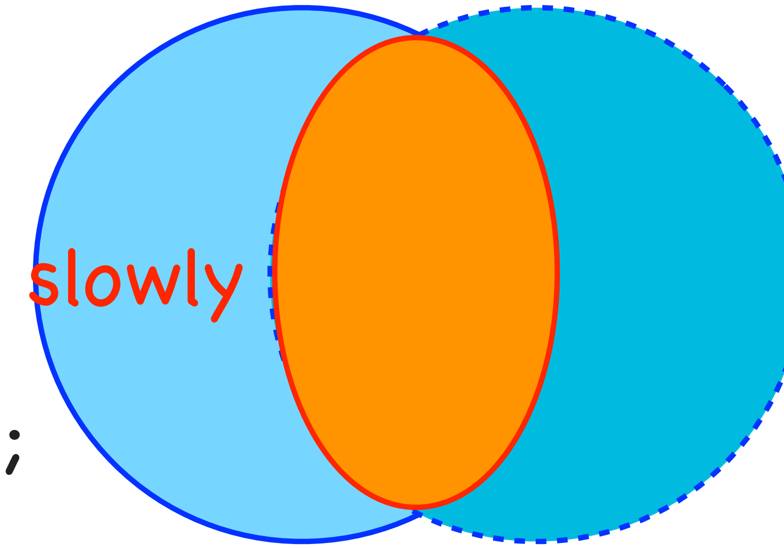
  ⇒ the fluid accelerates more in the impact parameter direction

  (cf. the Euler equation $\rho\left[\frac{\partial\vec{v}}{\partial t} + (\vec{v}\cdot\vec{\nabla})\vec{v}\right] = -\vec{\nabla}p$)

  ☛ anisotropic momentum distribution, $v_2 = \langle\cos 2\varphi\rangle > 0$,
  with $\langle\cdot\rangle$ an average over many particles and events.

What about collisions at lower energies?

Phenomenology, lesson 5: do not forget your assumptions!



- At $t = 0$, the nuclei collide
- At lower energies, the nuclei remnants leave slowly
- In the medium, there is a non-zero pressure; outside there are the remnants, which block any expansion along the impact parameter direction.

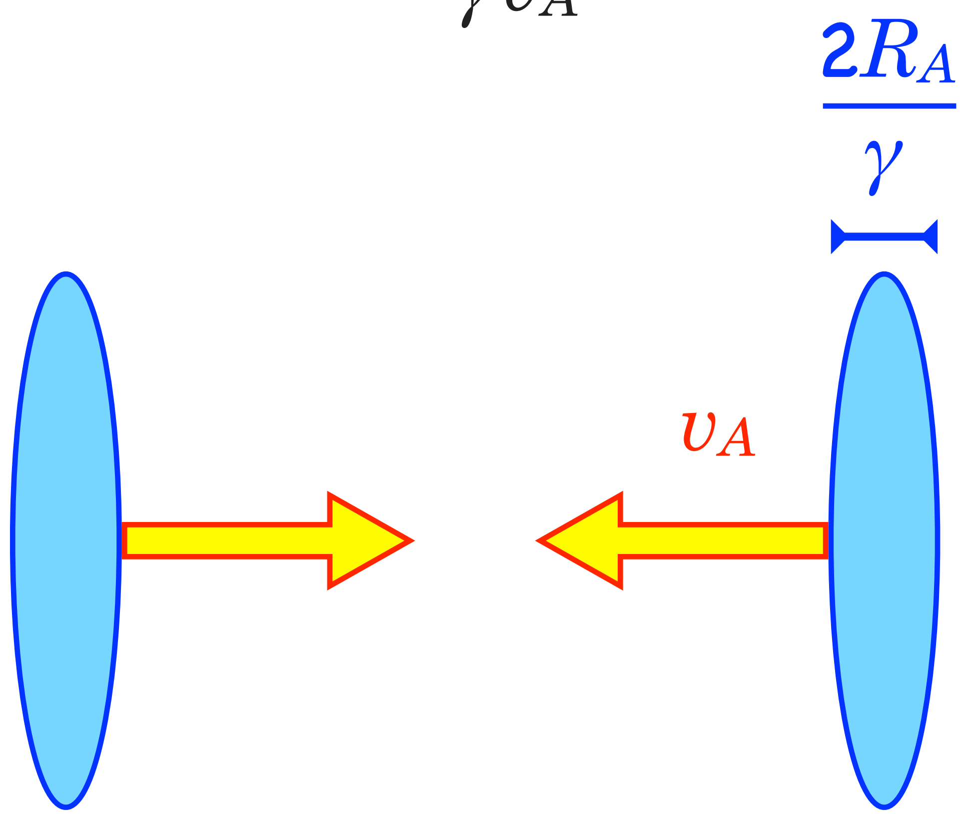
  ⇒ the fluid mostly expands perpendicularly to the impact parameter direction, i.e. at $\varphi = \pm 90°$

  ☛ anisotropic momentum distribution, $v_2 = \langle \cos 2\varphi \rangle < 0$.

☞ Need to compare the time scale for the development of anisotropic flow with that for the crossing of the nuclei!

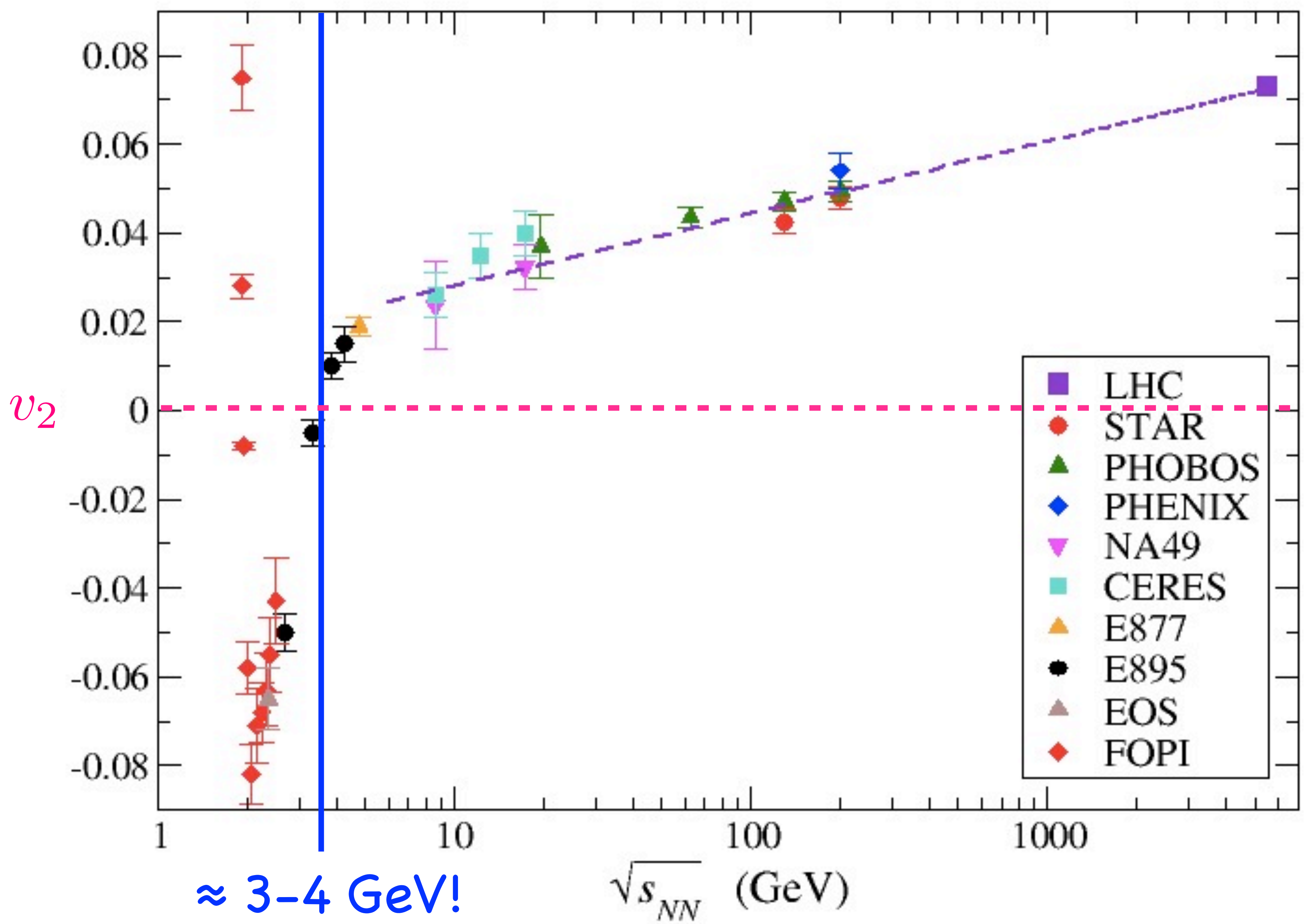
Anisotropic flow: a proof of Lorentz contraction!

- Typical time scale for the development of anisotropic flow: $\frac{R_A}{c_s}$, with c_s the speed of sound in the medium ($\lesssim c/\sqrt{3}$);
- Crossing time of the two nuclei: $\frac{2R_A}{\gamma v_A}$, with γ the Lorentz contraction factor.



$\frac{R_A}{c_s} \approx \frac{2R_A}{\gamma v_A}$ for $\gamma \approx 2$ $\Leftrightarrow$ (reduced) center-of-mass energy $\sqrt{s_{\rm NN}} \approx 4$ GeV

Anisotropic flow:
a proof of Lorentz contraction!



The change of sign of v_2 happens precisely at the predicted energy!