

String theory inspired approaches to heavy-ion collisions

Nicolas BORGHINI

Universität Bielefeld

String theory inspired approaches to heavy-ion collisions

- What is the purpose of colliding heavy nuclei at high energy?
- Phenomenological picture of the time evolution of a collision
 - Relation to the underlying 4-dimensional quantum gauge theory
- Insights from gravity/gauge correspondence
 - Classical gravity in 5 dimensions
 - Examples of predictions from the duality

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Why high-energy heavy-ion collisions?

- 🌐 What is the purpose of colliding heavy nuclei* at high energy?
 - 🌐 Because we can!
 - 🌐 To create a **medium** with an extraordinarily high **energy density**, i.e. possibly a **new state of matter** with novel properties.

* In practice, fully ionized ^{208}Pb (LHC @ CERN),
 ^{197}Au or ^{238}U (RHIC = Relativistic Heavy Ion Collider @ Brookhaven, NY)

A few scales & units to keep in mind...

● Radius of nucleus with atomic mass number A : $R_A \approx 1.1 A^{1/3}$ fm

$$1 \text{ fm (femtometer / Fermi)} = 10^{-15} \text{ m}$$

☞ for ^{208}Pb , $R_{\text{Pb}} \approx 6,8$ fm

● The corresponding “natural” time scale is $\frac{R_{\text{Pb}}}{c} = 6,8 \text{ fm}/c (!) \approx 23 \text{ ys}$

$$1 \text{ fm}/c \approx 3,3 \text{ ys (yoctosecond)} = 3,3 \cdot 10^{-24} \text{ s}$$

● Mass of the ^{208}Pb nucleus $m_{\text{Pb}} \approx 208 m_{\text{N}}$

$$\text{with } m_{\text{N}} = 0,939 \text{ GeV}/c^2 = 1.67 \cdot 10^{-27} \text{ kg}$$

☞ typical length, time, mass scales: fm, fm/ c , GeV/ c^2 .

A few scales & units to keep in mind...

🌐 What does “high-energy collisions” mean?

👉 in 2010–2011 the kinetic energy of a ^{208}Pb nucleus at LHC was

$$E_{\text{kin}} = 287 \text{ TeV} = 208 \times 1.38 \text{ TeV} = 1481 m_{\text{Pb}} c^2$$

ultrarelativistic regime! $v_{\text{Pb}} = (1 - 0.23 \cdot 10^{-6}) c$

👉 in a single Pb–Pb collision at LHC, the available energy is $2E_{\text{kin}}$.

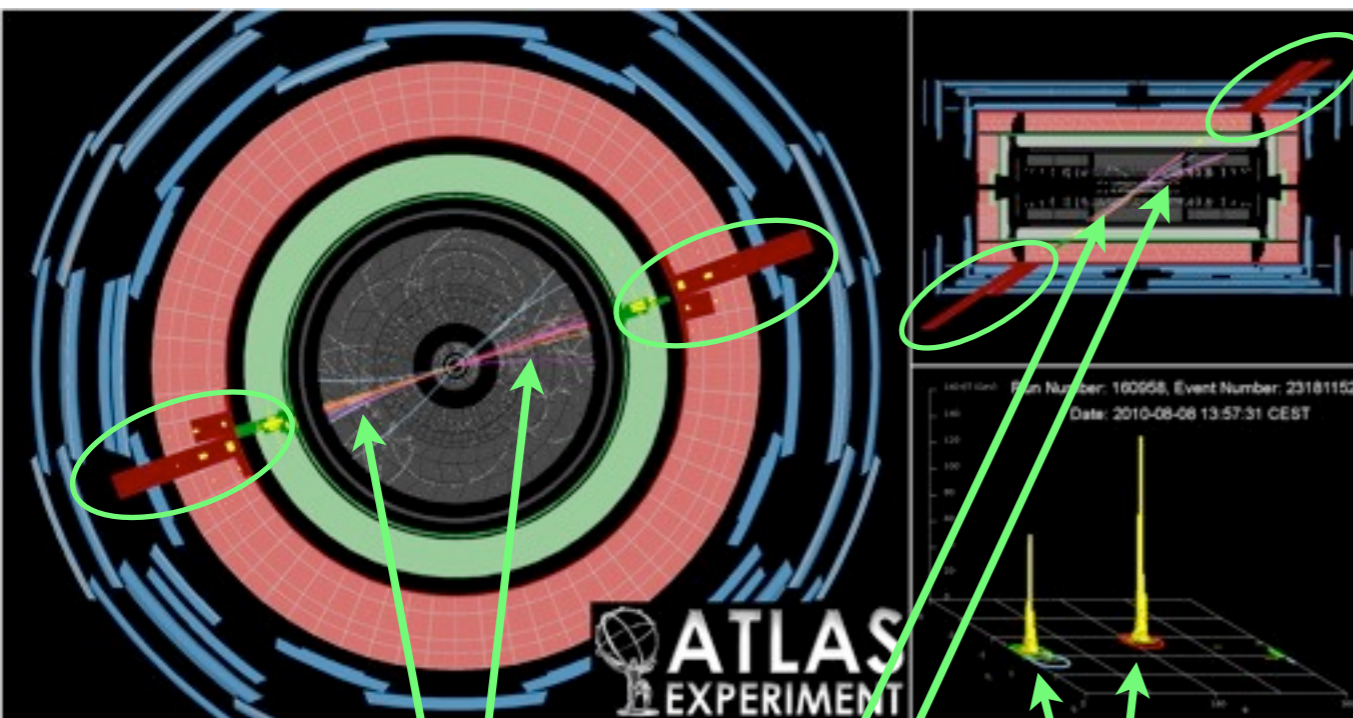
🌐 If 20% of this energy is deposited in a volume of about 1000 fm^3 , then the **energy density** in this volume is $e \approx 100 \text{ GeV/fm}^3$.

🌐 Such an energy density $e \approx 100 \text{ GeV/fm}^3$ amounts to a temperature $k_{\text{B}}T \approx 500 \text{ MeV}$, i.e. $T \approx 6 \cdot 10^{12} \text{ K}$ ($\gg 15 \cdot 10^6 \text{ K}$ at the center of the Sun!)

👉 **hot (& dense) medium**: “new state of matter”

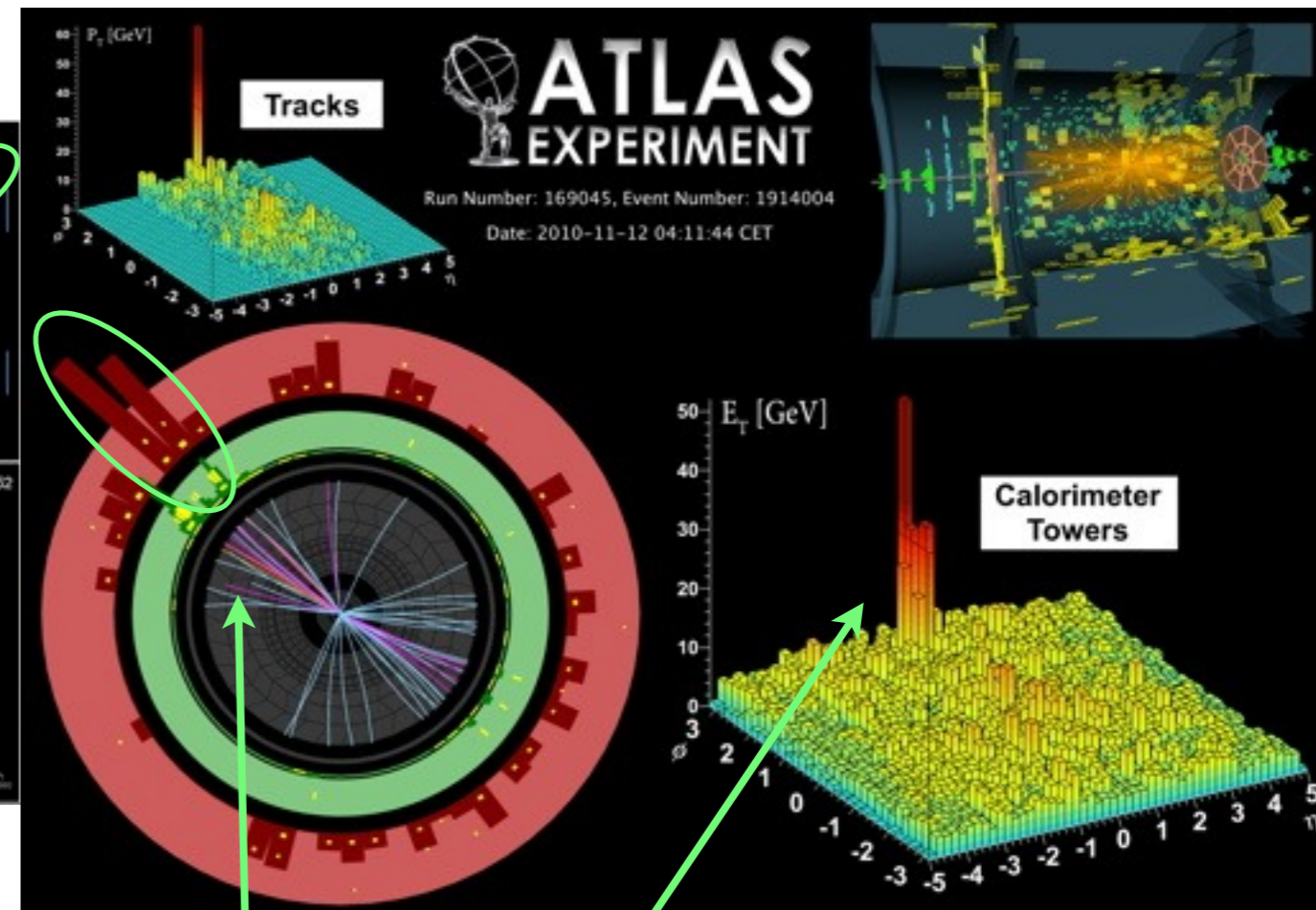
Is there evidence for this medium?

proton-proton collision
at the LHC



2 back-to-back "jets" of highly energetic particles, that deposit energy in calorimeters

Pb-Pb collision at the LHC

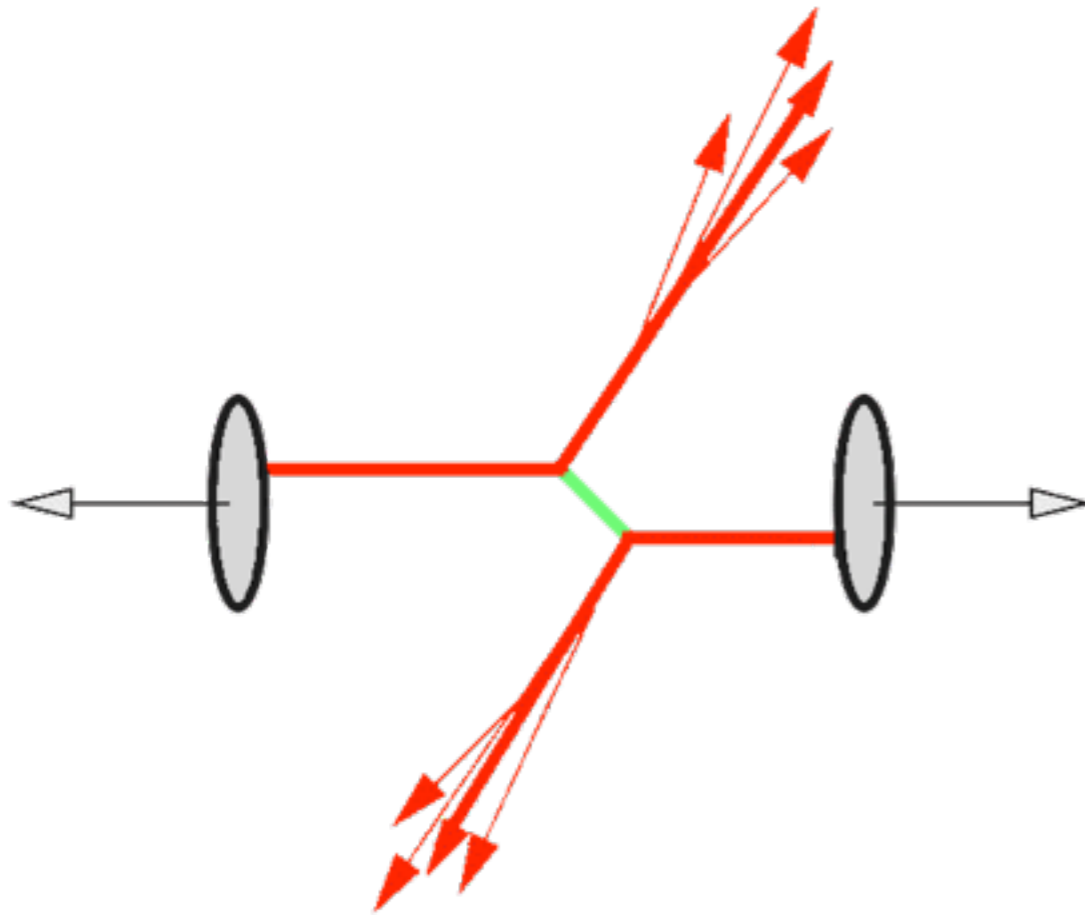


a single "jet", which has lost its back-to-back counterpart..

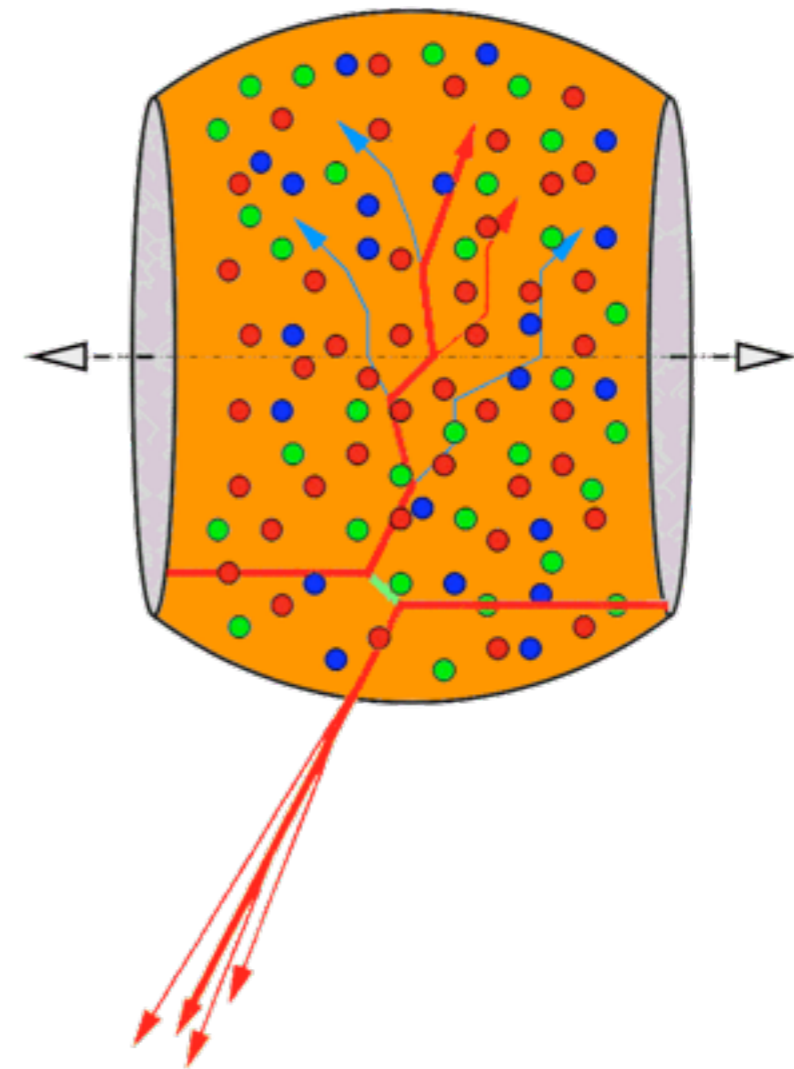
ATLAS Collaboration, [PRL 105 \(2010\) 252303](#)

Is there evidence for this medium?

proton-proton collision
at the LHC



Pb-Pb collision at the LHC



While propagating through the **hot and dense medium**, the “jet”-to-be has dissipated part of its energy, and does not emerge as a jet!

Is there evidence for this medium?

YES

There is a very opaque medium,
which can stop jets over short distances

What are its properties?

Propaganda...

HIGHEST MAN-MADE TEMPERATURE



TWEET 

FOR THE RECORD

WHO:
BROOKHAVEN NATIONAL
LABORATORY'S
RELATIVISTIC HEAVY ION
COLLIDER

WHAT:
HIGHEST MAN-MADE
TEMPERATURE

WHERE:
UNITED STATES

WHEN:
01 JAN 2001

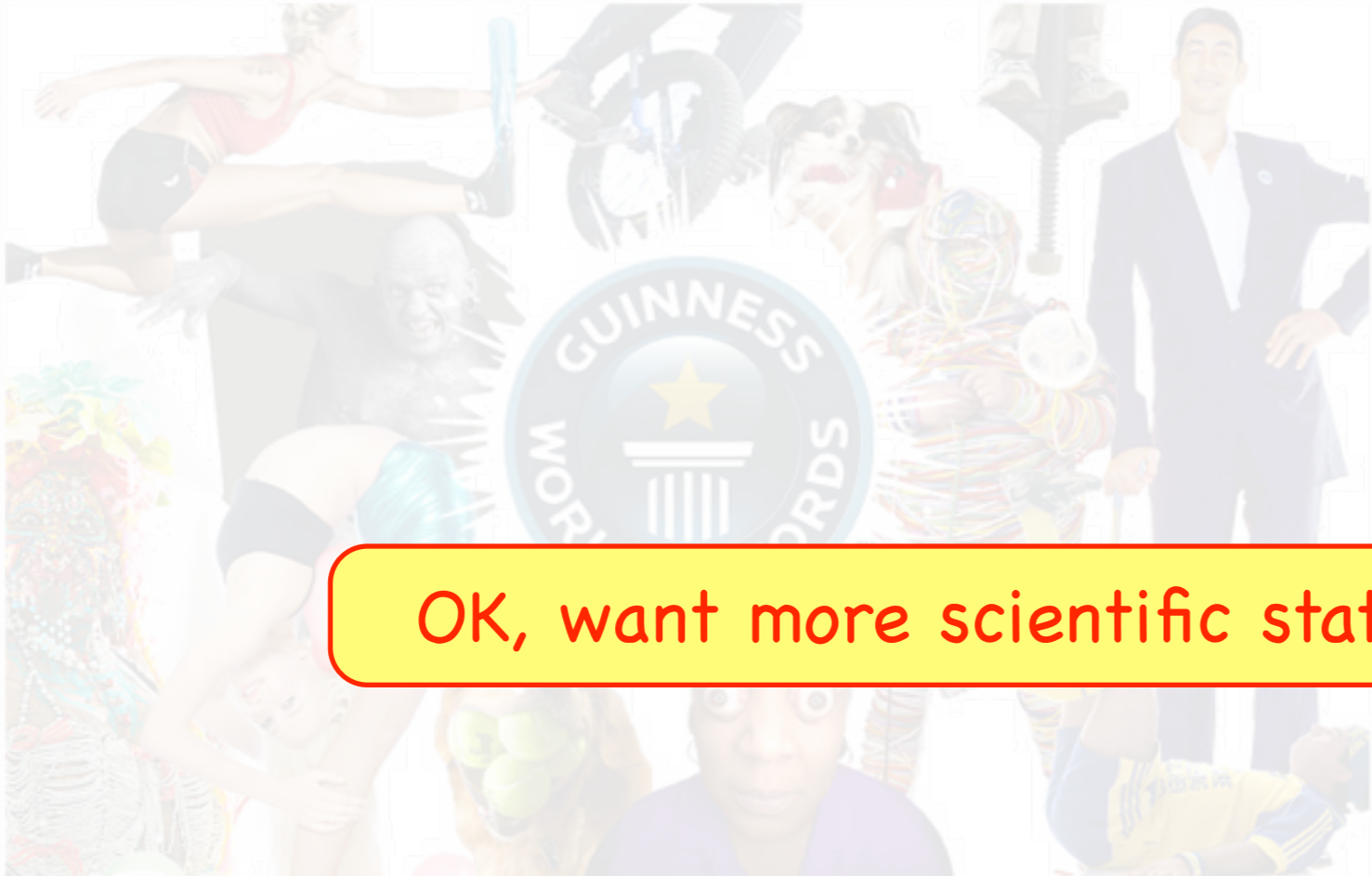
In February 2010, scientists at Brookhaven National Laboratory's Relativistic Heavy Ion Collider on Long Island, New York, USA, announced that they had smashed together gold ions at nearly the speed of light, briefly forming an exotic state of matter known as a quark-gluon plasma. This substance is believed to have filled the universe just a few microseconds after the Big Bang. During the experiments – which began in July 2001 and have taken a decade to authenticate – the plasma reached temperatures of around 4 trillion°C, some 250,000 times hotter than the centre of the Sun.

As of July 2012, even higher temperatures may have been achieved at Brookhaven, following the colliding of (heavier) uranium nuclei, but it will take a while to ascertain the temperatures reached; experiments using the Large Hadron Collider near Geneva, Switzerland, may have also achieved higher temperatures but are also yet to be determined.

<http://www.guinnessworldrecords.com/world-records/3000/highest-man-made-temperature>

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OK, want more scientific statements?

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<http://www.guinnessworldrecords.com/world-records/3000/highest-man-made-temperature>

Propaganda... reloaded(?)

Viewpoint

A “Little Bang” arrives at the LHC

Edward Shuryak

Department of Physics and Astronomy, Stony Brook University, Stony Brook, NY 11794, USA

Published December 13, 2010

The first experiments to study the quark-gluon plasma at the LHC reveal that even at the hottest temperatures ever produced at a particle accelerator, this extreme state of matter remains the best example of an ideal liquid.

ideal liquid: obeys the Euler equation of fluid dynamics

A Viewpoint on:

Elliptic Flow of Charged Particles in Pb-Pb Collisions at $\sqrt{s_{NN}} = 2.76$ TeV

K. Aamodt *et al.* (ALICE Collaboration)

Phys. Rev. Lett. **105**, 252302 (2010) – Published December 13, 2010

Observation of a Centrality-Dependent Dijet Asymmetry in Lead-Lead Collisions at $\sqrt{s_{NN}} = 2.76$ TeV with the ATLAS Detector at the LHC

G. Aad *et al.* (ATLAS Collaboration)



Phys. Rev. Lett. **105**, 252303 (2010) – Published December 13, 2010

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Evolution of the **medium** in a Pb–Pb collision at LHC

(a sketch!)

- At $t = 0$, the Pb nuclei collide: “event” (10^5 – 10^7 events in a month run)
 - some of their internal constituents are stopped and set free from the nuclei wavefunctions  **QCD** fields
 - at $t = 0^+$, the remnants of the nuclei fly away.
- First few fm/c: the liberated degrees of freedom form a “**fireball**”
 - which rapidly expands and cools down: **collective behavior**;
 - whose content (relevant degrees of freedom) evolves.
- At $t \approx 10$ – 20 fm/c, the fireball stops behaving collectively, particles fly freely to the detectors.
 about 2 – $10 \cdot 10^3$ particles per event

Evolution of the **medium** in a Pb-Pb collision at LHC

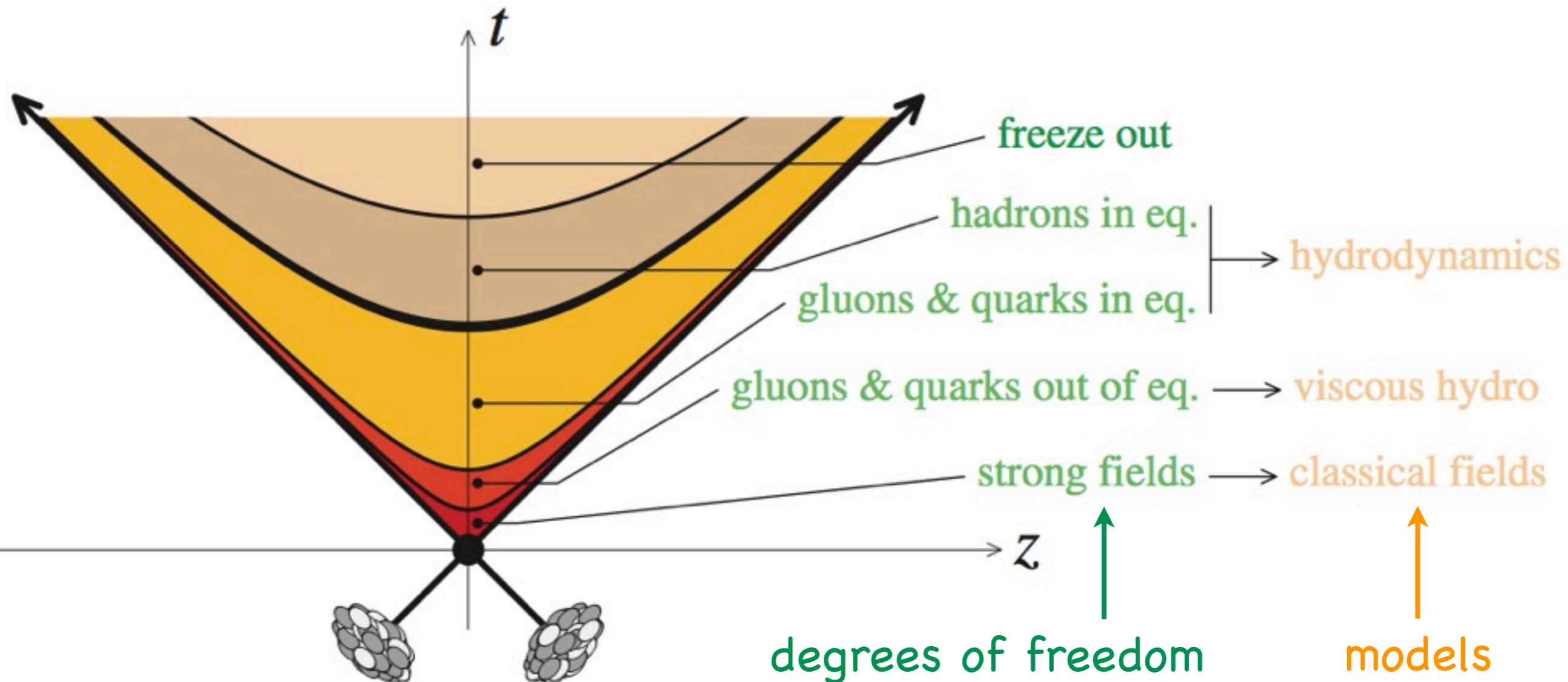


Figure taken from F.Gelis, [Int. J. Mod. Phys. A 28 \(2013\) 133001](#)

Models of the **medium** vs. theory

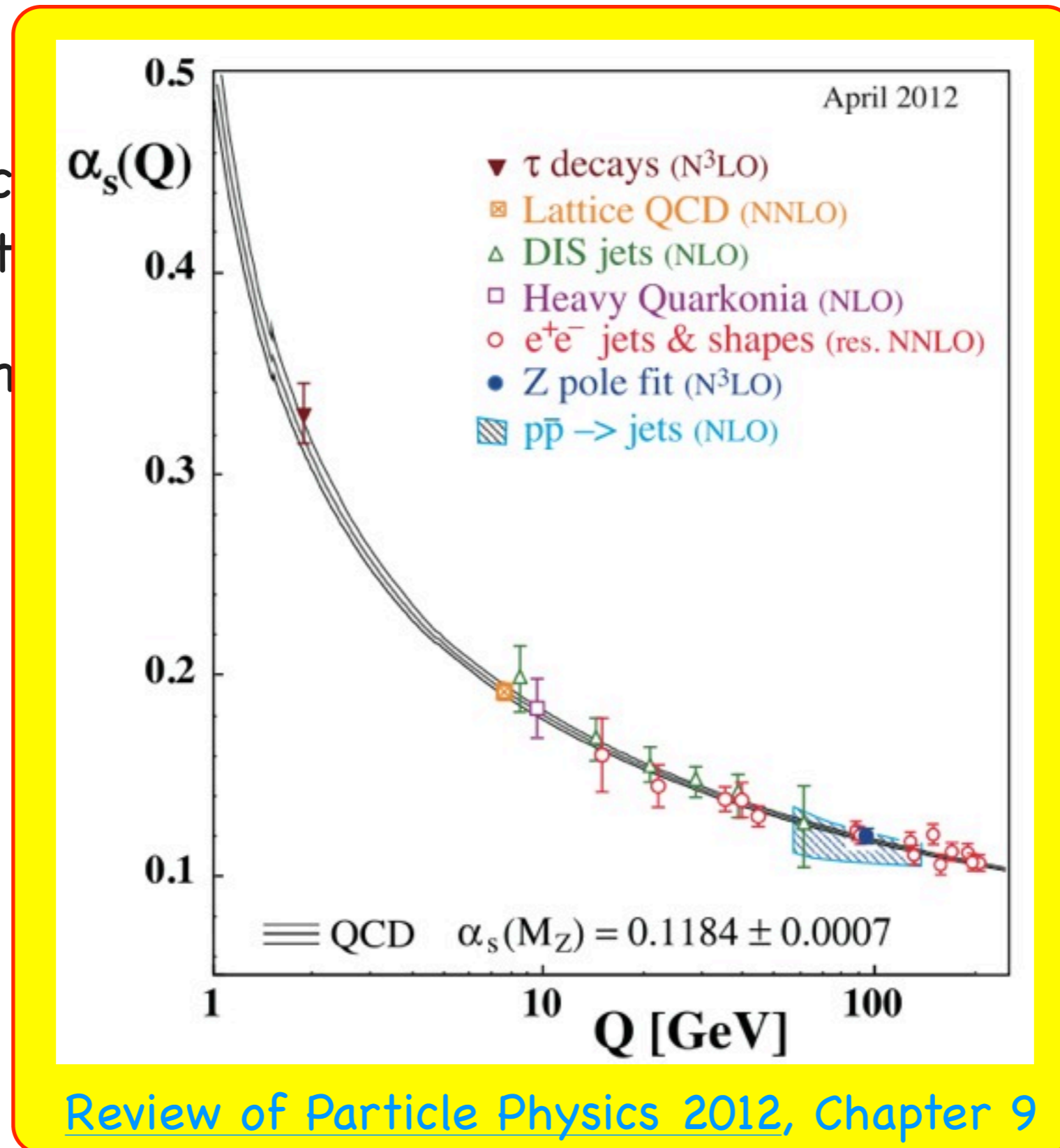
The **medium** consists of quarks and gluons: carriers of color charge, whose interaction is Quantum Chromodynamics (QCD).

➡ non-Abelian [SU(3)] gauge theory, with a running coupling constant

Models of the medium vs. theory

The medium consists of quarks and gluons whose interactions are described by a non-Abelian gauge theory

of color charge, the coupling constant



Models of the **medium** vs. theory

The **medium** consists of quarks and gluons: carriers of color charge, whose interaction is Quantum Chromodynamics (QCD).

➡ non-Abelian [SU(3)] gauge theory, with a running coupling constant

for a **medium** with $T \approx 200 \text{ MeV}$, $\alpha_s = \mathcal{O}(1)$: strong coupling!

... and the requirement is to study the “condensed-matter” aspects of this strongly-coupled theory.

Models of the **medium** vs. theory

A few questions to be answered:

- What are the properties (equation of state, transport coefficients...) of a **medium** of quarks and gluons in thermodynamical equilibrium — a quark-gluon plasma (**QGP**)?

- Is it even clear that the rapidly expanding system of quarks and gluons created in a high-energy nucleus-nucleus collision reaches some (local) equilibrium, that it "**thermalizes**"?

- ➡ Some of these issues can be addressed with "classical" approaches, for instance lattice gauge theory computations.

Physikalisches Kolloquium

A few questions

• What are the properties of a **medium** of a quark-gluon plasma

• Is it even close to (local) equilibrium

• Some of the theoretical approaches, for instance lattice

• (not coefficients...)
• at equilibrium —

• of quarks and gluons
• on reaches some

• "cal" approaches,

Prof. Dr. Frithjof Karsch

Bielefeld University and BNL, New York

The hottest man-made matter

The hottest man-made matter – the quark gluon plasma – is created in relativistic heavy ion collisions at the Relativistic Heavy Ion Collider (RHIC) in Brookhaven National Laboratory, USA, and the Large Hadron Collider (LHC) at CERN, Switzerland. Almost 40 years ago the existence of this novel form of strongly interacting matter has been predicted to arise as a consequence of two widely different features of the fundamental theory of strong interactions, Quantumchromodynamics (QCD). QCD describes the confinement of quarks and gluons at low temperatures as well as their asymptotic freedom at high temperature. At very high temperature one thus expects to find a gas of almost free quarks and gluons.

The experimental study of this novel form of strongly interacting matter shows that in the temperature range now accessible at RHIC and LHC it has properties that resemble more those of a fluid rather than a gas or plasma. The medium shows strong collective behavior; even heavy quarks equilibrate quickly and the medium seems to have a small electrical conductivity.

The theoretical study of strongly interacting matter and, in particular, the transition from the low to high temperature regime as well as the properties of hot and dense matter close to the transition requires large scale numerical simulations that are performed on today's fastest supercomputers as well as large clusters of graphics cards. I will present results from some of these calculations, in which the lattice gauge theory group in Bielefeld is heavily involved.

Montag, 10.06.2013, 16:15 Uhr

Ort: Hörsaal 6

Models of the **medium** vs. theory

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➡ Some of these issues can be addressed with “classical” approaches, for instance lattice gauge theory computations.

Others are beyond the reach of such methods, and will remain so for the next decade(s).

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Gauge/string duality

“Holographic” equivalence between D -dimensional $SU(N_c)$ gauge theory and some string theory on $(D+1)$ -dimensional spacetime.

Maldacena; Gubser, Klebanov, Polyakov; Witten 1998

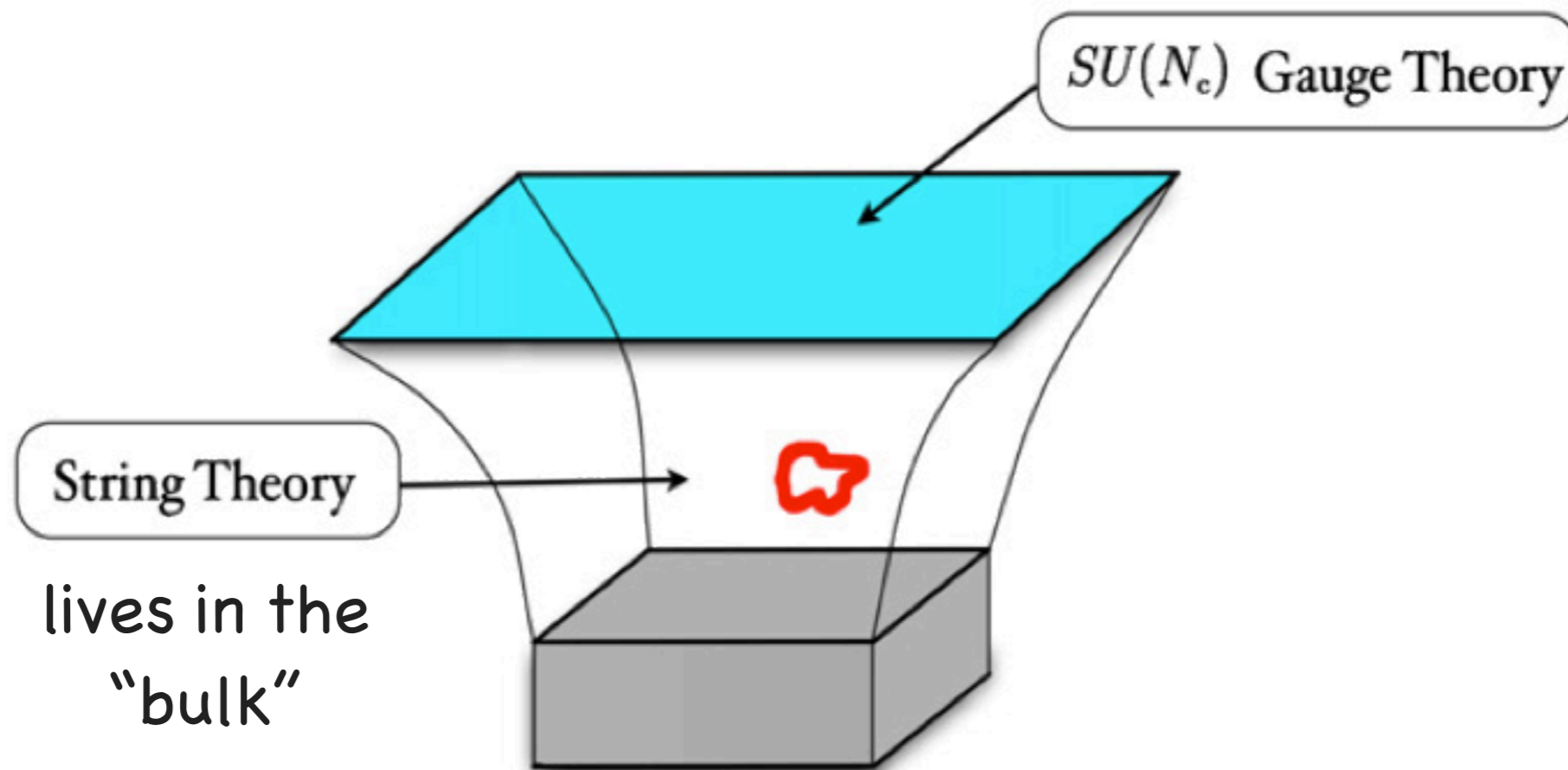


Figure taken from D.Mateos, [J. Phys. G 38 \(2011\) 124030](#)

Gauge/string duality

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Maldacena; Gubser, Klebanov, Polyakov; Witten 1998

... however only tractable – technical issue! – under 2 conditions:


◆ the string coupling constant should be arbitrarily small $g_s \rightarrow 0$

☞ classical string theory

◆ the string scale ℓ_s should be much smaller than the typical bulk scale L

☞ strings are pointlike, theory becomes gravity

\lambda = (L/\ell_s)^4 = g_{\text{YM}}^2 N_c with $g_s = \frac{1}{4\pi} g_{\text{YM}}^2$

$g_s \rightarrow 0, \ell_s/L \rightarrow 0$ ☞ $\left\{ \begin{array}{l} \lambda \rightarrow \infty: \text{strongly-coupled gauge theory} \\ N_c \rightarrow \infty \end{array} \right.$ 

Gauge/string duality

Equivalence in practice “only” between

- D -dimensional $SU(N_c)$ gauge theory with $N_c \rightarrow \infty$ and $\lambda \rightarrow \infty$
 - and classical gravity in $D+1$ dimensions.
- ➔ “gauge/gravity duality”

For **QCD**, with its 3 colors and asymptotic freedom, one may only hope for (qualitative) hints, in particular regarding quantities which exhibit some kind of “universality”.

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Thus, the dual gravity background of the high-temperature deconfined phase of any gauge theory is that of a black hole, whose temperature equals that of the gauge theory phase.

Witten, *Adv. Theor. Math. Phys.* **2** (1998) 505

Gauge/string duality

Equivalence

• D -dimens

• and class

• "gauge

For QCD, w

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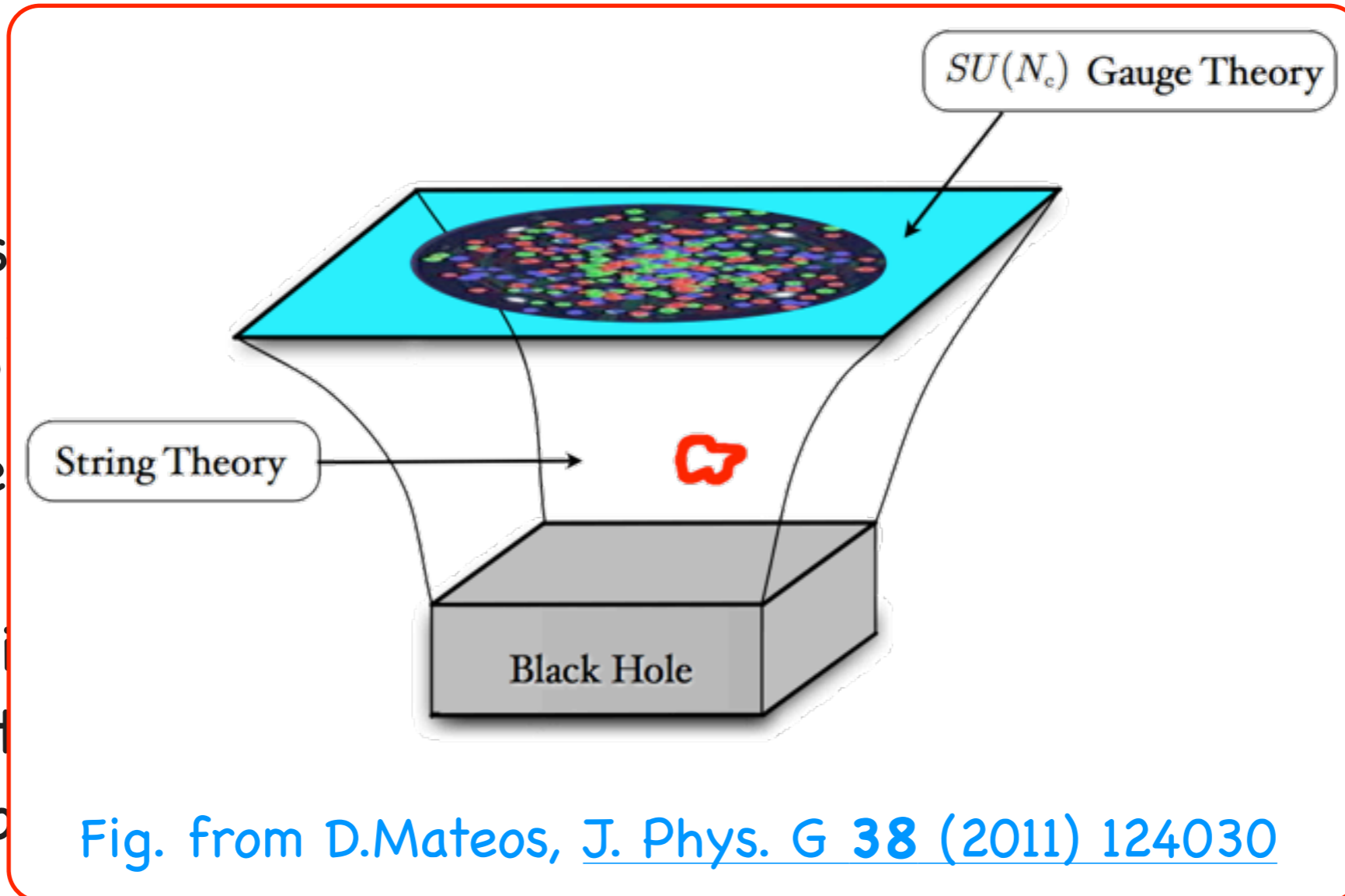


Fig. from D.Mateos, [J. Phys. G 38 \(2011\) 124030](#)

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Classical gravity in 5 dimensions

Metric tensor of the AdS₅ (anti de Sitter) solution of the Einstein eqs.

$$R_{\alpha\beta} - \frac{1}{2}Rg_{\alpha\beta}^{(5)} = \frac{6}{L^2}g_{\alpha\beta}^{(5)}$$

with cosmological term $\Lambda = \frac{D(D-1)}{2L^2} = \frac{6}{L^2}$ where $D = 4$

➡
$$ds^2 = \frac{L^2}{z^2} (-dt^2 + d\vec{x}^2 + dz^2) = \frac{L^2}{z^2} (\underbrace{\eta_{\mu\nu} dx^\mu dx^\nu}_{\text{flat metric on } \mathbb{R}^{1+3}} + dz^2)$$

$z = 0$ is the boundary, $z > 0$ the bulk, L the curvature radius.

Pictures of dS / AdS spaces in U.Moschella, [Sém. Poincaré 2005-1, 144](#)

Classical gravity in 5 dimensions

Metric tensor of the AdS₅ (anti de Sitter) solution of the Einstein eqs.

Why AdS₅?

Because it is the surface of equation

$$-t_1^2 - t_2^2 + x_1^2 + x_2^2 + x_3^2 + x_4^2 = -L^2$$

in the 6-dimensional space with metric

diag(-1,-1,+1,+1,+1,+1), and thus has the

symmetry $O(2,4)$...

with cosmologi



$ds^2 =$

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symmetry O(2,4)...

which is the symmetry group of the classical Maxwell equations
(Lorentz* + dilatations + special conformal transformations**)

*Lorentz(!) 1892; **Cunningham & Bateman 1909

➡ Duality AdS₅/conformal gauge theory in $D=4$ is not shocking

Pictures of dS / AdS spaces in U.Moschella, [Sém. Poincaré 2005-1, 144](#)

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$z = 0$ is the boundary, $z > 0$ the bulk, L the curvature radius.

Rem.: defining $r \equiv L^2/z$ to send the boundary to $r \rightarrow \infty$ can be useful:

$$ds^2 = \frac{r^2}{L^2} (-dt^2 + d\vec{x}^2) + \frac{L^2}{r^2} dr^2$$

Pictures of dS / AdS spaces in U.Moschella, [Sém. Poincaré 2005-1, 144](#)

Classical gravity in 5 dimensions

Metric tensor of AdS_5 with a black hole (but no matter):

$$ds^2 = \frac{L^2}{z^2} \left[- \left(1 - \frac{z^4}{z_h^4} \right) dt^2 + d\vec{x}^2 + \frac{dz^2}{1 - z^4/z_h^4} \right]$$

with z_h the location of the black hole horizon.

The Hawking temperature of the black hole is $T_H = \frac{1}{\pi z_h}$.

➡ Black hole entropy proportional to its "area": $A \propto \left(\frac{L^2}{z_h^2} \right)^3 z_h^3 \propto T_H^3$

Same scaling as the entropy density $s \propto T^3$ of a 3D-relativistic gas*

* Remember $s = \frac{\partial e}{\partial T}$ with energy density $e \propto T^4$ (Stefan-Boltzmann law)

Classical gravity in 5 dimensions

Metric tensor of AdS_5 with some graviton configuration, reinterpreted as a change of the background:

$$ds^2 = g_{\alpha\beta}^{(5)} dx^\alpha dx^\beta = \frac{L^2}{z^2} [g_{\mu\nu}(x^\rho, z) dx^\mu dx^\nu + dz^2]$$

(Fefferman-Graham coordinates*), with as before the requirement

$$R_{\alpha\beta} - \frac{1}{2} R g_{\alpha\beta}^{(5)} = \frac{6}{L^2} g_{\alpha\beta}^{(5)}$$

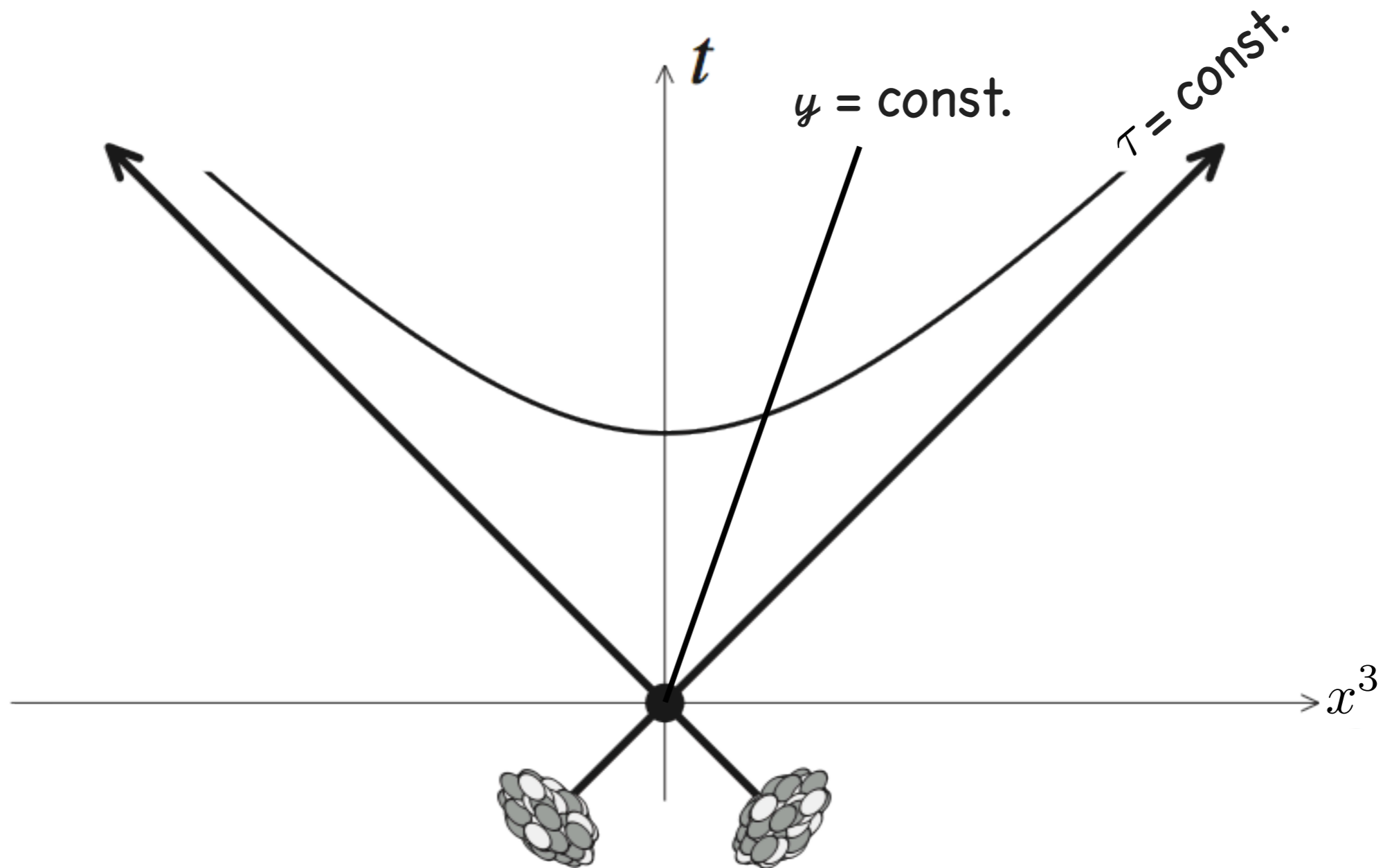
*Fefferman & Graham, [arXiv:0710.0919 \[math.DG\]](https://arxiv.org/abs/0710.0919)

Gauge/string duality & heavy ions

Geometry of a heavy-ion collision:

there exists a preferred axis, which suggests a change of coordinates

$(t, x^3) \rightarrow (\tau, y)$ with $t = \tau \cosh y$, $x^3 = \tau \sinh y$.



Gauge/string duality & heavy ions

Geometry of a heavy-ion collision:

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➡ on the flat Minkowski space of the gauge theory:

$$ds^2 = -d\tau^2 + d\mathbf{x}_\perp^2 + \tau^2 dy^2$$

that is, just for the sake of fun

$$\Gamma_{\tau y}^y = \Gamma_{y\tau}^y = \frac{1}{\tau}, \quad \Gamma_{yy}^\tau = \tau, \quad \Gamma_{\tau y}^\tau = \Gamma_{y\tau}^\tau = \Gamma_{yy}^y = 0$$

Gauge/string duality & heavy ions

Geometry of a heavy-ion collision:

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Physical assumption: at asymptotically high energies, the dynamics of the collision should be independent of y (\approx boosts along collision axis)

[Bjorken, Phys. Rev. D 27 \(1983\) 140](#)

Gauge/string duality & heavy ions

What can the duality say on boost-invariant flows?

Janik & Peschanski, [Phys. Rev. D 73 \(2006\) 045013](#)

Assuming boost invariance (i.e. no dependence on y) and no dependence on the transverse coordinates x^1, x^2 , the generic energy-momentum tensor only has non-vanishing components $T_{\tau\tau}, T_{yy}$ and $T_{11} = T_{22} \equiv T_{xx}$, functions of τ .

Energy-momentum conservation $T_{;\mu}^{\mu\nu} = 0$ and tracelessness $T_{\mu}^{\mu} = 0$ then yield

$$\begin{aligned}\tau \frac{dT_{\tau\tau}}{d\tau} + T_{\tau\tau} + \frac{1}{\tau^2} T_{yy} &= 0 \\ -T_{\tau\tau} + \frac{1}{\tau^2} T_{yy} + 2T_{xx} &= 0\end{aligned}$$

which can be solved in terms of a single function $e(\tau) \equiv T_{\tau\tau}(\tau)$

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Energy-momentum conservation $T_{;\mu}^{\mu\nu} = 0$ and tracelessness $T_{\mu}^{\mu} = 0$ then yield

$$T_{\mu\nu}(\tau) = \begin{pmatrix} e(\tau) & 0 & 0 & 0 \\ 0 & e(\tau) + \frac{1}{2}\tau e'(\tau) & 0 & 0 \\ 0 & 0 & e(\tau) + \frac{1}{2}\tau e'(\tau) & 0 \\ 0 & 0 & 0 & -\tau^3 e'(\tau) - \tau^2 e(\tau) \end{pmatrix}$$

with $e(\tau) \equiv T_{\tau\tau}(\tau)$, $e'(\tau) \equiv \frac{de(\tau)}{d\tau}$

Gauge/string duality & heavy ions

What can the duality say on boost-invariant flows?

Janik & Peschanski, [Phys. Rev. D 73 \(2006\) 045013](#)

➡ build the 5-dimensional background dual to the energy-momentum tensor

$$ds^2 = \frac{L^2}{z^2} \left[-e^{a(z,\tau)} d\tau^2 + e^{b(z,\tau)} d\mathbf{x}_\perp^2 + e^{c(z,\tau)} \tau^2 dy^2 + dz^2 \right]$$

with $a(z, \tau) = \sum_{k=0}^{\infty} a_k(\tau) z^{4+2k} = -e(\tau) z^4 + a_1(\tau) z^6 + a_2(\tau) z^8 + \dots$

and the requirement that $g_{\alpha\beta}^{(5)}$ be solution of the Einstein equations

$$R_{\alpha\beta} - \frac{1}{2} R g_{\alpha\beta}^{(5)} = \frac{6}{L^2} g_{\alpha\beta}^{(5)}$$

➡ $a_k(\tau)$ functions of $e(\tau)$ and its derivatives.

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where the $a_k(\tau)$ are functions of $e(\tau)$ and its derivatives.

Investigate the large proper time behavior $e(\tau) \sim \frac{1}{\tau^s}$ with (positivity of energy) $0 \leq s \leq 4$.

Solving, the 5D-background is only free of naked singularity for $s = \frac{4}{3}$.
physical assumption!

Gauge/string duality & heavy ions

What can the duality say on boost-invariant flows?

Janik & Peschanski, [Phys. Rev. D 73 \(2006\) 045013](#)

With $e(\tau) \underset{\tau \rightarrow \infty}{\sim} \frac{e_0}{\tau^{4/3}}$ the background metric becomes

$$ds^2 = \frac{L^2}{z^2} \left[\frac{(1 - \frac{e_0}{3} z^4 / \tau^{4/3})^2}{1 + \frac{e_0}{3} z^4 / \tau^{4/3}} d\tau^2 + \left(1 + \frac{e_0}{3\tau^{4/3}} z^4 \right) (d\mathbf{x}_\perp^2 + \tau^2 dy^2) + dz^2 \right]$$

Looks vaguely familiar? Remember slide 19, AdS₅ with black hole:

$$ds^2 = \frac{L^2}{\tilde{z}^2} \left[\frac{(1 - \tilde{z}^4 / z_h^4)^2}{1 + \tilde{z}^4 / z_h^4} dt^2 + \left(1 + \frac{\tilde{z}^4}{z_h^4} \right) d\vec{x}^2 + d\tilde{z}^2 \right]$$

with horizon at $z_h \propto 1/T_H$.

☞ AdS₅ with black hole with moving horizon at $z_h = \left(\frac{3}{e_0} \right)^{1/4} \tau^{1/3}$, i.e., extrapolating the static formula, $T_H \propto \tau^{-1/3}$ (?).

Gauge/string duality & heavy ions

What can the duality say on boost-invariant flows?

Janik & Peschanski, [Phys. Rev. D 73 \(2006\) 045013](#)

With $e(\tau) \underset{\tau \rightarrow \infty}{\sim} \frac{e_0}{\tau^{4/3}}$ the energy-momentum tensor becomes

$$T_{\mu\nu}(\tau) = \begin{pmatrix} e(\tau) & 0 & 0 & 0 \\ 0 & e(\tau) + \frac{1}{2}\tau e'(\tau) & 0 & 0 \\ 0 & 0 & e(\tau) + \frac{1}{2}\tau e'(\tau) & 0 \\ 0 & 0 & 0 & -\tau^3 e'(\tau) - \tau^2 e(\tau) \end{pmatrix}$$

Gauge/string duality & heavy ions

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...that is exactly the energy-momentum tensor of an ideal fluid, with equation of state energy density = 3 × pressure

At large times, the system “automatically” behaves like an ideal fluid!

☞ Behavior picked out by the regularity of the gravity dual...

see also Kajantie, Louko & Tahkokallio, [Phys. Rev. D 76 \(2007\) 106006](#)

Gauge/string duality & heavy ions

Further applications:

- Related idea (although strictly speaking not using the string/gauge duality): study flows on (Anti) de Sitter space(s) to deduce solutions of the **viscous fluid-dynamical equations** on \mathbb{R}^{1+3}

Gubser & Yarom, [Nucl. Phys. B 846 \(2011\) 469](#)

- Derivation of the **shear viscosity** of strongly-interacting matter

Policastro, Son & Starinets, [PRL 87 \(2001\) 081601](#); [JHEP 0209 \(2002\) 043](#)
+ many followers

The shear viscosity η in the finite-temperature gauge theory is related to the absorption cross-section of low-energy gravitons by an AdS_5 black hole.

In all gauge theories with a gravity dual, $\underbrace{\frac{\eta}{s} = \frac{1}{4\pi}}$ at leading order.

Fits to heavy-ion data: $\frac{\eta}{s} \approx \frac{1 \cdots 3}{4\pi} \dots$ extremely small:
"most ideal liquid"

Gauge/string duality & heavy ions

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- Derivation of the **shear viscosity** of strongly-interacting matter

[Policastro, Son & Starinets, PRL 87 \(2001\) 081601; JHEP 0209 \(2002\) 043](#)

- + further transport coefficients of the equations of (second order) dissipative relativistic fluid dynamics

[Baier, Romatschke, Son, Starinets & Stephanov, JHEP 0804 \(2008\) 100](#)

- ... thanks to correspondence between long-wavelength perturbations of black holes in AdS_5 and of deconfined gauged matter on its boundary

[Bhattacharyya, Hubeny, Minwalla & Rangamani, JHEP 0802 \(2008\) 045](#)

Gauge/string duality & heavy ions

Further applications:

- Computation of the coefficient (\hat{q}) characterizing jet energy loss

[Liu, Rajagopal & Wiedemann, JHEP 0703 \(2007\) 066](#)

Until now, only results for (quasi-)equilibrated gauge matter.

Far from equilibrium results:

- Estimate of the isotropization / thermalization times of the system arising from colliding two sheets* of strongly-interacting matter

[Chesler & Yaffe, PRL 106 \(2011\) 021601](#)

On the gravity side, numerical study of the formation of a horizon in the collision of two gravitational waves.

☞ Suggestive of much faster thermalization than estimates from pQCD...
handle with care!

*at high energies, the colliding nuclei are strongly Lorentz-contracted ($\gamma \approx 1500$ @ LHC) into pancakes

Gauge/string duality & heavy ions

Further applications:

- Computation of the coefficient (\hat{q}) characterizing jet energy loss

Liu, Rajagopal & Wiedemann, [JHEP 0703 \(2007\) 066](#)

Until now, only results for (quasi-)equilibrated gauge matter.

Far from equilibrium results:

- Estimate of the isotropization / thermalization times of the system arising from colliding two sheets* of strongly-interacting matter

Chesler & Yaffe, [PRL 106 \(2011\) 021601](#)

- How much entropy is released in such a process

Gubser, Pufu & Yarom, [Phys. Rev. D 78 \(2008\) 066014](#)

- Creation of lepton pairs or photons in the out-of-equilibrium stage

Baier, Steineder, Stricker, Taanila & Vuorinen,
[JHEP 1207 \(2012\) 094](#), [PRL 110 \(2013\) 101601](#), [arXiv:1304.3404](#)

Gauge/string duality & heavy ions

Further applications:

🌍 ... more every day!

Gauge/string duality & heavy ions

Further applications:

- ... more every day!

RAPID COMMUNICATIONS

Time-dependent heavy-quark potential at finite temperature from gauge-gravity duality
PHYSICAL REVIEW D 87, 101901(R) (2013)

Tomoya Hayata,^{1,2,*} Kanabu Nawa,^{2,†} and Tetsuo Hatsuda^{2,3,‡}

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(Received 3 December 2012; published 7 May 2013)

The potential between a heavy quark and an antiquark inside the quark-gluon plasma is studied on the basis of the gauge-gravity duality.

Gauge/string duality & heavy ions

Further applications:

● ... more every day!

RAPID COMMUNICATIONS

arXiv.org > hep-ph > arXiv:1306.2613

High Energy Physics - Phenomenology

Estimates for the Thermal Width of Heavy Quarkonia in Strongly Coupled Plasmas from Holography

Stefano I. Finazzo (Sao Paulo U.), Jorge Noronha (Sao Paulo U.)

(Submitted on 11 Jun 2013)

Time-depende

The gauge/gravity duality is used to investigate the imaginary part of the heavy quark potential (defined via the rectangular Wilson loop) in strongly coupled plasmas.

³Kavli Institute for the Phy

²The potential between a heavy basis of the gauge-gravity duality.

String theory inspired approaches to heavy-ion collisions

Correspondence between

- strongly-coupled gauge theories — especially at finite temperature — in 1+3 dimensions and
 - classical gravity — possibly with a black hole — in 1+4 dimensions
- allows one to perform computations in the latter framework, and to transpose their results to the former, where standard approaches are powerless.

String theory inspired approaches to heavy-ion collisions

HUGE literature, including some reviews / lectures, as for instance

- Gubser & Karch, Ann. Rev. Nucl. Part. Sci. **59** (2009) 145
- Casalderrey-Solana, Liu, Mateos, Rajagopal & Wiedemann, arXiv:1101.0618
- Adams, Carr, Schaefer, Steinberg & Thomas, New J. Phys. **14** (2012) 115009
- De Wolfe, Gubser, Rosen & Teaney, arXiv:1304.7794
- Heller, Janik & Peschanski, Acta Phys. Pol. **39** (2008) 3183
- Janik, Lect. Notes Phys. **828** (2011) 147

⋮